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PREFACE.

The publication of Volume XIII. has been unavoidably delayed so long mainly with the idea of printing two years' transactions in one volume. It has, however, been thought better to issue each year's transactions separately. Volume XIV. will be ready in a month or two, and in future each year's transactions will be prepared for issue at the following Annual Meeting.

Royal Society of Victoria.

1876.

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Royal Society of Victoria.

ANNIVERSARY ADDRESS

OF

The President,

MR. R. L. J. ELLERY, F.R.A.S., Government Astronomer.

(Delivered to the Members of the Royal Society, at their Annual *Conversazione*, held on Thursday, 10th August, 1876.)

YOUR EXCELLENCY AND GENTLEMEN OF THE
ROYAL SOCIETY,

The 12th Rule of our Society, relating to the time at which the Presidential Address shall be delivered, has of late years been more honoured in the breach than in the observance; every year it has got a little later—this year later than ever, and the usual phrase—"We meet to inaugurate our session," has become inappropriate. I must confess, however, that this bad habit has come into fashion since I have had the honour of being your President; the remedy, therefore, is obvious. We meet this evening to commemorate the entry of the Society into its 19th session by a social gathering of our members and their friends, as has been our custom for several years past, and the only really formal business of the evening provided for by our rules—the delivery of address—now devolves on me as your President.

In doing this I wish to be as brief as possible. Since I had last the honour of addressing you, about two years ago, you have done me the honour of twice re-electing me your President; and now, perhaps, is a fitting occasion to assure you how highly I appreciate the confidence you thus place in me. I have sometimes felt I should like to be relieved of the responsibility and anxiety of the position, and make room for a better man; but as each year has come around I have found myself nominated and re-elected without protesting against the honours you heap upon my head. I need scarcely tell you, gentlemen, that I take the greatest interest in the welfare of this Society, and I shall always be ready, as long as I have good health, to do my best for its good and advancement, whatever position I may hold in its ranks.

You will be glad to learn that the financial position of our Society is now better that it has been for some years. Our revenue proper is not much larger than heretofore, but the resumption of the small annual grant from the Government has enabled your Council to carry on the printing and other work of the Society in a satisfactory manner without getting into debt. We have now on our rolls 122 members, and I am glad to see among our junior members gentlemen who have been educated in the colony, who, from their acquirements and scientific training, I have reason to hope will become most useful acquisitions to the Society.

It has been usual for the President to refer in his address to the papers and other matters which have occupied our meetings held since the preceding *conversazione;* but, as the Transactions are now published and issued soon after each meeting, I think it will be unnecessary to refer to them on this occasion; suffice it to say that there have been six meetings held since our annual gathering last year, at which ten papers and other communications were contributed, which, in most cases, led to interesting and instructive discussions. While on this subject I may mention that I found during my late holiday in Europe that Scientific Societies there are subject to the same phenomenon as we, unfortunately, sometimes witness—namely, paucity of attendance at some of the ordinary meetings. There, as here, unless the business of the meetings is unusually interesting and sensational, a few only of the more earnest members attend; and I have been present at several meetings of some of the highest and oldest societies in London where the attendance has been no better than it is in this hall. Small attendances must not, however, be taken as any sign of the want of vitality, for the real functions of this and similar societies are but exhibited in the encouragement and inducement they afford to investigation and experiment, and in the resulting permanent knowledge embodied in their transactions. The small attendance at some of our ordinary meetings, when the business has been of less immediate interest, has induced the Council to arrange that some of them should be of a less formal and more of a conversational character, at which exhibits of new apparatus, intelligence of scientific or other progress, accounts of experiments or observations, not necessarily original, had been received and discussed; and this plan, so far as has been tried, has been found satisfactory.

I believe the functions of this Society might possibly be extended with advantage in the direction of brief special lectures for the demonstration of new or interesting facts in physical or other science. Such a course has already been thought of, and I believe is well worthy of putting into practice.

The books in the library have now been thoroughly arranged and classified, and the binding of the periodicals has been commenced, and will be proceeded with from time to time. As regards our publications, I may state that Volume XII. has been published and issued, and that all the earlier papers contributed during our present session are printed and distributed, and the rest in the printer's hands.

The building and grounds of the Society are in a much better condition than has been the case for some years past. The repairs to the fencing, and the growth of the trees, with the periodical attention given to the ground generally, have much improved the aspect of affairs. The interior of the building is in a good state of repair, but the appearance of the exterior is exceedingly unsightly. The necessity of getting it stuccoed has been constantly under the notice of the Council, but hitherto the state of the finances has not been such as to warrant it in making the necessary expenditure, more especially as they had the assurance of the architect that the building would not suffer for want of stuccoing for some time to come. The Council are of opinion, however, that if for no other reason than appearance sake, it is highly desirable to get this work done as soon as the funds will admit.

Leaving the more domestic affairs of the Society, I wish now to call your attention to some of the noteworthy facts connected with the past year's history of scientific progress.

In Astronomy there appears little of more than passing interest to arrest our attention; it almost seems as if a lull had fallen on this department of science after the unusual activity caused by the transit of Venus in December, 1874. This is apparent only, for nearly all the national observatories have been busily engaged, each in its own particular direction. This is true also as regards our own observatory, for while I have nothing sensational to refer to, our principal work—determination of the positions of stars, and the revision of Sir John Herschel's nebulæ with the great telescope—has gone on without intermission. Our great telescope has new rivals vying with it in probing the great depths of the universe. At the Paris observatory a large Newtonian reflector (almost of exactly similar dimensions to our Cassegrainian) has been lately completed, and is now at work; at Washington the great refractor of 26 inches aperture and 31 feet focal length is actively employed, and in some trials on nebular observation has proved itself no insignificant rival to the large apertures of our and the other three large reflectors; and further, the maker of the Melbourne telescope is now engaged in the construction of another enormous refractor for the Vienna Observatory, which is to be 27 inches aperture and about 33 feet focal length. Now that it is likely there will be more busy eyes and large telescopes occupied on the fainter celestial objects, to the observation of which our reflector has been principally devoted, it becomes all the more necessary that what has already been accomplished here should become known. At present very little of the results of the work of the great telescope has been published. I am now, however, in hopes that this will soon be done, as a method of doing it has been decided upon, and the only cause of delay now is the want of means. This, I have no reasonable doubt, will shortly be forthcoming, when a good account will be given of how this magnificent instrument has been employed since its erection. The final results of the observations of the transit of Venus have not yet been obtained; the laborious calculations involved will probably delay it for some time longer. It is believed, however, from approximate results already arrived at, that the sun's distance, from these observations, will be found to be somewhere between the distance obtained by the transit of Venus in 1769 (corrected by Stone), and the distance obtained by the parallax of Mars in 1862; that is, somewhere between 91,580,000 and 91,240,000 miles. The number of the planetoids (the small planets which occupy the gap between the orbits of Mars and Jupiter) already discovered is 161. Most of these bodies are so minute that their detection among the myriads of small stars is a matter of considerable difficulty, even to accomplished observers; but, nevertheless, a systematic search for new members of this group with telescopes of adequate power, appears to be always rewarded by discovery. The "Lunar tables," as they are called, are a series of numbers representing the position, distance, &c., of the moon from day to day or hour to hour calculated for some years in advance, and are of the utmost importance in practical astronomy, navigation, and determination of geographical position generally. It is, however, a remarkable fact that all tables hitherto computed become erroneous after the lapse of years, so that the places given no longer represent the moon's actual position, and this would seem at first sight all the more remarkable because her position is and has been continually observed by nearly all the principal national observatories. But the complexity of influences to which she is subjected in her motion through space, coupled with the fact that her mass is probably physically unsymmetrical, makes it an extremely difficult problem to form a theory, taking all these disturbing influences into account, so that tables founded on it shall give the moon's precise position at very distant dates. The tables in the *American Nautical Almanac* of Professor Pierce seem however, to be the best yet computed. The veteran Astronomer Royal of England, Sir George Airy, who is now in his seventy-sixth year, has lately undertaken to work out a new lunar theory to replace those which experience has shown to be insufficient. He reports that his task is well advanced towards completion, and I am sure all scientific men at least will wish him health and vigour to complete this great self-imposed task for the good of the whole civilised world.

In Physical Science also there is nothing of more than ordinary interest to refer to. Mr. Crookes' investigations on the action of light and heat on bodies *in vacuo* have been interesting in the highest degree, and although the supposition that the remarkable phenomena exhibited indicated the existence of a new force, which was at first entertained by some, has not been sustained by further investigation and experiment, his researches in this direction have, at least, opened up a new and interesting, if not useful field, in physical science. Concerning this, Mr. Foord will probably have a few words to say in the course of the evening, more especially in reference to a very interesting little apparatus known as Crookes' radiometer. Some little sensation has been excited lately by the supposed discovery of a new force, allied to electricity, and called etheric force. Some peculiar phenomena, observed with respect to induced electric currents, have been the origin of this supposition. There can be no doubt, however, that they are simply induction phenomena, perhaps not hitherto thoroughly investigated, although certainly known, but which with the present tendency to discover *new forces* have been precipitately put in that category.

Although the science of Chemistry advances steadily from year to year, it is not quite always that discoveries of popular interest are included among its newer acquisitions; the newly-discovered metal "gallium" is, however, sufficiently remarkable to demand a brief notice on this occasion. Formerly, the processes of humid analysis, including electrolysis, were the only means available for the discovery of new elementary substances; of late years the much more delicate and searching method of spectrum analysis has enabled us to discover—first, *rubidium* and *cæsium*, then *thallium*, afterwards *indium*, and now by its means *gallium* has been recognised, and has since been separated. All these are elements; they are all metals, each possessing definite chemical and other properties. Gallium was discovered in August, 1875, by M. Lecoq Boisbanbrau while examining with the spectroscope a *blende* (a sulphide of zinc) from a mine in the Pyrenees. He observed new and hitherto unrecognised lines in the spectrum, which have enabled him to pursue, and eventually to separate, and obtain specimens of, the new metal. The chemical and physical properties of this new substance are in some measure ascertained now that the metal has become tangible; but the delicacy of the means by which this has been brought about may be estimated from the statement that the earliest experiment in which the nature of the spectrum of this new metal was established was made on a quantity something less than the 1·5,000th part of a grain, dissolved in a very small drop of liquid. The melting point of pure gallium is stated to be so low as to warrant our regarding it as being with mercury, in the category of metals, fluid at ordinary atmospheric temperatures; nor are its already ascertained chemical relations less interesting. It has been shown that elementary bodies may be arranged, according to their combining equivalents, into groups of three, or "triads," in which the combining equivalent of the middle element is the numerical mean of the two others, but in more than one of these groups the middle term is wanting. From what has been ascertained concerning gallium, it appears highly probable that it will be found to fill one of these gaps—that, namely, between aluminium and indium; and it has been moreover suggested that a wanting element with a combining equivalent, the mean of these of silicon and tin, should be sought in the field of natural combinations respectively of arsenic and titanium. These foreshadowings of the existence of elements new to science of definite characters and positions in the great chemical scheme suggest a comparison with discoveries in another domain of human knowledge—with those, namely, which predicted and led to the discovery of the planet Neptune.

Some very interesting discussions on the efficacy of the intravenous injection of ammonia in cases of bites by Australian snakes have recently taken place at the Medical Society of Victoria, and perhaps there is no other subject that has cropped up in medical and surgical science during the past year which will have more interest for Australians than this. It cannot be said that the result of these discussions, or of the experiments which led to them, is altogether satisfactory, although there can be no doubt that in the evidence adduced, and the exchange of opinion, the knowledge of the whole question has been considerably advanced. When Professor Halford proved that a powerful agent like ammonia could, under certain conditions, be passed directly into the circulating blood, and so carried mechanically to the heart, and probably the nervous centres, without much danger, and that its effect in his hands appeared to be that animals apparently dying, from snake bites especially, were rapidly re-vitalised as it were, it naturally occurred to him as an appropriate remedy to try on the snake-poisoned human subject. This, as you know, was done, and the patient recovered; many other cases of a more or less similar nature occurred, where recovery from what at the time seemed a hopeless condition was apparently brought about by the injection of ammonia; and the opinion of a large number of intelligent medical men was in favour of the adoption of this treatment for such cases. Other equally intelligent medical men had doubts of the efficacy of this remedy, and eventually a committee of the Medical Society was appointed to carry out a series of experiments to test the value of ammonia injection in snake poison. Their report was so utterly adverse to the ordinarily received opinion, that a very animated and interesting discussion took place at several meetings of the Society, but the balance of opinion was still in favour of ammonia injection as a remedy under certain conditions, and I have no doubt it will still be resorted to in nearly every case of snake bite where the life seems in imminent danger. The question naturally suggests itself in every case that survives after the treatment by ammonia, "Would death have occurred without it?" This, of course, cannot be proved; but the same may be said of all remedies used in medicine or surgery. There can be no doubt, from what transpired at these discussions, that in many cases treated with ammonia the patient was poisoned with alcohol; but who shall say whether the snake or alcohol poison was killing? and if ammonia will save from both, so much the better. However important the intravenous injection of ammonia may be considered in the treatment of snake poison, I think its value as a therapeutic agent in other cases of endangered human life, as shown by some of the collateral evidence given in the discussions referred to, gives broader significance to the whole question than was apparently involved in the late experiments and controversy; and it is to be hoped that both Professor Halford and other of our medical men will extend their investigations and experiments, not only with the view to obtain a more precise idea of the *modus operandi* of this and other agents introduced directly into the circulation, but also with the view of thoroughly testing the value of this method of applying remedies in urgent cases. The comparatively modern method of endermic injection has become an inestimable blessing to suffering humanity, and enables the physician and surgeon to confidently use remedies which, administered in the ordinary way to enter the system by digestion, often only afforded relief at the expense of after-exhaustion of vital powers. If, therefore, further investigation should prove that the intravenous injection of remedies can be as safely and as advantageously used in some cases as the hypodermic injection is in others, it will constitute one of the most important steps in medical science achieved in modern times.

In connexion with this subject there is a matter which is exciting some considerable attention in England just now—I refer to the movement against vivisection. It is, of course, well known that experiments on living animals are frequently made by physiologists and others with the view of extending our knowledge of the vital functions of anatomy, and the action of chemical and other substances, in all cases ostensibly for the benefit of the human race. Of late years, however, a popular belief has grown up among a certain class in England that vivisection and torture of animals was practised to a very large extent in that country without adequate reason, and by persons not influenced by the highest motives, and very strenuous efforts were made to put a stop to such practices. The general public, however, are now convinced that this belief was erroneous in a great measure, and the statements as to the prevalence of the practice exaggerated; for while well-known and eminent physiologists did resort to vivisection in prosecuting their investigations, it was nearly always with that regard for the suffering or life of God's creatures which must necessarily influence all truly scientific men. The amount of vivisection practised was very small, and cases of wanton cruelty or needless experiment were found to be exceedingly few. While repudiating any sympathy with that indiscriminate sentimentality which characterised the more violent part of this movement, I am of opinion that some legislation on the matter is highly desirable to protect the earnest investigator on the one hand from the undue interference of sentimental busybodies, and to prevent an unnecessary resort to vivisection or experiment on animals, or carelessness or cruelty in the practice of it when necessary on the other. There has been a Royal Commission, which has inquired into the subject, and Lord Carnarvon has introduced a bill into the British Parliament, which, I think, will be hailed by all right thinking men as a just and righteous provision. The provisions of the bill are categorically given in *Nature*, and are as follow :—"1. Experiments must be performed with a view only to the advancement, by new discovery, of knowledge which will be useful for saving or prolonging human life, or alleviating human suffering ; 2, In a registered place; 3, By a person holding a licence from one of Her Majesty's principal Secretaries of State ; 4, The animal must, during the whole experiment, be under the complete influence of some anæsthetic, not urari ; and 5, Must be killed before it recovers from the influence of the anæsthetic ; 6, The experiment shall not be performed for demonstrational purposes ; 7, Nor for the purpose of attaining manual skill."

In former addresses I have on several occasions alluded to the subject of Meteorology somewhat at length, and have, I trust, kept you *au courant* with the most important points in connexion with the advancement of this branch of knowledge. To us in Australia the value of a better knowledge of the laws that govern the weather can scarcely be overrated, as our prosperity depends so largely on the amount and period of rainfall. Not that it is possible, by any amount of knowledge, to largely modify our climate ; it may become, nevertheless, possible, by systematic investigation, to foresee the approach of great disturbances of the atmosphere, or even critical seasons, and to be forewarned is to be forearmed. I do not think we have data extended over sufficient period or area in Australia to enable any one to safely make any deductions yet. I believe, however, that with the data we already possess, aided by a system of observations over as much of the coast-line as possible, combined with others at representative localities in the interior, and especially in those parts under the influence of the monsoons, we should be able to ascertain some of the more general laws which govern the weather in Australia, and which will go a long way to help towards the chief desideratum—obtaining a forewarning of storms, and even critical periods and seasons. To this end I have lately invited the co-operation of the directors of Australian observatories in establishing a uniform system of inter-colonial weather telegraphy, which I hope will be in full operation before our next *conversazione.* In America a most complete system has been in operation for some years, which I described to you on a former occasion. This system has been most successful, and it is stated that 80 per cent. of the predictions—which are published nearly every day—for the several districts over which the observations extend, turn out to be correct. These predictions, however, only refer to the weather from day to day, and not to any lengthened period; but even with this limitation it becomes of immense practical value, and no doubt commensurate with the very large national expenditure which is devoted to it.

A movement has lately been made in England which promises to be of the utmost importance not only simply as regards science, but also in an educational aspect. I refer to the loan collection of scientific apparatus which has been collected at the museums at Kensington, the public exhibition of which was privately opened by the Queen on May 13th. The proposition for this collection originated in England, where it was made to the Lords of the Committee of the Council on Education, was approved, and assumed a definite shape through the efforts of a committee including over 130 names of the most distinguished men of science. Although the display is in London, the movement is essentially international. Belgium, France, Germany, Italy, the Netherlands, Norway, Russia, Austria and Hungary, Spain, and United States, have undertaken to contribute, and have opened their museums and scientific storehouses in order that the collection shall be as complete as possible. Whatever intellectual pursuit is aided by instrumental means will be duly represented in this collection; and there will be brought together not only the instruments of research used at the present time, but many invaluable specimens of the tools with which the early pioneers of human knowledge first began to question Nature. The Astrolabe of Tycho Brahe, the telescope of Galileo, will be seen together with the magnificent astronomical instruments of the present day, prominent among which are models of the great Melbourne reflector and the gigantic Vienna refractor of 27 inches aperture. The various sections are so arranged that in many cases the history of the progress in the respective sciences is more plainly shown than could be done by a written book; while throughout can be contrasted specimens of the earliest apparatus used in any branch of science with the refined appliances of the present day—Newton's simple optical apparatus with the exquisite prisms and spectroscopes of to-day; Dalton's crude balance with the magnificent weighing-machines of the present time, with the unimpeachable weights of pure quartz. It would occupy too much time to speak of this subject with any justice to its importance. The value, however, of this movement cannot be over-estimated, although—as science as yet unfortunately only interests the few—it may not be so universally appreciated as we could hope. The *Times*, in an article on the opening of this exhibition, says:—"The exhibition which Her Majesty the Queen privately visits and opens to-day is one of which not only England, but Europe, may be justly proud. Pride, however, is not the only sentiment we English should feel; for at last, if even only for a brief space, we have, under the name of a loan collection of scientific apparatus, a Science

Museum as complete as those in which we have already enshrined our art and literature. For at least six months therefore we shall not only be as rich in this respect as France, Germany, Italy, Holland, and Switzerland, but far richer, since those nations, with an enthusiasm and goodwill which command our universal gratitude, have spoiled their ancient treasure-houses, their laboratories, and private collections, in order that science may be worthily represented among us now that our Government has consented to provide a home, however temporary, for her."

In conclusion, I would return for a few moments to the immediate affairs of our Society.

I have already referred to the smallness of attendance at some of our ordinary meetings, and to certain propositions for the improvement of the working of the Society. I would, however, exhort our scientific and literary members, and more especially our younger ones, to renewed activity. It cannot be supposed in a small community like ours that enough scientific workers in original investigations can be found to keep this Society in active operation with entirely new matter; but if our legitimate functions be fully exercised I can see no reason why we should not have busy sessions and full meetings. The fields of investigation are only too numerous; the further we advance in knowledge the wider they become; the more science contributes to the welfare, convenience, or luxury of the community, the more is demanded of it. So our young scientists have no lack of scope for their inquiries.

It should be clearly understood that accounts and results of experiments, the discovery or improvement of mechanical appliances, suggestions of new modes of investigation or observation, simple observations in natural history, astronomy, chemistry, physiology, medicine, or surgery, besides matters pertaining to the advancement of literature and art, all come within the proper province of this Society.

There is surely, then, enough to do. I have often found that most interesting and valuable information has been withheld because of a fear that it was of too trivial a nature, not original, or not sufficiently scientific. It is easier to make mistakes in this direction than in the opposite, for as a rule the Council will always exercise its discretion for the exclusion of contributions manifestly unworthy the attention of the members. If we each do our best for the advancement of knowledge we shall all do something, and I am sure the result will redound to the credit of this Society, as well as of the country we now belong to.

ART. I.—*On Practical Geodesy.*

BY MARTIN GARDINER, C.E.

[Read 11th May, 1876.]

THE method of investigation employed in this paper is of a purely elementary character, and in this respect it differs from that usually adopted by the most distinguished geometers who have written on the subject. The method introduced by Legendre, Delambre, and Puissant, and which has been followed by Airy and others, is characterised chiefly by the subsidiary use of the higher calculus and interminable series.

The elementary method here pursued leads to simpler and more comprehensive formulæ, and at the same time affords a clearer insight into the various relations between latitudes, azimuths, differences of longitude, length and circular measure of geodesic arc, angles of depression of the chord, &c. Its power of improving and extending the science in one of its most useful directions can be judged of from the numerous new results arrived at, and a comparison between them and those hitherto evolved by means of the higher calculus.

The errors which have been shewn to exist in some of the investigations and formulæ given in the "account" of the principal triangulation of Great Britain and Ireland, will no doubt attract the attention of Engineers and Surveyors engaged on trigonometrical surveys in India and elsewhere.

Let P_o be the pole of reference of the spheroidal earth;
„ C_o be the centre of the earth;
„ S_o, S_{oo}, be any two stations on the earth's surface;
„ Z_o, Z_{oo}, be the points in which the normals at the respective stations S_o, S_{oo}, cut the earth's polar axis.

The planes $S_oZ_oS_{oo}$, $S_{oo}Z_{oo}S_o$, are "the normal-chordal planes." And any plane whatever which contains the chord of the geodesic arc S_oS_{oo} shall be referred to as a chordal plane.

The polar and equatorial radii of the earth being 20855233, and 20926348 feet, it is easy to show that for arcs on its surface not more than 528000 feet or 100 miles in length, we may consider the traces of the two normal-chordal planes as equals in length and circular measure to that of the "true geodesic" or shortest arc between the stations.

Conceive two unit spheres described, having S_o, S_{oo}, as centres. Let $C_{\prime}$, $S_{\prime}$, I, P, be the points in which the sphere S_o is pierced by the productions of the lines C_oS_o, Z_oS_o, $S_{oo}S_o$, through the centre S_o, and by the line S_oP parallel to and in the same direction as the polar axis C_oP_o.

Let $C_{\prime\prime}$, $S_{\prime\prime}$, $I_{\prime\prime}$, $P_{\prime\prime}$, be the points in which the sphere S_{oo} is pierced by the productions of the lines C_oS_{oo}, $Z_{oo}S_{oo}$, by the chord S_oS_{oo} taken in the direction $S_{oo}S_o$, and by the line $S_{oo}P_{\prime\prime}$ parallel to and in the same direction as the polar radius C_oP_o.

Then evidently the points P, C, $S_{\prime}$, are in the trace, on the unit sphere S_o, of the earth's meridian plane through S_o; and $P_{\prime\prime}$, $C_{\prime\prime}$, $S_{\prime\prime}$, are in the trace, on the unit sphere S_{oo}, of the earth's meridian plane through the station S_{oo}.

The arc $P_{\prime\prime}I_{\prime\prime}$ is equal to the arc PI, each of them being the measure of the angle which the chord joining the stations makes with the earth's polar axis.

The angle $P_{\prime\prime}S_{\prime\prime}I_{\prime\prime}$ is the azimuth of the station S_o as observed at the station S_{oo}; and the angle $PS_{\prime}I$ is the supplement of the azimuth of the station S_{oo} as observed at the station S_o. The arcs $PS_{\prime}$, $P_{\prime\prime}S_{\prime\prime}$, are the geographic colatitudes of the stations S_oS_{oo},—such as can be measured directly by means of the Zenith Sector.

The arcs $PC_{\prime}$, $PC_{\prime\prime}$, are the geocentric colatitudes of the stations.

Now conceive the unit sphere S_{oo} moved by direct translation along the chord, carrying its lines and points rigidly fixed, until its centre coincides with the centre S_o of the unit sphere S_o. It is evident that the points $I_{\prime\prime}$, $P_{\prime\prime}$, will coincide with I, P, and that the points I, $C_{\prime}$, $C_{\prime\prime}$ lie in one great circle of the sphere S_o. It is also evident that the points $P_{\prime}$, $S_{\prime\prime}$, $C_{\prime\prime}$ lie in one great circle of the unit sphere S_o, and that the spherical angle $S_{\prime}PS_{\prime\prime}$ or $C_{\prime}PC_{\prime\prime}$ is equivalent to the difference of longitude of the stations S_oS_{oo}.

Let $p_{\prime}$, $p_{\prime\prime}$, be the points in which the lines PS_o, $P_{\prime\prime}S_{oo}$, parallel to the polar axis, pierce the earth's equator. Then it is evident that the plane angle $p_{\prime}C_{\circ}p_{\prime\prime}$ is equivalent to the difference of longitude of the stations.

It is also evident that the plane angles $C_{\circ}p_{\prime}p_{\prime\prime}$, $C_{\circ}p_{\prime\prime}p_{\prime}$ are equals respectively to the spherical angle $S_{\prime}PI$, and the supplement of the spherical angle $S_{\prime\prime}PI$.

Let $D_{\prime}$, $D_{\prime\prime}$ be the points in which the great circles $IS_{\prime\prime}$, $IS_{\prime}$ cut the great circles $PS_{\prime}C_{\prime}$, $PS_{\prime\prime}C_{\prime\prime}$ respectively. It is evident the arc $S_{\prime}S_{\prime\prime}$ is the measure of the angle which the normals make with each other.

The arc $S_{\prime}D_{\prime\prime}$ is the measure of the plane angle $S_{\circ}Z_{\circ}S_{\circ\circ}$; the arc $S_{\prime\prime}D_{\prime}$ is the measure of the plane angle $S_{\circ\circ}Z_{\circ\circ}S_{\circ}$; the arcs $S_{\prime}C_{\prime}$, $S_{\prime\prime}C_{\prime\prime}$ are the measures of "the angles of the vertical" at the stations $S_{\circ}S_{\circ\circ}$; the spherical angle $S_{\prime}IS_{\prime\prime}$ is equal to the angle between the two normal-chordal planes.

And if O, $E_{\prime}$, $E_{\prime\prime}$ be the points in which the great circle of the unit sphere having I as pole cuts the arcs $S_{\prime}S_{\prime\prime}$, $S_{\prime}D_{\prime\prime}$, $S_{\prime\prime}D_{\prime}$ respectively; it is evident that the arcs $S_{\prime}E_{\prime}$, $S_{\prime\prime}E_{\prime\prime}$ are the measures of the angles of depression of the geodesic chord $S_{\circ}S_{\circ\circ}$ below the tangent planes to the spheroidal earth at the respective stations $S_{\circ}S_{\circ\circ}$; and they are the complements of the angles which the normals make with the chord.

The spherical angles $S_{\prime\prime}S_{\prime}D_{\prime\prime}$, $S_{\prime}S_{\prime\prime}D_{\prime}$ are equivalents to the angles which any plane parallel to the two normals makes with the two normal-chordal planes.

And the spherical angles $S_{\prime\prime}D_{\prime}D_{\prime\prime}$, $S_{\prime}D_{\prime\prime}D_{\prime}$ are equivalents to the angles which any plane parallel to the two lines $S_{\circ}Z_{\circ\circ}$, $S_{\circ\circ}Z_{\circ}$, makes with the normal-chordal planes.

The interpretation of the other points, lines, angles, and planes of the figure can present no difficulty, and no further elucidation is necessary here; but in order to avoid misconceptions, it should be remembered that all through this paper (when *two* stations only are considered) we will consider the latitude of the station $S_{\circ}$ greater or not less than the latitude of the station $S_{\circ\circ}$,—as indicated in the figure.

NOTATION.

$l_{\prime}$, $l_{\prime\prime}$	denote the	latitudes of the stations $S_{\circ}$, $S_{\circ\circ}$, respectively.
l', l''	„	colatitudes, or the arcs $PS_{\prime}$, $PS_{\prime\prime}$ „
L', L''	„	arcs $PD_{\prime}$, $PD_{\prime\prime}$.
$A_{\prime}$, $A_{\prime\prime}$	„	azimuths or angles $PS_{\prime}D_{\prime\prime}$, $PS_{\prime\prime}D_{\prime}$.
$A_{\circ}$, $A_{\circ\circ}$	„	angles $PS_{\prime}S_{\prime\prime}$, $PS_{\prime\prime}S_{\prime}$ of the triangle $S_{\prime}PS_{\prime\prime}$.
$D_{\prime}$, $D_{\prime\prime}$	„	„ $PD_{\prime}S_{\prime\prime}$, $PD_{\prime\prime}S_{\prime}$.
$z_{\prime}$, $z_{\prime\prime}$	„	arcs $S_{\prime}D_{\prime\prime}$, $S_{\prime\prime}D_{\prime}$.

$\alpha_{,}$, $\alpha_{,,}$ denote the angles of depression of the chord, or arcs $S_{,}E_{,}$, $S_{,,}E_{,,}$.
$\delta_{,}$, $\delta_{,,}$,, the small arcs $S_{,}D_{,}$, $S_{,,}D_{,,}$.
$\Omega_{,}$, $\Omega_{,,}$,, angles $S_{,,}S_{,}D_{,,}$, $S_{,}S_{,,}D_{,}$.
$R_{,}$, $R_{,,}$,, normals $S_{\circ}Z_{\circ}$, $S_{\circ\circ}Z_{\circ\circ}$, terminating in polar axis.
$Q_{,}$, $Q_{,,}$,, lines $S_{\circ}Z_{\circ\circ}$, $S_{\circ\circ}Z_{\circ\circ}$.
$\phi_{,}$, $\phi_{,,}$,, angles $IPS_{,}$ and supplement of $IPS_{,,}$.
s, k ,, lengths of geodesic arc and chord respectively.
ν denotes the arc $S_{,}S_{,,}$, or the angle between the normals.
Σ ,, circular measure of the geodesic arc s.
θ ,, arc PI, or angle between the chord and polar axis.
Δ ,, angle $S_{,}IS_{,,}$ between the normal-chordal planes.
a ,, length of the earth's equatorial radius.
b ,, ,, ,, ,, polar radius.
e ,, earth's eccentricity.

1. Values of geodetic constants, in accordance with the dimensions of the earth as finally adopted by the Ordnance Department of Great Britain and Ireland.

$\mathrm{a} = 20926348$ feet $\qquad \log.\ \mathrm{a} = 7{\cdot}3206934433$

$\mathrm{b} = 20855233$ feet $\qquad \log.\ \mathrm{b} = 7{\cdot}3192150463$

$e = {\cdot}0823719976978 \qquad \log.\ e = \bar{2}{\cdot}9157795987$

$e^2 = {\cdot}0067851460047 \qquad \log.\ e^2 = \bar{3}{\cdot}8315591974$

$(1—e^2) = {\cdot}9932148539953 \qquad \log.\ (1—e^2) = \bar{1}{\cdot}9970432059$

$\left(\frac{1}{1—e^2}\right) = 1{\cdot}0068314987210 \qquad \log.\ \left(\frac{1}{1—e^2}\right) = 0\ 0029567941$

$\left(\frac{e^2}{1—e^2}\right) = {\cdot}0068314987230 \qquad \log.\ \left(\frac{e^2}{1—e^2}\right) = \bar{3}{\cdot}8345159915$

☞ The geodetic tables above referred to give also the logs. to 8 places of decimals of the normals terminating in the polar axis for all latitudes from the equator to the pole. The well-known formula by means of which any of these normals is expressed in terms of the latitude to which it pertains is—

$$R = \frac{\mathrm{a}}{\sqrt{1—e^2 \sin^2 l}}$$

2. The following relations are evident from the figure—

$$C_{\circ}p_{,} = R_{,} \cos l_{,}\,; \qquad C_{\circ}p_{,,} = R_{,,} \cos l_{,,} \tag{1}$$
$$S_{\circ}p_{,} = R_{,} (1—e^2) \sin l_{,}\,; \qquad S_{\circ\circ}p_{,,} = R_{,,} (1—e^2) \sin l_{,,} \tag{2}$$
$$C_{\circ}Z_{\circ} = R_{,}\, e^2 \sin l_{,}\,; \qquad C_{\circ}Z_{\circ\circ} = R_{,,}\, e^2 \sin l_{,,} \tag{3}$$
$$Q^2_{,} = (C_{\circ}p_{,})^2 + (S_{\circ}p_{,} + C_{\circ}Z_{\circ\circ})^2 = R^2_{,} — 2R_{,}\, e^2 \sin^2 l_{,} + F \tag{4}$$
$$Q^2_{,,} = (C_{\circ}p_{,}) + (S_{\circ\circ}p_{,,} + C_{\circ}Z_{\circ})^2 = R^2_{,,} — 2R_{,,}\, e^2 \sin^2 l_{,,} + F \tag{5}$$

in which F is the same function of the latitudes in the equation (4) and (5).

$$S_{\circ}p_{\prime} - S_{\circ\circ}p_{\prime\prime} = (R_{\prime} \sin l_{\prime} - R_{\prime\prime} \sin l_{\prime\prime}) \cdot (1-e^2) \qquad (6)$$

$$C_{\circ}Z_{\circ} - C_{\circ}Z_{\circ\circ} = (R_{\prime} \sin l_{\prime} - R_{\prime\prime} \sin l_{\prime\prime}) \cdot e^2 \qquad (7)$$

$$S_{\circ}p_{\prime} - S_{\circ\circ}p_{\prime\prime} : Z_{\circ}Z_{\circ\circ} :: (1-e^2) : e^2 \qquad (8)$$

3. From the expressions for the magnitudes of $Q_{\prime}$, $Q_{\prime\prime}$, we have

$$R_{\prime}^2 + Q_{\prime}^2 = 2 \cdot R_{\prime}^2 (1 - e^2 \sin^2 l_{\prime}) + F = 2a^2 + F;$$
$$R_{\prime\prime}^2 + Q_{\prime\prime}^2 = 2 \cdot R_{\prime\prime}^2 (1 - e^2 \sin^2 l_{\prime\prime}) + F = 2a^2 + F.$$

And therefore it is obvious that we have the relation—

$$R_{\prime}^2 + Q_{\prime}^2 = R_{\prime\prime}^2 + Q_{\prime\prime}^2 \qquad (9)$$

Hence it follows that if N be the middle point of the segment $Z_{\circ}Z_{\circ\circ}$ of the polar axis intercepted by the normals, we have—

$$NS_{\circ} = NS_{\circ\circ} \qquad (10)$$

And from this it is obvious that the stations $S_{\circ}$, $S_{\circ\circ}$, are in the surface of a sphere whose centre is N, and that we have

$$\begin{gathered} R_{\prime} \; 7 \; Q_{\prime\prime} \\ R_{\prime\prime} \; 7 \; Q_{\prime} \\ \delta_{\prime\prime} \; 7 \; \delta_{\prime} \end{gathered} \qquad (11)$$

(See formulæ 81·A and 81·B in the sequel.)

4. If in each of the triangles $Z_{\circ}Z_{\circ\circ}S_{\circ}$, $Z_{\circ}Z_{\circ\circ}S_{\circ\circ}$, we express the base $Z_{\circ}Z_{\circ\circ}$ in terms of the other two sides and the included angle, it is evident from (9) that—

$$R_{\prime} \cdot Q_{\prime} \cdot \cos \delta_{\prime} = R_{\prime\prime} \cdot Q_{\prime\prime} \cdot \cos \delta_{\prime\prime} \qquad (12)$$

$$\therefore \frac{\cos \delta_{\prime}}{\cos \delta_{\prime\prime}} = \frac{R_{\prime\prime} \cdot Q_{\prime\prime}}{R_{\prime} \cdot Q_{\prime}}$$

$$\therefore R_{\prime\prime} \cdot Q_{\prime\prime} \; 7 \; R_{\prime} \cdot Q_{\prime} \qquad (13)$$

absolutely; but in all ordinary cases they are equals to at least 10 places of decimals in their logarithms.

5. It is evident that the plane through the middle point N, of the segment $Z_{\circ}Z_{\circ\circ}$, perpendicular to the geodesic chord $S_{\circ}S_{\circ\circ}$, must bisect this chord or pass through its middle point M. And therefore, since the portions $NZ_{\circ}$, $NZ_{\circ\circ}$, of $Z_{\circ}Z_{\circ\circ}$, which lie on opposite sides of this plane are equals, it follows that the planes through $Z_{\circ}$, $Z_{\circ\circ}$, perpendicular to the geodesic chord $S_{\circ}S_{\circ\circ}$, cut it in points $T_{\circ}$, $T_{\circ\circ}$, equidistant from its middle point M. Hence—

$$\sin \alpha_{\prime} = \cos T_{\circ}S_{\circ}Z_{\circ} = \frac{S_{\circ}T_{\circ}}{R_{\prime}}$$

$$\sin \alpha_{\prime\prime} = \cos T_{\circ\circ}S_{\circ\circ}Z_{\circ\circ} = \frac{S_{\circ\circ}T_{\circ\circ}}{R_{\prime\prime}}$$

$$\therefore \frac{\sin \alpha_{,}}{\sin \alpha_{,,}} = \frac{R_{,,}}{R_{,}} \tag{14}$$

And since we suppose $l_{,}$ greater than $l_{,,}$, we know that $R_{,}$ is greater than $R_{,,}$; and hence we learn that the angle of depression $\alpha_{,,}$ adjacent to the station having the lesser latitude is greater than the angle of depression $\alpha_{,}$ adjacent to the station having the greater latitude.

6. We have, evidently—

$$\frac{S_{\circ}T_{\circ}}{S_{\circ\circ}T_{\circ}} = \frac{S_{\circ\circ}T_{\circ\circ}}{S_{\circ}T_{\circ\circ}}$$

or, which is the same—

$$\frac{\tan \alpha_{,}}{\tan (z_{,} - \alpha_{,})} = \frac{\tan \alpha_{,,}}{\tan (z_{,,} - \alpha_{,,})} \tag{15}$$

Now it is evident that each side of this equation is greater than unity; and $\therefore$ when $z_{,}$ and $z_{,,}$ are each less than a quadrant, we have—

$$\begin{array}{l} \alpha_{,} \urcorner z_{,} - \alpha_{,} \\ \alpha_{,,} \urcorner z_{,,} - \alpha_{,,} \end{array} \tag{16}$$

7. If the latitudes $l_{,}$, $l_{,,}$ of any two stations (on the same side of the earth's equator) be of constant magnitudes, then, no matter how otherwise the stations may vary in position, it is evident that the points $Z_{\circ}$, $Z_{\circ\circ}$, in which the normals cut the polar axis, remain fixed. It is also evident that as regards the magnitudes of L', L'', $\delta_{,}$, $\delta_{,,}$ they too are constants, and the same as if the stations were on one meridian. Hence it is obvious that when $l_{,}$ is greater than $l_{,,}$, or, which is the same—when l'' is greater than l', we know that the first and third of the following are true—

$$\begin{array}{l} l'' \urcorner L'' \\ L'' \urcorner L' \\ L' \urcorner l' \end{array} \tag{17}$$

The truth of the second of these relations is easily seen. For drawing perpendiculars $S_{\circ}H_{\circ}$, $S_{\circ\circ}H_{\circ\circ}$, from the stations to the polar axis, it is evident we have—

$$\begin{aligned} \tan L'' &= S_{\circ\circ}H_{\circ\circ} \div (Z_{\circ\circ}H_{\circ\circ} + Z_{\circ\circ}Z_{\circ}) \\ \tan L' &= S_{\circ}H_{\circ} \div (Z_{\circ\circ}H_{\circ\circ} + H_{\circ\circ}H_{\circ}); \end{aligned}$$

and therefore since $S_{\circ\circ}H_{\circ\circ} \urcorner S_{\circ}H_{\circ}$, and that $Z_{\circ\circ}Z_{\circ} \angle H_{\circ\circ}H_{\circ}$,

$$\begin{array}{c} \tan L'' \urcorner \tan L' \\ L'' \urcorner L' \end{array}$$

Hence also (since each of the four arcs is less than 90°) we have

$$\begin{gathered}\sin l'' \urcorner \sin L'' \\ \sin L'' \urcorner \sin L' \\ \sin L' \urcorner \sin l'\end{gathered} \tag{18}$$

8. From the spherical triangles $D_{\prime}PS_{\prime\prime}$, $D_{\prime\prime}PS_{\prime}$, we have—

$$\begin{gathered}\sin L' \sin D_{\prime} = \sin l'' \sin A_{\prime\prime} \\ \sin L'' \sin D_{\prime\prime} = \sin l' \sin A_{\prime}\end{gathered}$$

$\therefore$

$$\begin{gathered}\sin D_{\prime} \urcorner \sin A_{\prime\prime} \\ \sin A_{\prime} \urcorner \sin D_{\prime\prime}\end{gathered} \tag{19}$$

And since each of the angles $(D_{\prime} + A_{\prime\prime})$, $(A_{\prime} + D_{\prime\prime})$, is less than 180°, it follows that—

$$\begin{gathered}D_{\prime} \urcorner A_{\prime\prime}, \text{ and that } A_{\prime\prime} \text{ is acute} \\ A_{\prime} \urcorner D_{\prime\prime}, \text{ and that } D_{\prime\prime} \text{ is acute}\end{gathered} \tag{20}$$

9. We shall now establish the following important relations between the azimuths and angles $D_{\prime}$, $D_{\prime\prime}$—

$$\begin{gathered}D_{\prime} \urcorner A_{\prime} \\ A_{\prime} \urcorner A_{\prime\prime} \\ A_{\prime\prime} \urcorner D_{\prime\prime}\end{gathered} \tag{21}$$

First, from the triangles $S_{\prime}PD_{\prime\prime}$, $S_{\prime\prime}PD_{\prime}$, we have—

$$\begin{gathered}\sin z_{\prime} \sin A_{\prime} = \sin L'' \sin \omega \\ \sin z_{\prime\prime} \sin A_{\prime\prime} = \sin L' \sin \omega\end{gathered}$$

But from (14), (15), and (16), it is evident that—

$$z_{\prime\prime} \urcorner z_{\prime} \tag{22}$$

And therefore, since sin L″ is greater than sin L′ we have—

$$\sin z_{\prime} \sin A_{\prime} \urcorner \sin z_{\prime\prime} \sin A_{\prime\prime}$$

$\therefore$

$$\frac{\sin A_{\prime}}{\sin A_{\prime\prime}} \urcorner 1.$$

Now, since $A_{\prime} + A_{\prime\prime}$ is less than 180°, and that angle $A_{\prime\prime}$ is acute (see 20), therefore it follows that—

$$A_{\prime} \urcorner A_{\prime\prime}$$

In order to shew that the first and third of the relations (21) are true, we may proceed thus—

Applying formula 4, page 158, of Serret's Trigonometry to the spherical triangle $S_{\prime}IS_{\prime\prime}$, and putting ϵ to represent the spherical excess of this triangle, we have—

$$\tan \tfrac{1}{2} (\triangle - \epsilon) = \frac{\sin \frac{1}{2} (a_{\prime} - a_{\prime\prime})}{\cos \frac{1}{2} (a_{\prime} + a_{\prime\prime})} \cdot \tan \tfrac{1}{2} \triangle \tag{23}$$

And, since $a_{\prime}-a_{\prime\prime}$ is negative, it follows $\triangle$ is less than ϵ; Hence also—

$$\text{angle } IS_{\prime}S_{\prime\prime} + \text{angle } IS_{\prime\prime}S_{\prime} \urcorner 180^\circ$$

$$\text{angle } S_{\prime}S_{\prime\prime}D_{\prime} \urcorner \text{angle } S_{\prime\prime}S_{\prime}D_{\prime\prime}$$

$$\text{or,} \quad \Omega_{\prime\prime} \urcorner \Omega_{\prime} \tag{24}$$

We have also—

$$A_{\prime} + A_{\prime\prime} = PS_{\prime}S_{\prime\prime} + PS_{\prime\prime}S_{\prime} + (\epsilon - \triangle)$$

$$\& \therefore \quad A_{\prime} + A_{\prime\prime} \urcorner A_{\circ} + A_{\circ\circ} \tag{25}$$

Now the triangle $S_{\prime}ID_{\prime}$ is evidently such that—

$$\text{angle } IS_{\prime}D_{\prime} + \text{angle } ID_{\prime}S_{\prime} \angle 180^\circ$$

$$\text{but,} \quad \text{angle } PD_{\prime}S_{\prime\prime} + \text{angle } ID_{\prime}S_{\prime} = 180^\circ$$

$$\therefore, \quad \text{angle } PD_{\prime}S_{\prime\prime} \urcorner \text{angle } IS_{\prime}D_{\prime}$$

$$\text{or,} \quad D_{\prime} \urcorner A_{\prime}$$

And the triangle $S_{\prime\prime}ID_{\prime\prime}$ is evidently such that—

$$\text{angle } IS_{\prime\prime}D_{\prime\prime} + \text{angle } ID_{\prime}S_{\prime\prime} \urcorner 180$$

$$\text{but,} \quad \text{angle } PD_{\prime\prime}S_{\prime} + \text{angle } ID_{\prime\prime}S_{\prime\prime} = 180$$

$$\therefore, \quad \text{angle } IS_{\prime\prime}D_{\prime\prime} \urcorner \text{angle } PD_{\prime\prime}S_{\prime}$$

$$\text{or,} \quad A_{\prime\prime} \urcorner D_{\prime\prime}$$

10. From equation (14) or, $\frac{\sin a_{\prime}}{\sin a_{\prime\prime}} = \frac{R_{\prime\prime}}{R_{\prime}}$, we have—

$$\frac{\sin a_{\prime\prime} - \sin a_{\prime}}{\sin a_{\prime\prime} + \sin a_{\prime}} = \frac{R_{\prime} - R_{\prime\prime}}{R_{\prime} + R_{\prime\prime}}$$

$$\frac{\tan \frac{1}{2}(a_{\prime\prime} - a_{\prime})}{\tan \frac{1}{2}(a_{\prime\prime} + a_{\prime})} = \frac{R_{\prime} - R_{\prime\prime}}{R_{\prime} + R_{\prime\prime}} \tag{26}$$

$$\tan \tfrac{1}{2}(a_{\prime\prime} - a_{\prime}) = \frac{R_{\prime} - R_{\prime\prime}}{R_{\prime} + R_{\prime\prime}} \tan \tfrac{1}{2} \Sigma \tag{27}$$

From this equation it is evident that when the latitudes are of constant magnitudes, then the greater the circular measure Σ of the intervening geodesic arc is, the greater will be the difference of the angles of depression of the chord. But although $a_{\prime\prime}-a_{\prime}$ increases or decreases according as Σ increases or decreases, it is nevertheless evident, from (14), that both $a_{\prime\prime}$ and $a_{\prime}$ increase or decrease as $a_{\prime\prime} + a_{\prime}$ or Σ increases or decreases.

Moreover, it is evident that when the latitudes are constants—

$$\frac{\cos a_{\prime}}{\cos a_{\prime\prime}} \text{ increases as } \Sigma \text{ increases} \tag{28}$$

$$\frac{\tan a_{\prime}}{\tan a_{\prime\prime}} \text{ decreases as } \Sigma \text{ increases} \tag{29}$$

However, it is proper to observe that even for a geodesic arc on the earth's spheroidal surface whose circular measure is as great as $1^{\circ}_{\prime\prime}\ 30'$, and the latitudes of whose extremities differ by as much as 1°, we may, with due respect to the utmost attainable precision in geodetic surveying in Victoria, assume—

$$\frac{\cos \alpha_{\prime}}{\cos \alpha_{\prime\prime}} = 1 \tag{30}$$

For by means of (27) it can be easily shown that even in this extreme case $\alpha_{\prime\prime} - \alpha_{\prime}$ is less than a sixth part of a second, and that the logarithms of $\cos \alpha_{\prime}$ and $\cos \alpha_{\prime\prime}$ will be the same to 8 places of decimals, and differ in the ninth place by less than 4. Hence also, in the actual practice of trigonometrical surveying, we may, for some purposes, assume—

$$\frac{\alpha_{\prime\prime}}{\alpha_{\prime}} = \frac{\tan \alpha_{\prime\prime}}{\tan \alpha_{\prime}} = \frac{\sin \alpha_{\prime\prime}}{\sin \alpha_{\prime}} = \frac{R_{\prime}}{R_{\prime\prime}} \tag{31}$$

$$\alpha_{\prime\prime} - \alpha_{\prime} = \frac{R_{\prime} - R_{\prime\prime}}{R_{\prime} + R_{\prime\prime}} \cdot \Sigma \tag{32}$$

their logs. being the same to at least 8 places of decimals. Formulæ 27 and 32 will be found very useful in the computation of the angles of depression of the chord of the geodesic arc; but, when worked by means of logarithms, the best way is to find, in the first instance, an angle x such that—

$$\tan x = \frac{R_{\prime}}{R_{\prime\prime}} \tag{33}$$

and then equations (27) and (32) can be written in the forms—

$$\tan \tfrac{1}{2} (\alpha_{\prime\prime} - \alpha_{\prime}) = \tan (x - 45^{\circ}) \cdot \tan \tfrac{1}{2} \Sigma \tag{34}$$

$$\alpha_{\prime\prime} - \alpha_{\prime} = \tan (x - 45^{\circ}) \cdot \Sigma'' \tag{35}$$

And since the angle $x - 45^{\circ}$ can never be more than a few seconds in magnitude we have, in lieu of 35—

$$\alpha_{\prime\prime} - \alpha_{\prime} = \Sigma'' \cdot (x - 45^{\circ}) \sin 1'' \tag{36}$$

Moreover, it is evident, that in actual practice, we infer—from (31) and (15)—that—

$$\frac{\alpha_{\prime}}{z_{\prime} - \alpha_{\prime}} = \frac{\alpha_{\prime\prime}}{z_{\prime\prime} - \alpha_{\prime\prime}} \text{ approximately} \tag{37}$$

and ∴

$$\frac{z_{\prime}}{z_{\prime\prime}} = \frac{\alpha_{\prime}}{\alpha_{\prime\prime}} = \frac{\sin \alpha_{\prime}}{\sin \alpha_{\prime\prime}} = \frac{R_{\prime\prime}}{R_{\prime}} \tag{38}$$

shewing that the auxiliary angle x of (33) has its tangent equal to the ratio of the angles of depression of the chord, and also equal to the ratio of the arcs $z_{\prime\prime}$ and $z_{\prime}$.

11. Again, from the triangle $S_{\prime}IS_{\prime\prime}$, we have, rigorously—

$$\frac{\sin\Omega_{\prime}}{\sin\Omega_{\prime\prime}} = \frac{\cos\alpha_{\prime\prime}}{\cos\alpha_{\prime}} \qquad (39)$$

Hence it follows that for any pair of mutually visible stations, such as occur in trigonometrical surveying, we may assume—

$$\left.\begin{aligned}\frac{\sin\Omega_{\prime}}{\sin\Omega_{\prime\prime}} &= 1\,;\\ \frac{\tan\Omega_{\prime}}{\tan\Omega_{\prime\prime}} &= 1\,;\\ \frac{\cos\Omega_{\prime}}{\cos\Omega_{\prime\prime}} &= 1\,;\end{aligned}\right\}\ \text{their logarithms being the same to at least 8 places of decimals.} \qquad (40)$$

(See formulæ (30) and remarks as to its approximate accuracy.)

12. From what has been already shewn or observed, it is evident—

$$\Omega_{\prime\prime} - \Omega_{\prime} = \epsilon - \Delta \qquad (41)$$

and ∴, we have from (23)—

$$\tan\tfrac{1}{2}(\Omega_{\prime\prime} - \Omega_{\prime}) = \frac{\sin\frac{1}{2}(\alpha_{\prime\prime} - \alpha_{\prime})}{\cos\frac{1}{2}\Sigma}\cdot\tan\tfrac{1}{2}\Delta \qquad (42)$$

∴

$$\Omega_{\prime\prime} - \Omega_{\prime} = \frac{\sin\frac{1}{2}(\alpha_{\prime\prime} - \alpha_{\prime})}{\cos\frac{1}{2}\Sigma}\cdot\Delta \qquad (43)$$

and, since $\alpha_{\prime\prime} - \alpha_{\prime}$ is but a fraction of a second, even when Σ is as much as $1^{\circ}_{\prime\prime}\ 30'$; and that Δ can be but a few seconds in all cases that occur; it is easy to prove that, in the actual practice of trigonometrical surveying, the angle $\Omega_{\prime\prime} - \Omega_{\prime}$ will never exceed the $\frac{1}{10000}$ part of a second. And from this and equations (40) it follows that we can regard

$$\Omega_{\prime\prime} = \Omega_{\prime} = \Omega$$

In the account of the trigonometrical survey of Great Britain and Ireland, the magnitude of $\Omega_{\prime\prime} - \Omega_{\prime}$ is shewn to be always less than $\frac{1}{10000}$ part of a second; but it is not shewn that the ratio of the sines or tangents of the angles $\Omega_{\prime\prime}$, $\Omega_{\prime}$, may be regarded as equal to unity for all pairs of mutually visible stations: yet this is necessary, as, in some instances, $\Omega_{\prime\prime}$ and $\Omega_{\prime}$ are extremely small arcs.

13. And if we put $\Xi_{\prime}$ and $\Xi_{\prime}$ to represent the small spherical angles $S_{\prime\prime}D_{\prime}D_{\prime\prime}$, $S_{\prime}D_{\prime\prime}D_{\prime}$, it is evident that, in like manner, we have—

$$\Xi_{\prime\prime} - \Xi_{\prime} = \frac{\sin\frac{1}{2}(D_{\prime}E_{\prime\prime} - D_{\prime\prime}E_{\prime})}{\cos\frac{1}{2}(D_{\prime}E_{\prime\prime} + D_{\prime\prime}E_{\prime})}\cdot\Delta \qquad (44)$$

and it can be easily shewn that the difference of the angles $\Xi_{\prime\prime}$ and $\Xi_{\prime}$ is as extremely small as the difference of the angles $\Omega_{,,}$ and $\Omega_{,}$, and that they too can be regarded as equal to each other. Moreover, the points $D_{,}OD_{,,}$ are on one great circle.

14. Now, since for all pairs of mutually visible stations on the earth's spheroidal surface, we have—

$$A_{,} + A_{,,} = A_{\circ} + A_{\circ\circ}$$

and that we can express the angle ω in terms of the angles $A_{\circ} + A_{\circ\circ}$ and the sides l', l'', of the triangle $S_{,}PS_{,,}$; therefore by substituting, in such expression, $A_{,} + A_{,,}$ for its equivalent, we have—

$$\tan \tfrac{1}{2}\,\omega = \frac{\cos \frac{1}{2}(l'' - l')}{\cos \frac{1}{2}(l'' + l')} \cdot \cot \tfrac{1}{2}(A_{,} + A_{,,})$$

$$\therefore \qquad (45)$$

$$\tan \tfrac{1}{2}\,\omega = \frac{\cos \frac{1}{2}(l_{,} - l_{,,})}{\sin \frac{1}{2}(l_{,} + l_{,,})} \cdot \cot \tfrac{1}{2}(A_{,} + A_{,,})$$

This formulæ is known as *Dalby's Theorem*, for the history of which see the "Account of the Principal Triangulation of Great Britain and Ireland," page 236.

15. By applying Delambre's analogies to the same spherical triangle $S_{,}PS_{,,}$, we find in like manner—

$$\sin \tfrac{1}{2}(A_{,} + A_{,,}) = \frac{\cos \frac{1}{2}\,\omega}{\cos \frac{1}{2}\,\nu} \cdot \cos \tfrac{1}{2}(l'' - l') \qquad (46)$$

$$\cos \tfrac{1}{2}(A_{,} + A_{,,}) = \frac{\sin \frac{1}{2}\,\omega}{\cos \frac{1}{2}\,\nu} \cdot \cos \tfrac{1}{2}(l'' + l') \qquad (47)$$

and $\therefore$

$$\tan \tfrac{1}{2}(A_{,} + A_{,,}) = \frac{\cos \frac{1}{2}(l'' - l')}{\cos \frac{1}{2}(l'' + l')} \cdot \cot \tfrac{1}{2}\,\omega$$

$$(48)$$

$$\cot \tfrac{1}{2}(A_{,} + A_{,,}) = \frac{\cos \frac{1}{2}(l'' + l')}{\cos \frac{1}{2}(l'' - l')} \cdot \tan \tfrac{1}{2}\,\omega$$

☞ From (48) it is evident that when the latitudes of the stations are of constant magnitudes, then the *greater* the difference of longitude ω is, the *less* will the sum of the two azimuths be.

"CONVERGENCE OF MERIDIANS."

The stations being supposed on the same side of the earth's equator, the sum of the azimuths $A_{,} + A_{,,}$ is always less than 180°; and it is customary to call the defect or

$$180^{\circ} - (A_{,} + A_{,,})$$

the "convergence" of the meridians as respects the stations. Putting C to denote this convergence, it is evident from 48 that we have—

$$\tan \tfrac{1}{2}\,C = \frac{\sin \frac{1}{2}(l_{,} + l_{,,})}{\cos \frac{1}{2}(l_{,} - l_{,,})} \cdot \tan \tfrac{1}{2}\,\omega$$

And should the latitudes of the stations be equal, then putting l for the common value, we have the rigorous formula

$$\tan \tfrac{1}{2} C = \sin l \cdot \tan \tfrac{1}{2} \omega$$

or, since the tangents of small angles are proportional to the numbers of seconds in the angles, we have, approximately—

$$C'' = \sin l \cdot \omega''$$

in which C'' and ω'' represent the seconds in the "convergence" of meridians, and in the difference of the longitude of the stations.

16. And applying Todhunter's formula pertaining to spherical excess (see page 72, formula 3, of his trigonometry) to the same spherical triangle, we at once obtain the useful relations—

$$\left.\begin{aligned} \cot \tfrac{1}{2} l' \cdot \cot \tfrac{1}{2} l'' &= -\frac{\cos \frac{1}{2}(A_{,} + A_{,,} - \omega)}{\cos \frac{1}{2}(A_{,} + A_{,,} + \omega)} \\ \tan \tfrac{1}{2} l' \cdot \tan \tfrac{1}{2} l'' &= -\frac{\cos \frac{1}{2}(A_{,} + A_{,,} + \omega)}{\cos \frac{1}{2}(A_{,} + A_{,,} - \omega)} \end{aligned}\right\} \tag{49}$$

It is evident that instead of $\frac{1}{2} l'$ and $\frac{1}{2} l''$, we may write $(45° - \frac{1}{2} l_{,})$ and $(45° - \frac{1}{2} l_{,,})$ in formulæ (49).

17. From the spherical triangles $S_{,}PI$, $S_{,,}PI$, we have—

$$\sin \phi_{,} = \frac{\sin A_{,} \cos \alpha_{,}}{\sin \theta}; \qquad \sin \phi_{,,} = \frac{\sin A_{,,} \cos \alpha_{,,}}{\sin \theta}$$

$\therefore$

$$\frac{\sin A_{,}}{\sin A_{,,}} = \frac{\sin \phi_{,}}{\sin \phi_{,,}} \cdot \frac{\cos \alpha_{,,}}{\cos \alpha_{,}}$$

But from the plane triangle $p_{,}C_{o}p_{,,}$, we have—

$$\frac{\sin \phi_{,}}{\sin \phi_{,,}} = \frac{R_{,,} \cos l_{,,}}{R_{,} \cos l_{,}}$$

$\therefore$ also the rigorous formula—

$$\frac{\sin A_{,}}{\sin A_{,,}} = \frac{R_{,,} \cos l_{,,}}{R_{,} \cos l_{,}} \cdot \frac{\cos \alpha_{,,}}{\cos \alpha_{,}} \tag{50}$$

And since for any pair of mutually visible stations, such as occur in trigonometrical surveying, we may assume $\frac{\cos \alpha_{,,}}{\cos \alpha_{,}} = 1$,

$\therefore$ we have—

$$\frac{\sin A_{,}}{\sin A_{,,}} = \frac{R_{,,} \cos l_{,,}}{R_{,} \cos l_{,}} \tag{51}$$

$$\frac{\sin A_{,}}{\sin A_{,,}} = \frac{\cos l_{,,}}{\cos l_{,}} \sqrt{\frac{1 - e^2 \sin^2 l_{,}}{1 - e^2 \sin^2 l_{,,}}} \tag{52}$$

$$\frac{\sin^2 A_{,}}{\sin^2 A_{,,}} = \frac{(1 - e^2) \tan^2 l_{,} + 1}{(1 - e^2) \tan^2 l_{,,} + 1} \tag{53}$$

(true to at least 8 decimals places in their logs.)

☞ From either of these we at once perceive that, with respect to mutually visible stations, *the ratio of the sines of the azimuths* will remain sensibly constant when the latitudes of the stations are of constant magnitudes, no matter how the difference of longitude or the intervening geodesic arc may vary in magnitude.

18. If we find an angle σ such that—

$$\tan \sigma = \frac{R_{,,} \cos l_{,,}}{R_{,} \cos l_{,}} \qquad (54)$$

then from 51, we derive—

$$\frac{\tan \frac{1}{2} (A_{,} - A_{,,})}{\tan \frac{1}{2} (A_{,} + A_{,,})} = \tan (\sigma - 45^\circ) \qquad (55)$$

$$\therefore \tan \tfrac{1}{2} (A_{,} - A_{,,}) = \tan \tfrac{1}{2} (A_{\circ} + A_{\circ\circ}) \cdot \tan (\sigma - 45^\circ) \quad (56)$$

$$\tan \tfrac{1}{2} (A_{,} - A_{,,}) = \frac{\cos \frac{1}{2} (l_{,} - l_{,,})}{\sin \frac{1}{2} (l_{,} + l_{,,})} \cdot \tan (\sigma - 45^\circ) \cdot \cot \tfrac{1}{2} \omega \quad (57)$$

☞ From this equation it is evident that when the latitudes are constants, then the greater ω is, the *less* will the difference of the azimuths be. We already know that, in such case, the *less* also will be the sum of the azimuths, and ∴ the less will each of the azimuths be.

19. It is evident that $A_{\circ} - A_{\circ\circ} = A_{,} - A_{,,} + 2\,\Omega$ and ∴

$$\tan \left\{ \tfrac{1}{2} (A_{,} - A_{,,}) + \Omega \right\} = \frac{\sin \frac{1}{2} (l_{,} - l_{,,})}{\cos \frac{1}{2} (l_{,} + l_{,,})} \cdot \cot \tfrac{1}{2} \omega \qquad (58)$$

and from this and (57) it is evident that when the latitudes of the stations are constants in magnitude, we have

$$\frac{\tan \left\{ \frac{1}{2} (A_{,} - A_{,,}) + \Omega \right\}}{\tan \frac{1}{2} (A_{,} - A_{,,})} = \text{constant.}$$

And since the greater the difference of longitude of the stations is, the less $A_{,} - A_{,,}$ must be; ∴ the greater ω is, the less will Ω be.

20. From the spherical triangle $S_{,}PS_{,,}$, we have

$$\frac{\sin (A_{,,} - \Omega)}{\sin (A_{,} + \Omega)} = \frac{\sin l'}{\sin l''}$$

$$\therefore \tan \Omega = \frac{\sin A_{,,} \sin l'' - \sin A_{,} \sin l'}{\cos A_{,,} \sin l'' + \cos A_{,} \sin l'} \qquad (59)$$

☞ In such cases as occur in trigonometrical surveying the angle Ω will range from zero to a limiting value of about $10'_{,,}\, 00''$. In the case of the worked-out example in the sequel, the value of Ω is $7'_{,,}\, 22''$ nearly.

21. From the spherical triangles $S_{,}PI$, $S_{,,}PI$, we have—

$$\sin \theta \sin \phi_{,} = \sin A_{,} \cos \alpha_{,}$$
$$\sin \theta \sin \phi_{,,} = \sin A_{,,} \cos \alpha_{,,}$$

Multiplying both sides of these equations by the chord k, and remembering that the projection $k_{\circ}$ of the chord on the plane of the equator is equal to $k.\ \sin\theta$, we have—

$$\begin{aligned} k\cdot\sin A_{\prime}\cos a_{\prime} &= k_{\circ}\cdot\sin\phi_{\prime} \\ k\cdot\sin A_{\prime\prime}\cos a_{\prime\prime} &= k_{\circ}\cdot\sin\phi_{\prime\prime} \end{aligned}$$

But from the plane triangle $p_{\prime}C_{\circ}p_{\prime\prime}$ we know that

$$k_{\circ} = \frac{R_{\prime}\cos l_{\prime}\sin\omega}{\sin\phi_{\prime}} = \frac{R_{\prime\prime}\cos l_{\prime\prime}\sin\omega}{\sin\phi_{\prime\prime}}$$

∴ we have—

$$\begin{aligned} k\cdot\sin A_{\prime}\cos a_{\prime} &= R_{\prime\prime}\cos l_{\prime\prime}\sin\omega \\ k\ .\ \sin A_{\prime\prime}\cos a_{\prime\prime} &= R_{\prime}\cos l_{\prime}\sin\omega \end{aligned} \tag{60}$$

And, since $k = 2s\cdot\sin\frac{1}{2}\Sigma \div \Sigma\cdot\sin 1''$, we have—

$$\begin{aligned} \frac{2s\cdot\sin A_{\prime}\sin\frac{1}{2}\Sigma\cdot\cos a_{\prime}}{\Sigma\cdot\sin 1''} &= R_{\prime\prime}\cos l_{\prime\prime}\sin\omega \\ \frac{2s\cdot\sin A_{\prime\prime}\sin\frac{1}{2}\Sigma\cdot\cos a_{\prime\prime}}{\Sigma\cdot\sin 1''} &= R_{\prime}\cos l_{\prime}\sin\omega \end{aligned} \tag{61}$$

And since for any pair of mutually visible stations $\cos a_{\prime} = \cos a_{\prime\prime} = \cos\frac{1}{2}\Sigma$,

$$\begin{aligned} \frac{s\cdot\sin A_{\prime}\cdot\sin\Sigma}{\Sigma\cdot\sin 1''} &= R_{\prime\prime}\cos l_{\prime\prime}\sin\omega \\ \frac{s\cdot\sin A_{\prime\prime}\sin\Sigma}{\Sigma\cdot\sin 1''} &= R_{\prime}\cos l_{\prime}\sin\omega \end{aligned} \tag{62}$$

When the geodesic arc s is such that its circular measure Σ is not more than 1°, we immediately deduce the relations—

$$\begin{aligned} \omega &= \frac{s\cdot\sin A_{\prime}}{R_{\prime\prime}\cdot\cos l_{\prime\prime}\cdot\sin 1''} \\ \omega &= \frac{s\cdot\sin A_{\prime\prime}}{R_{\prime}\cdot\cos l_{\prime}\cdot\sin 1''} \end{aligned} \tag{63}$$

☞ In Chambers' "Practical Mathematics," and in the article on "Geodesy" in Spon's Dictionary of Engineering, the formulæ (63) are given in an erroneous form which must inevitably lead to incompatible results when applied in trigonometrical surveying. The erroneous formulæ given there and elsewhere are—

$$\omega = \frac{s\cdot\sin A_{\prime}}{R_{\prime}\cdot\cos l_{\prime\prime}\cdot\sin 1''} = \frac{s\cdot\sin A_{\prime\prime}}{R_{\prime\prime}\cdot\cos l_{\prime}\cdot\sin 1''}$$

(See note 6 to problem 10 given in the sequel.)

22. From 50 or 60 we have—

$$\frac{\cos a_{\prime}}{\cos a_{\prime\prime}} = \frac{R_{\prime\prime}\cos l_{\prime\prime}\sin A_{\prime\prime}}{R_{\prime}\cos l_{\prime}\sin A_{\prime}} \tag{64}$$

But (14)
$$\frac{\sin \alpha_{,}}{\sin \alpha_{,,}} = \frac{R_{,,}}{R_{,}} \tag{65}$$

$$\therefore \quad \frac{\tan \alpha_{,}}{\tan \alpha_{,,}} = \frac{\cos l_{,} \sin A_{,}}{\cos l_{,,} \sin A_{,,}} \tag{66}$$

From these we can easily express the squares of the sines, cosines, and tangents of the angles of depression of the chord in terms of the two latitudes and two azimuths; but it is obvious that such expressions must assume the indefinite form $\frac{0}{0}$ when the latitudes are equal, or $R_{,} = R_{,,}$. And from (64) and (27), we have—

$$\tan \tfrac{1}{2}(\alpha_{,,}+\alpha_{,}) = \left(\frac{R_{,}+R_{,,}}{R_{,}-R_{,,}}\right)^{\frac{1}{2}} \cdot \left(\frac{R_{,,} \cos l_{,,} \sin A_{,,} - R_{,} \cos l_{,} \sin A_{,}}{R_{,,} \cos l_{,,} \sin A_{,,} + R_{,} \cos l_{,} \sin A_{,}}\right)^{\frac{1}{2}}$$

$$\tan \tfrac{1}{2}(\alpha_{,,}-\alpha_{,}) = \left(\frac{R_{,}-R_{,,}}{R_{,}+R_{,,}}\right)^{\frac{1}{2}} \cdot \left(\frac{R_{,,} \cos l_{,,} \sin A_{,,} - R_{,} \cos l_{,} \sin A_{,}}{R_{,,} \cos l_{,,} \sin A_{,,} + R_{,} \cos l_{,} \sin A_{,}}\right)^{\frac{1}{2}}$$

The expression for $\tan \frac{1}{2} \Sigma$ or $\tan \frac{1}{2} (a_{,,} + a_{,})$, given in (67), is of a like character. It assumes the indefinite form $\frac{0}{0}$ when $R_{,} = R_{,,}$; which is the case on a spheroid when the latitudes of the stations are equal, and always the case on a sphere, no matter how the stations may be situated with respect to each other.

23. From the triangles $D_{,}S_{,}I$, $D_{,,}S_{,,}I$, we have—

$$\left.\begin{aligned} \frac{\cos \alpha_{,}}{\cos (z_{,,} - \alpha_{,,})} &= \frac{\sin D_{,}}{\sin A_{,}} \\ \frac{\cos \alpha_{,,}}{\cos (z_{,} - \alpha_{,})} &= \frac{\sin D_{,,}}{\sin A_{,,}} \end{aligned}\right\} \tag{69}$$

$$\left.\begin{aligned} \sin D_{,} &= \frac{\cos l_{,,} \sin \omega}{\sin z_{,}} \\ \sin D_{,,} &= \frac{\cos l_{,} \sin \omega}{\sin z_{,,}} \end{aligned}\right\} \tag{70}$$

And from these we at once obtain the relations—

$$\left.\begin{aligned} \cot z_{,} &= \frac{\sin A_{,,} \cos \alpha_{,,}}{\cos l_{,} \sin \omega \cos \alpha_{,}} - \tan \alpha_{,} \\ \cot z_{,,} &= \frac{\sin A_{,} \cos \alpha_{,}}{\cos l_{,} \sin \omega \cos \alpha_{,,}} - \tan \alpha_{,,} \end{aligned}\right\} \tag{71}$$

If in these we substitute the values of $\sin \omega$ from (60) we have—

$$\left.\begin{aligned} \tan z_{,} &= \frac{k \cdot \cos \alpha_{,}}{R_{,} - k \sin \alpha_{,}} \\ \tan z_{,,} &= \frac{k \cdot \cos \alpha_{,,}}{R_{,,} - k \cdot \sin \alpha_{,,}} \end{aligned}\right\} \tag{72}$$

From the triangles $S_{o}S_{oo}Z_{o}$, $S_{oo}S_{o}Z_{oo}$, we have—

$$\left.\begin{aligned}\sin z_{,} &= \frac{k\cdot\cos(z_{,}-\alpha_{,})}{R_{,}}\\ \sin z_{,,} &= \frac{k\cdot\cos(z_{,,}-\alpha_{,,})}{R_{,,}}\end{aligned}\right\} \qquad (73)$$

And for stations which do not differ in latitude by more than 1°, we know that $\cos(z_{,}-\alpha_{,})$, $\cos(z_{,,}-\alpha_{,,})$, and $\cos\frac{1}{2}\Sigma$, are the same to 8 places of decimals in their logarithms; ∴ for such stations we have the closely approximate formulæ—

$$\left.\begin{aligned}\sin z_{,} &= \frac{k\cdot\cos\frac{1}{2}\Sigma}{R_{,}}\\ \sin z'' &= \frac{k\cdot\cos\frac{1}{2}\Sigma}{R''}\end{aligned}\right\} \qquad (74)$$

But in order to find $z_{,}$ and $z_{,,}$ in the actual practice of trigonometrical surveying (the latitudes of the two stations being such as do not differ by more than 1°) we have the well-known simple formulæ—

$$\left.\begin{aligned}z_{,} &= \frac{s}{R_{,}\cdot\sin 1''}\\ z_{,,} &= \frac{s}{R_{,,}\cdot\sin 1''}\end{aligned}\right\} \qquad (75)$$

which enable us to find $z_{,}$ and $z_{,,}$ to within $\frac{1}{1000}$ part of a second of rigorous accuracy. This can be easily seen from the following—

We have the rigorously true equation—

$$R_{,}\cdot Q_{,}\cdot\cos\delta_{,} = R_{,,}\cdot Q_{,,}\,.\cos\delta_{,,}$$

in which (as is shewn in the sequel) $\delta_{,}$ and $\delta_{,,}$ are always each less than 16 seconds, and differ from each other by less than 0·2″; and as we know that under such circumstances the logs. of $\cos\delta_{,}$ and $\cos\delta_{,,}$ will be the same to 10 places of decimals, ∴ we can assume—

$$R_{,}\cdot Q_{,} = R_{,,}\cdot Q_{,,}$$

$$\text{But}\qquad R_{,}^{2} + Q_{,}^{2} = R_{,,}^{2} + Q_{,,}^{2}\quad\text{absolutely,}$$

$$\therefore\qquad R_{,} = Q_{,,}\quad\text{nearly}$$

$$R_{,,} = Q_{,}\ \text{nearly}$$

Hence if $I_{,}$, $I_{,,}$ be put to represent the bases of the isosceles triangles having the angles $z_{,}$, $z_{,,}$, as vertical angles, and sides equal to $R_{,}$, $R_{,,}$, respectively, we have—

$$\begin{aligned}I_{,}^{2} &= R_{,}^{2} + R_{,}^{2} - 2\,R_{,}^{2}\cos z_{,}\\ &= R_{,}^{2} + Q_{,,}^{2} - 2\,R_{,}\cdot Q_{,,}\cos z_{,}\\ &= k^{2}\end{aligned}$$

and ∴, obviously, we have $z_{,} = \dfrac{s}{R_{,} \cdot \sin 1''}$

And,
$$\begin{aligned} I_{,,}^2 &= R_{,,}^2 + R_{,,}^2 - 2\, R_{,,}^2 \cdot \cos z_{,,} \\ &= R_{,,}^2 + Q_{,}^2 - 2\, R_{,,} \cdot Q_{,} \cdot \cos z_{,,} \\ &= k^2 \end{aligned}$$

∴, obviously, we have $z_{,,} = \dfrac{s}{R_{,,} \sin 1''}$

Nevertheless it is evident that the perpendicular let fall from the station $S_{\circ}$ on the line $S_{\circ\circ}Z_{\circ}$, lies inside the triangle $S_{\circ}Z_{\circ}S_{\circ\circ}$, and that the perpendicular let fall from $S_{\circ\circ}$ on the line $S_{\circ}Z_{\circ\circ}$ lies inside the triangle $S_{\circ}Z_{\circ\circ}S_{\circ\circ}$; and ∴ that $I_{,} \urcorner k$, and also $I_{,,} \urcorner k$; and that, with respect to absolute accuracy, we have—

$$z_{,} \urcorner \frac{s}{R_{,} \sin 1''}; \qquad z_{,,} \urcorner \frac{s}{R_{,,} \sin 1''}.$$

However, the values of $z_{,}$ and $z_{,,}$ as given by (75) are such that for a distance of a degree along the meridian they cannot differ from the absolutely true values by as much as $\frac{6}{10}$ of an inch of error in the length of s would cause. (See "Account of," &c., page 247.)

☞ It is no easy matter to guard against inferring that $z_{,,}$ can never be greater than $\dfrac{s}{\rho \cdot \sin 1''}$ or $(a_{,,} + a_{,})$. But that $z_{,,}$ can be greater than $a_{,,} + a_{,}$ may be easily seen in the following manner:—

It has been already shewn that in all cases in which $l_{,}$ is greater than $l_{,,}$, we must have $D_{,}$ greater than $A_{,}$. Now if we suppose the point $S_{\circ}$ fixed on the spheroidal earth (and ∴ $S_{,}$ also fixed on the unit sphere), and that the point $S_{\circ\circ}$ (which has $S_{,,}$ as corresponding point on the unit sphere) assumes at first a position such that $l_{,} = l_{,,}$, and then moves continuously along the meridian in which it is situated, making $l_{,,}$ less and less until the angle $A_{,}$ becomes $= 90°$, then of course $D_{,}$ from being equal to $A_{,}$ at the commencement must have increased continuously until at length it exceeded 90°. And it is evident that at one state of the implicated entities, the angle $D_{,}$ was 90°, and $A_{,}$ less than 90°, and ∴ that in such state $\sin A_{,}$ was less than $\sin D_{,}$. But if we were to assume that $z_{,,}$ should be always less than $a_{,,} + a_{,}$, or never greater than $a_{,,} + a_{,}$, then $ID_{,}$ should be always greater than $IS_{,}$, and ∴ $\sin A_{,}$ always greater than $\sin D_{,}$, which we know to be absurd.

☞ Moreover, it is evident that by putting V to represent the particular value of the angle A, when unequal to $D_{,}$ but such that $\sin A_{,} = \sin D_{,}$ (in which case $A_{,}$ is acute and $D_{,}$ obtuse) it is evident that—

whenever $A_{,} \mathrel{7} V$, then will $z_{,,} \angle a_{,,} + a_{,}$ or Σ
whenever $A_{,} \angle V$, then will $z_{,,} \mathrel{7} a_{,,} + a_{,}$ or Σ

Hence:—If $S_{\circ\circ}$ be any fixed point within any convex closed curve on the earth's spheroidal surface, and $Z_{\circ\circ}$ the point in which the normal to the surface at $S_{\circ\circ}$ cuts the polar axis: then there are 4 real points $S_{\circ}$ on this curve, and 4 only, such that the angle $S_{\circ\circ}Z_{\circ\circ}S_{\circ}$ subtended at $Z_{\circ\circ}$ is equal to the sum of the angles $a_{,,}$, $a_{,}$, of depression of the chord $S_{\circ\circ}S_{\circ}$ below the tangent planes at $S_{\circ\circ}$, $S_{\circ}$. Viz.—The two points in which the curve is cut by the plane X through $S_{\circ\circ}$ which is perpendicular to the polar axis; and the two points lying on the same side of X, and such that the azimuth of $S_{\circ}$ taken at $S_{\circ\circ}$ is acute, and the azimuth of $S_{\circ\circ}$ taken at $S_{\circ}$ is also acute but greater than the other, and approaching very nearly to 90° owing to the earth's small ellipticity.

24. From the triangles $S_{,,}PD_{,}$, $S_{,}PD_{,,,}$ we have—

$$\begin{aligned} \sin L' &= \frac{\sin z_{,,} \sin A_{,,}}{\sin \omega} \\ \sin L'' &= \frac{\sin z_{,} \sin A_{,}}{\sin \omega} \end{aligned} \tag{76}$$

$$\begin{aligned} \cos L' &= \cos z_{,,} \cos l'' + \sin z_{,,} \sin l'' \cos A_{,,} \\ \cos L'' &= \cos z_{,} \cos l' + \sin z_{,} \sin l' \cos A_{,} \end{aligned} \tag{77}$$

$$\begin{aligned} \cot L' &= \frac{\cot A_{,,} \sin \omega + \cos l'' \cos \omega}{\sin l''} \\ \cot L'' &= \frac{\cot A_{,} \sin \omega + \cos l' \cos \omega}{\sin l'} \end{aligned} \tag{78}$$

And since L′ and L″ are the circular measures of the angles between the lines $S_{\circ}Z_{\circ\circ}$, $S_{\circ\circ}Z_{\circ}$, and the polar axis, we have evidently—

$$\begin{aligned} \cot L' &= e^2 \cdot \frac{R_{,,} \sin l_{,,}}{R_{,} \cos l_{,}} + (1 - e^2) \tan l_{,} \\ \cot L'' &= e^2 \cdot \frac{R_{,} \sin l_{,}}{R_{,,} \cos l_{,,}} + (1 - e^2) \tan l_{,,} \end{aligned} \tag{79}$$

25. By letting fall perpendiculars from $Z_{\circ\circ}$, $Z_{\circ}$, on the

normals $R_{\prime}$, $R_{\prime\prime}$, we easily find the following expressions for $\delta_{\prime}$ and $\delta_{\prime\prime}$—

$$\tan \delta_{\prime} = \frac{e^2 (R_{\prime} \sin l_{\prime} - R_{\prime\prime} \sin l_{\prime\prime}) \cos l_{\prime}}{R_{\prime} - e^2 (R_{\prime} \sin l_{\prime} - R_{\prime\prime} \sin l_{\prime\prime}) \sin l_{\prime}}$$

$$\tan \delta_{\prime\prime} = \frac{e^2 (R_{\prime} \sin l_{\prime} - R_{\prime\prime} \sin l_{\prime\prime}) \cos l_{\prime\prime}}{R_{\prime\prime} + e^2 (R_{\prime} \sin l_{\prime} - R_{\prime\prime} \sin l_{\prime\prime}) \sin l_{\prime\prime}} \qquad (80)$$

And from the plane triangles whose bases are $Z_o Z_{oo}$, and vertices S_o, S_{oo}, we have—

$$\sin \delta_{\prime} = \frac{e^2 (R_{\prime} \cos l' - R_{\prime\prime} \cos l'') \sin L'}{R_{\prime}}$$

$$\sin \delta_{\prime\prime} = \frac{e^2 (R_{\prime} \cos l' - R_{\prime\prime} \cos l'') \sin L''}{R_{\prime\prime}} \qquad (81)$$

Again, from the triangles $S_o S_{oo} Z_o$, $S_o S_{oo} Z_{oo}$, it is evident that—

$$\frac{R_{\prime}}{Q_{\prime\prime}} = \frac{\cos (z_{\prime} - \alpha_{\prime})}{\cos \alpha_{\prime}}; \qquad \frac{R_{\prime\prime}}{Q_{\prime}} = \frac{\cos (z_{\prime\prime} - \alpha_{\prime\prime})}{\cos \alpha_{\prime\prime}}; \qquad (81\text{A})$$

and, to 8 places of decimals in their logarithms, we have—

$$\frac{R_{\prime}}{Q_{\prime\prime}} = \frac{R_{\prime\prime}}{Q_{\prime}} = 1. \qquad (81\text{B})$$

Hence, from the triangles $Z_o Z_{oo} S_o$, $Z_o Z_{oo} S_{oo}$, we have the relations—

$$\frac{\sin L'}{\sin l'} = \frac{R_{\prime}}{R_{\prime\prime}}; \qquad \frac{\sin L''}{\sin l''} = \frac{R_{\prime\prime}}{R_{\prime}}.$$

such that their logs. are the same to 7 places of decimals. And if in the first and second of (81) we substitute for $\frac{R_{\prime}}{R_{\prime\prime}}$, and $\frac{R_{\prime\prime}}{R_{\prime}}$ the above equivalents, we have with an accuracy to at least 7 places of decimals in their logs.—

$$\sin \delta_{\prime} = e^2 (\sin L' \cos l' - \cos l'' \sin l')$$
$$\sin \delta_{\prime\prime} = e^2 (\cos l' \sin l'' - \sin L'' \cos l'') \qquad (82)$$

which we may write in the forms—

$$\sin \delta_{\prime} = e^2 \left\{ - \cos l'' \sin (L' - \delta_{\prime}) + \sin L' \cos (L' - \delta_{\prime}) \right\}$$

$$\sin \delta_{\prime\prime} = e^2 \left\{ \cos l' \sin (L'' + \delta_{\prime\prime}) - \sin L'' \cos (L'' + \delta_{\prime\prime}) \right\}$$

And if we expand these and regard $\cos \delta_{\prime} = \cos \delta_{\prime\prime} = 1$ (which we can do since $\delta_{\prime}$ or $\delta_{\prime\prime}$ is always less than 20″) we easily find—

$$\sin \delta_{\prime} = \frac{e^2 \cdot (\cos L' - \cos l'') \sin L'}{(1 - e^2) + e^2 (\cos L' - \cos l'') \cos L'}$$

$$\sin \delta_{,,} = \frac{e^2 \cdot (\cos l' - \cos L'') \sin L''}{(1 - e^2) - e^2 (\cos l' - \cos L'') \cos L''}$$

which we may write in the forms—

$$\begin{aligned}\sin \delta_{,} &= \frac{2 \cdot e^2 \cdot \sin \tfrac{1}{2} (l'' + L') \sin \tfrac{1}{2} (l'' - L') \sin L'}{(1 - e^2) + 2 \cdot e^2 \cdot \sin \tfrac{1}{2} (l'' + L') \sin \tfrac{1}{2} (l'' - L') \cos L'} \\ \sin \delta_{,,} &= \frac{2 \cdot e^2 \cdot \sin \tfrac{1}{2} (L'' + l') \sin \tfrac{1}{2} (L'' - l') \sin L''}{(1 - e^2) - 2 \cdot e^2 \cdot \sin \tfrac{1}{2} (L'' + l') \sin \tfrac{1}{2} (L'' - l') \cos L''}\end{aligned} \tag{83}$$

(to be used when extreme accuracy is desired.)

Hence evidently (since $\delta_{,}$ or $\delta_{,,}$ is always less than 20 seconds) we have—

$$\begin{aligned}\sin \delta_{,} &= 2 \left(\frac{e^2}{1 - e^2}\right) \sin L' \sin \tfrac{1}{2} (l'' + L') \sin \tfrac{1}{2} (l'' - L') \\ \sin \delta_{,,} &= 2 \left(\frac{e^2}{1 - e^2}\right) \sin L'' \sin \tfrac{1}{2} (L'' + l') \sin \tfrac{1}{2} (L'' - l')\end{aligned} \tag{84}$$

giving $\delta_{,}$ in excess, and $\delta_{,,}$ too small. However, in all ordinary cases, they give values of $\delta_{,}$, $\delta_{,,}$, correct to $\frac{1}{1000}$ part of one second. And since—

$$\begin{aligned}\sin \tfrac{1}{2} (l'' + L') \sin \tfrac{1}{2} (l'' - L') &= \sin (D_{,} - A_{,,}) \cdot \frac{\sin^2 \tfrac{1}{2} z_{,,}}{\sin \omega} \\ &= \tfrac{1}{2} \cdot \sin (D_{,} - A_{,,}) \tan \tfrac{1}{2} z_{,,} \cdot \frac{\sin L'}{\sin A_{,,}} \\ \sin \tfrac{1}{2} (L'' + l') \sin \tfrac{1}{2} (L'' - l') &= \sin (A_{,} - D_{,,}) \cdot \frac{\sin^2 \tfrac{1}{2} z_{,}}{\sin \omega} \\ &= \tfrac{1}{2} \cdot \sin (A_{,} - D_{,,}) \tan \tfrac{1}{2} z_{,} \cdot \frac{\sin L''}{\sin A_{,}}\end{aligned}$$

Therefore we have the equally approximate relations—

$$\begin{aligned}\sin \delta_{,} &= 2 \left(\frac{e^2}{1 - e^2}\right) \sin L' \cdot \frac{\sin (D_{,} - A_{,,})}{\sin \omega} \cdot \sin^2 \tfrac{1}{2} z_{,,} \\ &= \left(\frac{e^2}{1 - e^2}\right) \sin^2 L' \cdot \frac{\sin (D_{,} - A_{,,})}{\sin A_{,,}} \tan \tfrac{1}{2} z_{,,} \\ &= 2 \left(\frac{e^2}{1 - e^2}\right) \sin l'' \cdot \frac{\sin A_{,,} \sin (D_{,} - A_{,,})}{\sin D_{,} \sin \omega} \cdot \sin^2 \tfrac{1}{2} z_{,,} \\ &= \left(\frac{e^2}{1 - e^2}\right) \frac{\sin A_{,,} \sin (D_{,} - A_{,,})}{\sin^2 \omega} \cdot \sin^2 z_{,,} \cdot \tan \tfrac{1}{2} z_{,,} \\ &= \left(\frac{e^2}{1 - e^2}\right) \sin^2 l'' \cdot \frac{\sin A_{,,} \sin (D_{,} - A_{,,})}{\sin^2 D_{,}} \cdot \tan \tfrac{1}{2} z_{,,}\end{aligned} \tag{85}$$

$$\begin{aligned}
\sin\,\delta_{\prime\prime} &= 2\left(\frac{e^2}{1-e^2}\right)\sin\,L''\,\frac{\sin\,(A_{\prime}-D_{\prime\prime})}{\sin\,\omega}\cdot\sin^2\tfrac{1}{2}\,z_{\prime}\\
&= \left(\frac{e^2}{1-e^2}\right)\sin^2 L''\cdot\frac{\sin\,(A_{\prime}-D_{\prime\prime})}{\sin\,A_{\prime}}\cdot\tan\tfrac{1}{2}\,z_{\prime}\\
&= 2\left(\frac{e^2}{1-e^2}\right)\sin\,l'\cdot\frac{\sin\,A_{\prime}\,\sin\,(A_{\prime}-D_{\prime\prime})}{\sin\,D_{\prime\prime}\,\sin\,\omega}\cdot\sin^2\tfrac{1}{2}\,z_{\prime}\\
&= \left(\frac{e^2}{1-e^2}\right)\frac{\sin\,A_{\prime}\,\sin\,(A_{\prime}-D_{\prime\prime})}{\sin^2\omega}\,\sin^2 z_{\prime}\cdot\tan\tfrac{1}{2}\,z_{\prime}\\
&= \left(\frac{e^2}{1-e^2}\right)\sin^2 l'\cdot\frac{\sin\,A_{\prime}\,\sin\,(A_{\prime}-D_{\prime\prime})}{\sin^2 D_{\prime\prime}}\,\tan\tfrac{1}{2}\,z_{\prime}
\end{aligned}\qquad(86)$$

And since the arcs $z_{\prime}$, $z_{\prime\prime}$, do not exceed 1° in the usual cases of trigonometrical surveys, we have, with sufficient accuracy for some purposes—

$$\begin{aligned}
\delta_{\prime} &= \left(\frac{e^2}{1-e^2}\right)\cdot\sin\,L'\cdot\sin\tfrac{1}{2}\,(l''+L')\cdot(l''-L')\\
&= \tfrac{1}{2}\left(\frac{e^2}{1-e^2}\right)\cdot\sin\,L'\cdot\frac{\sin\,(D_{\prime}-A_{\prime\prime})}{\sin\,\omega}\cdot z^2_{\prime\prime}\cdot\sin\,1''\\
&= \tfrac{1}{2}\left(\frac{e^2}{1-e^2}\right)\cdot\sin^2 L'\,\frac{\sin\,(D_{\prime}-A_{\prime\prime})}{\sin\,A_{\prime\prime}}\cdot z_{\prime\prime}\\
&= \tfrac{1}{2}\left(\frac{e^2}{1-e^2}\right)\frac{\sin\,A_{\prime\prime}\,\sin\,(D_{\prime}-A_{\prime\prime})}{\sin\,D_{\prime}\,\sin\,\omega}\cdot\sin\,l''\cdot z^2_{\prime\prime}\cdot\sin\,1''\\
&= \tfrac{1}{2}\left(\frac{e^2}{1-e^2}\right)\frac{\sin\,A_{\prime\prime}\,\sin\,(D_{\prime}-A_{\prime\prime})}{\sin^2\omega}\cdot\sin^2 z_{\prime\prime}\cdot z_{\prime\prime}\\
&= \tfrac{1}{2}\left(\frac{e^2}{1-e^2}\right)\frac{\sin\,A_{\prime\prime}\,\sin\,(D_{\prime}-A_{\prime\prime})}{\sin^2 D_{\prime}}\cdot\sin^2 l''\cdot z_{\prime\prime}
\end{aligned}\qquad(87)$$

$$\begin{aligned}
\delta_{\prime\prime} &= \left(\frac{e^2}{1-e^2}\right)\cdot\sin\,L''\cdot\sin\tfrac{1}{2}\,(L''+l')\cdot(L''-l')\\
&= \tfrac{1}{2}\left(\frac{e^2}{1-e^2}\right)\cdot\sin\,L''\cdot\frac{\sin\,(A_{\prime}-D_{\prime\prime})}{\sin\,\omega}\cdot z^2_{\prime}\cdot\sin\,1''\\
&= \tfrac{1}{2}\left(\frac{e^2}{1-e^2}\right)\cdot\sin^2 L''\cdot\frac{\sin\,(A_{\prime}-D_{\prime\prime})}{\sin\,A_{\prime}}\cdot z_{\prime}\\
&= \tfrac{1}{2}\left(\frac{e^2}{1-e^2}\right)\frac{\sin\,A_{\prime}\,\sin\,(A_{\prime}-D_{\prime\prime})}{\sin\,D_{\prime\prime}\,\sin\,\omega}\cdot\sin\,l'\cdot z^2_{\prime}\cdot\sin\,1''\\
&= \tfrac{1}{2}\left(\frac{e^2}{1-e^2}\right)\frac{\sin\,A_{\prime}\,\sin\,(A_{\prime}-D_{\prime\prime})}{\sin^2\omega}\cdot z^3_{\prime}\cdot\sin^2 1''\\
&= \tfrac{1}{2}\left(\frac{e^2}{1-e^2}\right)\frac{\sin\,A_{\prime}\,\sin\,(A_{\prime}-D_{\prime\prime})}{\sin^2 D_{\prime\prime}}\cdot\sin^2 l'\cdot z_{\prime}
\end{aligned}\qquad(88)$$

☞ Referring to the approximate relation—

$$\frac{\sin l'}{\sin L'} = \frac{\sin L''}{\sin l''}$$

made use of in arriving at the preceding values of $\delta_{\prime}$, $\delta_{\prime\prime}$, it may be proper to observe that we must not always use it as if it were rigorously true. If so used we should, as a consequence, have—

$$\frac{\sin A_{\prime}}{\sin D_{\prime}} = \frac{\sin D_{\prime\prime}}{\sin A_{\prime\prime}}$$

and therefore the first side of this equation always less than unity, which we know to be absurd. Hence we perceive that the adoption of the above approximate relation is equivalent to assuming that between the limits of the possible values of $A_{\prime}$ from the state in which $A_{\prime} = D_{\prime\prime}$ to that in which $A_{\prime} = V$, we have $\sin D_{\prime} = \sin A_{\prime}$, and $\sin A_{\prime\prime} = \sin D_{\prime\prime}$ so nearly true that their logarithms are the same to 7 places of decimals. However, we will now shew how those small angular differences can be computed.

26. It is evident that the amount by which the angle $A_{\prime\prime}$ exceeds $D_{\prime\prime}$ is truly expressed by the spherical excess of the small triangle $S_{\prime}S_{\prime\prime}D_{\prime\prime}$. It is also evident that the amount by which the angle $D_{\prime}$ exceeds $A_{\prime}$ is expressed by the spherical excess of the small triangle $S_{\prime}S_{\prime\prime}D_{\prime}$. Hence (see formula 4, page 158, Serrets', &c.)—

$$\left.\begin{aligned}
\cot \tfrac{1}{2} A_{\prime\prime} &= \cot \tfrac{1}{2} D_{\prime\prime} \cdot \frac{\cos \frac{1}{2}(z_{\prime} + \delta_{\prime\prime})}{\cos \frac{1}{2}(z_{\prime} - \delta_{\prime\prime})} \\
\tan \tfrac{1}{2} A_{\prime\prime} &= \tan \tfrac{1}{2} D_{\prime\prime} \cdot \frac{\cos \frac{1}{2}(z_{\prime} - \delta_{\prime\prime})}{\cos \frac{1}{2}(z_{\prime} + \delta_{\prime\prime})} \\
\tan \tfrac{1}{2} A_{\prime} &= \tan \tfrac{1}{2} D_{\prime} \cdot \frac{\cos \frac{1}{2}(z_{\prime\prime} + \delta_{\prime})}{\cos \frac{1}{2}(z_{\prime\prime} - \delta_{\prime})} \\
\cot \tfrac{1}{2} A_{\prime} &= \cot \tfrac{1}{2} D_{\prime} \cdot \frac{\cos \frac{1}{2}(z_{\prime\prime} - \delta_{\prime})}{\cos \frac{1}{2}(z_{\prime\prime} + \delta_{\prime})}
\end{aligned}\right\} \quad (89)$$

We have also (see formula 3, page 158, of Serrets' Trigonometry) rigorously—

$$\left.\begin{aligned}
\tan \tfrac{1}{2}(A_{\prime\prime} - D_{\prime\prime}) &= \frac{\tan \frac{1}{2} z_{\prime} \tan \frac{1}{2} \delta_{\prime\prime} \sin D_{\prime\prime}}{1 - \tan \frac{1}{2} z_{\prime} \tan \frac{1}{2} \delta_{\prime\prime} \cos D_{\prime\prime}} \\
\tan \tfrac{1}{2}(D_{\prime} - A_{\prime}) &= \frac{\tan \frac{1}{2} z_{\prime\prime} \tan \frac{1}{2} \delta_{\prime} \sin D_{\prime}}{1 + \tan \frac{1}{2} z_{\prime\prime} \tan \frac{1}{2} \delta_{\prime} \cos D_{\prime}}
\end{aligned}\right\} \quad (90)$$

And the angles $\frac{1}{2}(A_{\prime\prime} - D_{\prime\prime})$, $\frac{1}{2}(D_{\prime} - A_{\prime})$, being but fractions of a second; and the values of $\tan \frac{1}{2} z_{\prime} \cdot \tan \frac{1}{2} \delta_{\prime\prime}$

$\cos D_{,,}$, and $\tan \frac{1}{2} z_{,,} \cdot \tan \frac{1}{2} \delta_{,} \cdot \cos D_{,}$ being so extremely small, it is evident we can find the values of the angles $A_{,,}$ and $A_{,}$ to the $\frac{1}{10000}$ part of a second by means of the ameliorated formulæ—

$$\begin{aligned} \tan \tfrac{1}{2} (A_{,,} - D_{,,}) &= \sin D_{,,} \tan \tfrac{1}{2} z_{,} \cdot \tan \tfrac{1}{2} \delta_{,,} \\ \tan \tfrac{1}{2} (D_{,} - A_{,}) &= \sin D_{,} \tan \tfrac{1}{2} z_{,,} \cdot \tan \tfrac{1}{2} \delta_{,} \end{aligned} \tag{91}$$

We can also arrive at these in the following manner—

From formula (1), implicating spherical excess, on page 157 of Serrets' Trigonometry, we have—(since in actual practice of surveying the logs. of $\cos \frac{1}{2} \nu$, $\cos \frac{1}{2} z_{,}$, $\cos \frac{1}{2} z_{,,}$, are the same to 6 or 7 places of decimals)—

$$\begin{aligned} \sin \tfrac{1}{2} (A_{,,} - D_{,,}) &= \sin D_{,,} \cdot \tan \tfrac{1}{2} z_{,} \cdot \sin \tfrac{1}{2} \delta_{,,} \\ \sin \tfrac{1}{2} (D_{,} - A_{,}) &= \sin D_{,} \cdot \tan \tfrac{1}{2} z_{,,} \cdot \sin \tfrac{1}{2} \delta_{,} \end{aligned} \tag{92}$$

$\therefore$ also

$$\begin{aligned} A_{,,} - D_{,,} &= \sin D_{,,} \tan \tfrac{1}{2} z_{,} \cdot \delta_{,,} \\ D_{,} - A_{,} &= \sin D_{,} \tan \tfrac{1}{2} z_{,,} \cdot \delta_{,} \end{aligned} \tag{93}$$

or,

$$\begin{aligned} A_{,,} - D_{,,} &= \tfrac{1}{2} \cdot z_{,} \cdot \delta_{,,} \cdot \sin 1'' \cdot \sin D_{,,} \\ D_{,} - A_{,} &= \tfrac{1}{2} \cdot z_{,,} \cdot \delta_{,} \cdot \sin 1'' \cdot \sin D_{,} \end{aligned}$$

And from these and formulæ (87) and (88), we easily find—

$$\begin{aligned} A_{,,} - D_{,,} &= \tfrac{1}{4} \cdot \frac{e^2}{1 - e^2} \cdot \sin l' \cdot \sin L'' \sin (A_{,} - D_{,,}) \cdot z_{,}^2 \times \sin 1'' \\ &= \tfrac{1}{4} \cdot \frac{e^2}{1 - e^2} \cdot \sin^2 l' \cdot \frac{\sin A_{,} \sin (A_{,} - D_{,,})}{\sin D_{,,}} \cdot z_{,}^2 \times \sin 1'' \\ &= \tfrac{1}{4} \cdot \frac{e^2}{1 - e^2} \cdot \sin l' \cdot \frac{\sin A_{,} \sin (A_{,} - D_{,,})}{\sin \omega} \cdot z_{,}^3 \times \sin^2 1'' \end{aligned}$$

$$\begin{aligned} D_{,} - A_{,} &= \tfrac{1}{4} \cdot \frac{e^2}{1 - e^2} \cdot \sin l'' \cdot \sin L' \cdot \sin (D_{,} - A_{,,}) \cdot z_{,,}^2 \times \sin 1'' \\ &= \tfrac{1}{4} \cdot \frac{e^2}{1 - e^2} \cdot \sin^2 l'' \cdot \frac{\sin A_{,,} \sin (D_{,} - A_{,,})}{\sin D_{,}} \cdot z_{,,}^2 \times \sin 1'' \\ &= \tfrac{1}{4} \cdot \frac{e^2}{1 - e^2} \cdot \sin l'' \cdot \frac{\sin A_{,,} \sin (D_{,} - A_{,,})}{\sin \omega} \cdot z_{,,}^3 \times \sin^2 1'' \end{aligned}$$

☞ In the "Account of the Principal Triangulation of Great Britain and Ireland" (see pages 248, 249, formulæ 32 and 36), the following erroneous expressions are given—

$$\begin{aligned} D_{,,} - A_{,,} &= \tfrac{1}{4} \cdot \frac{e^2}{1 - e^2} \cdot \cos^2 l_{,} \sin 2A_{,} \cdot z^2_{,} \times \sin 1'' \\ D_{,} - A_{,} &= \tfrac{1}{4} \cdot \frac{e^2}{1 - e^2} \cdot \cos^2 l_{,,} \sin 2A_{,,} \cdot z^2_{,,} \times \sin 1'' \end{aligned} \tag{96}$$

with respect to which we may observe—

1°. From them we should infer that $D_{,,} - A_{,,}$ and $D_{,} - A_{,}$ have finite values when the latitudes of the stations are equal; but we know, in any such case, that the angles $D_{\prime\prime}$, $A_{\prime\prime}$, $D_{\prime}$, $A_{\prime}$, are equal.

2°. From the first of the equations we should infer that $A_{\prime\prime}$ is less than $D_{\prime\prime}$ when $A_{\prime}$ is acute; but we know that $A_{\prime\prime}$ must be always greater than $D_{\prime\prime}$, when $l_{\prime}$ is greater than $l_{\prime\prime}$, or when $A_{\prime}$ is greater than $A_{\prime\prime}$.

3°. In the example 1 worked out in this paper, we have, by using correct formulæ—

$$A_{\prime\prime} - D_{\prime\prime} = 0'' \cdot 1334; \quad D_{\prime} - A_{\prime} = 0'' \cdot 1334.$$

But if we were to use the above erroneous formulæ, we would find the values—

$$A_{\prime\prime} - D_{\prime\prime} = 0'' \cdot 1315; \quad D_{\prime} - A_{\prime} = 0'' \cdot 1352.$$

☞ On page 676 the formula 96 is misprinted: $\frac{1}{\sin 1''}$ being there used instead of $\sin 1''$.

27. From (46) and (47) it is easy to deduce the following expression—

$$\sin \tfrac{1}{2} \nu = \frac{\sqrt{\cos \frac{1}{2} (A_{\prime} + A_{\prime\prime} + x) \cos \frac{1}{2} (A_{\prime} + A_{\prime\prime} - x)}}{\cos \frac{1}{2} (A_{\prime} + A_{\prime\prime})}$$

in which the angle x is found from—

$$\sin \tfrac{1}{2} x = \sin \tfrac{1}{2} (l_{\prime} + l_{\prime\prime}) \cdot \sin \tfrac{1}{2} \omega.$$

28. The perpendicular from Z_{oo} to the line $S_{oo}Z_{o}$ is equal $Z_{o}Z_{oo} \cdot \sin L''$; and $\therefore$ it is evident that the perpendicular from Z_{oo} on the normal-chordal plane $S_{o}S_{oo}Z_{o}$ is equal to $Z_{o}Z_{oo} \cdot \sin L'' \cdot \sin D_{\prime\prime}$. But the perpendicular from Z_{oo} on the chord $S_{o}SS_{oo}$ is evidently equal to $R_{\prime\prime} \cdot \cos \alpha_{\prime\prime}$. Hence, obviously—

$$\sin \triangle = \frac{Z_{o}Z_{oo} \cdot \sin L'' \cdot \sin D_{\prime\prime}}{R_{\prime\prime} \cdot \cos \alpha_{\prime\prime}}$$

But,

$Z_{o}Z_{oo} = e^2 (R_{\prime} \sin l_{\prime} - R_{\prime\prime} \sin l_{\prime\prime})$; $\sin L'' \sin D_{\prime\prime} = \cos l_{\prime} \sin A_{\prime}$; and

$$\cos \alpha_{\prime\prime} = \frac{R_{\prime} \cos l_{\prime} \sin \omega}{k \cdot \sin A_{\prime\prime}}$$

Hence we have—

$$\sin \triangle = e^2 \cdot k \cdot \frac{\sin A_{\prime} \sin A_{\prime\prime}}{\sin \omega} \cdot \left(\frac{\sin l_{\prime}}{R_{\prime\prime}} - \frac{\sin l_{\prime\prime}}{R_{\prime}}\right) \qquad (98)$$

$$\sin \triangle = k \cdot \frac{R^2_{\prime} - R^2_{\prime\prime}}{R_{\prime} \cdot R_{\prime\prime}} \cdot \frac{\sin A_{\prime} \sin A_{\prime\prime}}{\sin \omega} \cdot (R_{\prime} \sin l' + R_{\prime\prime} \sin l_{\prime\prime})^{-1}$$

$$\sin \triangle = \frac{(R^2_{\prime} - R^2_{\prime\prime})^{\frac{1}{2}}}{R_{\prime} \sin l_{\prime} + R_{\prime\prime} \sin l_{\prime\prime}} \cdot (\cos^2 l_{\prime\prime} \sin^2 A_{\prime\prime} - \cos^2 l_{\prime} \sin^2 A_{\prime})$$

These expressions are rigorously true, and can be used in other investigations.

We have also from the triangles $\mathrm{IS}_{,}\mathrm{D}_{,,}$, $\mathrm{IS}_{,,}\mathrm{D}_{,,,}$—

$$\sin \triangle = \frac{\sin \delta_{,} \cdot \sin \mathrm{D}_{,}}{\cos \frac{1}{2} \Sigma} = \frac{\sin \delta_{,,} \sin \mathrm{D}_{,,}}{\cos \frac{1}{2} \Sigma} \qquad (101)$$

☞ In the "Account of the Principal Triangulation of Great Britain and Ireland," the following expressions are given—

$$\begin{aligned} \triangle &= e^2 \cdot \sin 2\,\mathrm{A}_{,} \cdot \cos^2 (l_{,} + l_{,,}) \cdot \tfrac{1}{2} \Sigma \\ \triangle &= e^2 \cdot \sin 2\,\mathrm{A}_{,,} \cdot \cos^2 (l_{,} + l_{,,}) \cdot \tfrac{1}{2} \Sigma \end{aligned} \qquad (102)$$

That this formula is erroneous is easily seen: for independent of the oversight committed in assuming that sin 2 $\mathrm{A}_{,}$ is equal to sin 2 $\mathrm{A}_{,,}$, we know that any expression representing $\triangle$ must vanish when the latitudes $l_{,}$, $l_{,,}$, are equal; and this is not the case with formulæ (102).

29. When the stations $\mathrm{S}_{\circ}$, $\mathrm{S}_{\circ\circ}$, are mutually visible (not more than 100 miles apart), it is evident that if from the middle point of the arc ν we conceive perpendicular arcs drawn to the circles $\mathrm{S}_{,}\mathrm{D}_{,,}$, $\mathrm{S}_{,,}\mathrm{D}_{,}$, they will form two right angled spherical triangles (having vertices at $\mathrm{S}_{,}$ and $\mathrm{S}_{,,}$), which may be considered equals in all respects. It is evident that two of the sides of either of these triangles are equals to $\frac{1}{2}\,\nu$ and $\frac{1}{2}\,\Sigma$, and that the third side of either may be regarded as equal to $\frac{1}{2} \triangle$.

From this relation connecting the angle between the normals, the angle between the normal-chordal planes, and the circular measure of the geodesic arc between the stations, we have—

$$\cos \tfrac{1}{2}\,\nu = \cos \tfrac{1}{2} \triangle \cdot \cos \tfrac{1}{2} \Sigma \qquad (103)$$

$$\sin \tfrac{1}{2} \triangle = \sin \tfrac{1}{2}\,\nu \cdot \sin \Omega \qquad (104)$$

$$\tan \tfrac{1}{2} \triangle = \sin \tfrac{1}{2} \Sigma \cdot \tan \Omega \qquad (105)$$

$$\tan \tfrac{1}{2} \Sigma = \tan \tfrac{1}{2}\,\nu \cdot \cos \Omega \qquad (106)$$

simple relations which will be found very useful in practical work of trigonometrical surveys.

30. The following expressions for the cosines, sines, and tangents of the angles of depression of the chord are rigorous with respect to any two stations on the earth's spheroidal surface; and the easy methods by which they have been deduced (from what has been already done) are omitted, as they can present no difficulty to the reader.

$$\cos \alpha_{\prime} = \frac{R_{\prime\prime} \cos l_{\prime\prime} \sin \omega}{k \cdot \sin A_{\prime}}$$
$$\cos \alpha_{\prime\prime} = \frac{R_{\prime} \cos l_{\prime} \sin \omega}{k \cdot \sin A_{\prime\prime}} \qquad (107)$$

$$\sin \alpha_{\prime} = \frac{R_{\prime} \cos l_{\prime} - R_{\prime\prime} \cos l_{\prime\prime} (\tan l_{\prime} \cot A_{\prime} \sin \omega + \cos \omega)}{k \cdot \cos l_{\prime}}$$
$$\sin \alpha_{\prime\prime} = \frac{R_{\prime\prime} \cos l_{\prime\prime} - R_{\prime} \cos l_{\prime} (\tan_{\prime\prime} \cot A_{\prime\prime} \sin \omega + \cos \omega)}{k \cdot \cos l_{\prime\prime}} \qquad (108)$$

$$\sin \alpha_{\prime} = \frac{R_{\prime}R_{\prime\prime} (\cos l_{\prime} \cos l_{\prime\prime} \cos \omega + (1-e^2) \sin l_{\prime} \sin l_{\prime\prime}) - a^2}{k \cdot R_{\prime}}$$
$$\sin \alpha_{\prime\prime} = \frac{R_{\prime}R_{\prime\prime} (\cos l_{\prime} \cos l_{\prime\prime} \cos \omega + (1-e^2) \sin l_{\prime} \sin l_{\prime\prime}) - a^2}{k \cdot R_{\prime\prime}} \qquad (109)$$

$$\tan \alpha_{\prime} = \frac{R_{\prime} \sin A_{\prime}}{R_{\prime\prime} \cdot \cos l_{\prime\prime} \sin \omega} - \frac{\cot \omega \sin A_{\prime} + \sin l_{\prime\prime} \cos A_{\prime}}{\cos l_{\prime}}$$
$$\tan \alpha_{\prime\prime} = \frac{R_{\prime\prime} \sin A_{\prime\prime}}{R_{\prime} \cos l_{\prime} \sin \omega} - \frac{\cot \omega \sin A_{\prime\prime} + \sin l_{\prime\prime} \cos A_{\prime\prime}}{\cos l_{\prime\prime}} \qquad (110)$$

$$\tan \alpha_{\prime} = \frac{\cos l_{\prime}}{\sin A_{\prime\prime}} \cdot \frac{R_{\prime\prime} \sin A_{\prime\prime} \cos A_{\prime} + R_{\prime} \cos A_{\prime\prime} \sin A_{\prime}}{R_{\prime\prime} \sin l_{\prime\prime} + R_{\prime} \sin l_{\prime}}$$
$$\tan \alpha_{\prime\prime} = \frac{\cos l_{\prime\prime}}{\sin A_{\prime}} \cdot \frac{R_{\prime\prime} \sin A_{\prime\prime} \cos A_{\prime} + R_{\prime} \cos A_{\prime\prime} \sin A_{\prime}}{R_{\prime\prime} \sin l_{\prime\prime} + R_{\prime} \sin l_{\prime}} \qquad (111)$$

31. By equating the values of sin $\alpha_{\prime}$ given in (108), (109), we have an equation from which we can at once express cot $A_{\prime}$ in terms of the two latitudes and the difference of longitude ω. And equating the values of sin $\alpha_{\prime\prime}$ given in (108), (109), we can express cot $A_{\prime\prime}$ in terms of the two latitudes and difference of longitude. However, we can find other expressions for the cotangents of the azimuths, thus—

From the spherical triangles $S_{\prime}PD_{\prime\prime}$, $S_{\prime\prime}PD_{\prime}$, we have

$$\cot A_{\prime} = \frac{\cot L'' \cos l_{\prime} - \sin l_{\prime} \cos \omega}{\sin \omega}$$
$$\cot A_{\prime\prime} = \frac{\cot L' \cos l_{\prime\prime} - \sin l_{\prime\prime} \cos \omega}{\sin \omega}$$

And if in these we substitute the values of cot L″, cot L′, given in (79), we have—

$$\cot A_{\prime} = \frac{\frac{R_{\prime}}{R_{\prime\prime}} \cdot e^2 \sin l_{\prime} \cos l_{\prime} + (1-e^2) \sin l_{\prime\prime} \cos l_{\prime} - \sin l_{\prime} \cos l_{\prime\prime} \cos \omega}{\cos l_{\prime\prime} \sin \omega}$$
$$\cot A_{\prime\prime} = \frac{\frac{R_{\prime\prime}}{R_{\prime}} \cdot e^2 \sin l_{\prime\prime} \cos l_{\prime\prime} + (1-e^2) \sin l_{\prime} \cos l_{\prime\prime} - \sin l_{\prime\prime} \cos l_{\prime} \cos \omega}{\cos l_{\prime} \sin \omega} \qquad (112)$$

These have been arrived at by other means in the "Account of the Principal Triangulation of Great Britain and Ireland." Moreover, from the spherical triangle $S_{\prime}PS_{\prime\prime}$, we have—

$$\cot A_{\circ} = \frac{\sin l_{\prime\prime} \cos l_{\prime} - \sin l_{\prime} \cos l_{\prime\prime} \cos \omega}{\cos l_{\prime\prime} \sin \omega}$$

$$\cot A_{\circ\circ} = \frac{\sin l_{\prime} \cos l_{\prime\prime} - \sin l_{\prime\prime} \cos l_{\prime} \cos \omega}{\cos l_{\prime} \sin \omega}$$

$$\therefore \cot A_{\prime} - \cot A_{\circ} = \left(\frac{R_{\prime}}{R_{\prime\prime}} \sin l_{\prime} - \sin l_{\prime\prime}\right) \cdot \frac{e^2 \cdot \cos l_{\prime}}{\cos l_{\prime\prime} \sin \omega} \qquad (113)$$

$$\cot A_{\prime\prime} - \cot A_{\circ\circ} = \left(\frac{R_{\prime\prime}}{R_{\prime}} \sin l_{\prime\prime} - \sin l_{\prime}\right) \cdot \frac{e^2 \cdot \cos l_{\prime\prime}}{\cos l_{\prime} \sin \omega}$$

These also are given in the "Account of the Principal Triangulation of Great Britain and Ireland" (see page 231 of that work).

32. From (60) it is evident that for any pair of mutually visible stations, we have—

$$k = \frac{R_{\prime} \cos l_{\prime} \sin \omega}{\sin A_{\prime\prime} \cos \frac{1}{2} \Sigma}$$

$$k = \frac{R_{\prime\prime} \cos l_{\prime\prime} \sin \omega}{\sin A_{\prime} \cos \frac{1}{2} \Sigma} \qquad (114)$$

$$k = \frac{R_{\prime} R_{\prime\prime}}{(R^2_{\prime} - R^2_{\prime\prime})^{\frac{1}{2}}} \cdot \frac{\sin \omega}{\sin A_{\prime} \sin A_{\prime\prime}} \cdot (\cos^2 l_{\prime\prime} \sin^2 A_{\prime\prime} - \cos^2 l_{\prime} \sin^2 A_{\prime})$$

the last of which is rigorously accurate for any two stations on the earth's spheroidal surface, and a direct expression in terms of the two latitudes and difference of longitude; but it assumes the form $\frac{0}{0}$ when the latitudes $l_{\prime}$, $l_{\prime\prime}$, are equal.

33. From $\frac{\sin^2 \alpha_{\prime}}{\sin^2 \alpha_{\prime\prime}} = \frac{R^2_{\prime\prime}}{R^2_{\prime}} = \frac{1 - e^2 \sin^2 l_{\prime}}{1 - e^2 \sin^2 l_{\prime\prime}}$, we have the rigorous formulæ—

$$e^2 = \frac{\sin^2 \alpha_{\prime\prime} - \sin^2 \alpha_{\prime}}{\sin^2 l_{\prime} \sin^2 \alpha_{\prime\prime} - \sin^2 l_{\prime\prime} \sin^2 \alpha_{\prime}} \qquad (115)$$

$$\frac{b^2}{a^2} = \frac{\cos^2 l_{\prime} \sin^2 \alpha_{\prime\prime} - \cos^2 l_{\prime\prime} \sin^2 \alpha_{\prime}}{\sin^2 l_{\prime} \sin^2 \alpha_{\prime\prime} - \sin^2 l_{\prime\prime} \sin^2 \alpha_{\prime}} \qquad (116)$$

applying to any two stations whatever on the earth's spheroidal surface.

From (53) we have—

$$e^2 = \frac{\sin^2 A_{\prime\prime} \sec^2 l_{\prime} - \sin^2 A_{\prime} \sec^2 l_{\prime\prime}}{\sin^2 A_{\prime\prime} \tan^2 l_{\prime} - \sin^2 A_{\prime} \tan^2 l_{\prime\prime}} \qquad (117)$$

$$\frac{b^2}{a^2} = \frac{\sin^2 A_{\prime} - \sin^2 A_{\prime\prime}}{\sin^2 A_{\prime\prime} \tan^2 l_{\prime} - \sin^2 A_{\prime} \tan^2 l_{\prime\prime}} \qquad (118)$$

(Holding true to at least 8 places of decimals in their logarithms.)

The expressions for e^2 and $\frac{b^2}{a^2}$ in 115, 116, 117, 118, assume the form $\frac{0}{0}$ when the latitudes of the stations are equal. If the latitudes and mutual azimuths of *numerous pairs of suitable stations* be carefully found from actual observation with good instruments, &c., it is obvious that 117 and 118 will enable us to find the most probably correct or suitable value for the earth's eccentricity in the locality of the survey. And the great importance of having such a value of e will be obvious from the examples worked out in the sequel.

We can easily find other expressions for e^2 from 78 and 79, by substituting in (79) the values of $\frac{R_{\prime\prime}}{R_{\prime}}$ and $\frac{R_{\prime}}{R_{\prime\prime}}$ given in 51.

34. It may be seen, from a glance at the figure, that when the two stations have not the same latitude, a difference in the heights of the stations (with respect to the earth's spheroidal surface) will introduce errors into the observed values of the azimuths $A_{\prime}$, $A_{\prime\prime}$ and other azimuthal readings.

1°. It is evident that according as the station S_{00} is higher or lower than the station S_0 by the length h, so will the observed azimuth $A_{\prime}$ be too great or too small by an angle μ which the length expressed by $h \times \sin \triangle$ subtends at the distance s. And according as the station S_0 is higher or lower than the station S_{00} by the length h, so will the observed azimuth $A_{\prime\prime}$ be too small or too great by an angle μ which the length expressed by $h \times \sin \triangle$ subtends at the distance s.

2°. It is ∴ obvious that when the station S_0 is higher than the station S_{00} then will the azimuths $A_{\prime}$ and $A_{\prime\prime}$, as found by direct observation, be too small; and when the station S_{00} is higher than the station S_0 then will the azimuths $A_{\prime}$ and $A_{\prime\prime}$, as found by direct observation, be too large.

☞ To find the error of correction μ, we have—

$$\mu = \frac{h}{s} \cdot \triangle$$

Now, in an example given in the sequel, we have $s = 513{,}906$ feet, and $\triangle = 10''{\cdot}85$. And according as we suppose the station S_0 to be higher or lower than the station S_{00} by the length $h = 10{,}000$ feet, so will each of the azimuths $A_{\prime}$, $A_{\prime\prime}$, be too small or too great by

$$\mu = 0'' \cdot 211$$

35. We will now consider how the magnitude of the angle $\triangle$ varies when the stations $S_{\circ}$, $S_{\circ\circ}$, are supposed to be situated on two fixed parallels of latitude, and at such distances asunder as may or can occur in trigonometrical surveying.

From equation 100 we at once perceive that when the latitudes $l_{\prime}$, $l_{\prime\prime}$, are constants, the angle $\triangle$ between the normal-chordal planes increases or decreases according as

$$\cos^2 l_{\prime\prime} \sin^2 A_{\prime\prime} - \cos^2 l_{\prime} \sin^2 A_{\prime} \text{ increases or decreases.}$$

Or, if in this we substitute for $\sin^2 A_{\prime}$ its equivalent as given by equation 50, then we know that $\triangle$ increases or decreases according as the expression

$$\sin^2 A_{\prime\prime} \left(R^2_{\prime} \cos^2 l_{\prime\prime} - R^2_{\prime\prime} \cos^2 l_{\prime} \cdot \frac{\cos^2 \alpha_{\prime\prime}}{\cos^2 \alpha_{\prime}} \right) \text{ increases or decreases.}$$

Now $A_{\prime\prime}$ being the necessarily acute and lesser azimuth, we know that $\sin^2 A_{\prime\prime}$ increases as the azimuth $A_{\prime\prime}$ increases. And, since $\frac{\sin \alpha_{\prime\prime}}{\sin \alpha_{\prime}} = \frac{R_{\prime}}{R_{\prime\prime}}$ is constant, and that $\alpha_{\prime\prime}$ and $\alpha_{\prime}$ increase or decrease according as the difference of longitude ω increases or decreases, it is evident that $\frac{1 - \sin^2 \alpha_{\prime\prime}}{1 - \sin^2 \alpha_{\prime}}$ or $\frac{\cos^2 \alpha_{\prime\prime}}{\cos^2 \alpha_{\prime}}$ *decreases* according as the difference of longitude *increases*; and ∴ that $\triangle$ increases as ω and $A_{\prime\prime}$ increase up to that point at which the trace of the normal-chordal plane containing $R_{\prime\prime}$ touches the parallel of latitude on which $S_{\circ}$ is situated.

36. Other new and useful formulæ can be easily derived from the figure. For instance, from the spherical triangles $S_{\prime}PI$, $S_{\prime\prime}PI$,

$$\begin{aligned} \cos \theta &= \sin \alpha_{\prime} \sin l_{\prime} - \cos \alpha_{\prime} \cos l_{\prime} \cos A_{\prime} \\ \cos \theta &= - \sin \alpha_{\prime\prime} \sin l_{\prime\prime} + \cos \alpha_{\prime\prime} \cos l_{\prime\prime} \cos A_{\prime\prime} \end{aligned} \quad (119)$$

$$\begin{aligned} \therefore \sin \alpha_{\prime} \sin l_{\prime} + \sin \alpha_{\prime\prime} \sin l_{\prime\prime} = {} & \cos \alpha_{\prime} \cos l_{\prime} \cos A_{\prime} \\ & + \cos \alpha_{\prime\prime} \cos l_{\prime\prime} \cos A_{\prime\prime} \end{aligned} \quad (120)$$

and hence with close approximation to absolute accuracy, we have

$$\tan \alpha_{\prime} \sin l_{\prime} + \tan \alpha_{\prime\prime} \sin l_{\prime\prime} = \cos l_{\prime} \cos A_{\prime} + \cos l_{\prime\prime} \cos A_{\prime\prime}$$

but

$$\frac{\tan \alpha_{\prime}}{\tan \alpha_{\prime\prime}} = \frac{\cos l_{\prime} \sin A_{\prime}}{\cos l_{\prime\prime} \sin A_{\prime\prime}}$$

And from these we easily find

$$\begin{aligned} \tan \alpha_{\prime} &= \frac{\cos l_{\prime} \cos A_{\prime} + \cos l_{\prime\prime} \cos A_{\prime\prime}}{\cos l_{\prime} \sin l_{\prime} \sin A_{\prime} + \cos l_{\prime\prime} \sin l_{\prime\prime} \sin A_{\prime\prime}} \cdot \cos l_{\prime} \sin A_{\prime} \\ \tan \alpha_{\prime\prime} &= \frac{\cos l_{\prime} \cos A_{\prime} + \cos l_{\prime\prime} \cos A_{\prime\prime}}{\cos l_{\prime} \sin l_{\prime} \sin A_{\prime} + \cos l_{\prime\prime} \sin l_{\prime\prime} \sin A_{\prime\prime}} \cdot \cos l_{\prime\prime} \sin A_{\prime\prime} \end{aligned} \quad (121)$$

and ∴

$$\tan\tfrac{1}{2}\Sigma = \frac{\cos l_{\prime}\cos A_{\prime} + \cos l_{\prime\prime}\cos A_{\prime\prime}}{\sin 2l_{\prime}\sin A_{\prime} + \sin 2l_{\prime\prime}\sin A_{\prime\prime}} \cdot 2\sqrt{\cos l_{\prime}\cos l_{\prime\prime}\sin A_{\prime}\sin A_{\prime\prime}}$$

The expressions given for the tangents of the angles of depression of the geodesic chord in (110) and (111) implicate the assumed eccentricity of the earth, while the expressions (121) depend entirely on the observed latitudes and azimuths. If applied to the example 1 problem 1 given in the sequel (which may be regarded as an extreme case in trigonometrical surveying) it will be found that the resulting values of $\alpha_{\prime}$ and $\alpha_{\prime\prime}$ can be accurately determined to $\frac{1}{1000}$ part of one second,—their logs. holding true to 8 places of decimals.

By substituting in (111) the values $\frac{R_{\prime\prime}}{R_{\prime}}$ and $\frac{R_{\prime}}{R_{\prime\prime}}$ as given in (51), we easily rearrive at formulæ (121); and by like substitutions in (110), we easily find the following values for the tangents of the angles of depression of the chord — true to at least 8 places of decimals in their logs —

$$\tan\alpha_{\prime} = \frac{\sin A_{\prime\prime}}{\cos l_{\prime}\sin\omega} - \frac{\sin A_{\prime}\cot\omega + \cos A_{\prime}\sin l_{\prime}}{\cos l_{\prime}}$$

$$= \frac{\sin A_{\prime\prime}}{\cos l_{\prime}\sin\omega} - \cot z_{\prime}$$

$$\tan\alpha_{\prime\prime} = \frac{\sin A_{\prime}}{\cos l_{\prime\prime}\sin\omega} - \frac{\sin A_{\prime\prime}\cot\omega + \cos A_{\prime\prime}\sin l_{\prime\prime}}{\cos l_{\prime\prime}}$$

$$= \frac{\sin A_{\prime}}{\cos l_{\prime\prime}\sin\omega} - \cot z_{\prime\prime}$$

And when $\alpha_{\prime\prime}$ and $\alpha_{\prime}$ are found, we have $\Sigma = \alpha_{\prime\prime} + \alpha_{\prime}$.

However, there are other methods of finding approximate values of Σ, in terms of the latitudes, azimuths, and length of arc between the stations, &c.; but I defer their consideration for a future paper.

37. With respect to the figure it may be observed that if $F_{\prime}$ and $F_{\prime\prime}$ be the points in which the chordal plane $NS_{o}S_{oo}$ cuts the arcs $PS_{\prime}$, $PS_{\prime\prime}$, it is evident that the arc $PF_{\prime}$ is divided harmonically in $S_{\prime}$, $D_{\prime}$, and that the arc $PF_{\prime\prime}$ is divided harmonically in $D_{\prime\prime}$, $S_{\prime\prime}$. For the anharmonic ratio of the points $PF_{\prime}S_{\prime}D_{\prime}$ is the same as that of the pencil of straight lines $S_{o}\cdot(PF_{\prime}S_{\prime}D_{\prime})$, and ∴ the same as that of the four points ∞, N, Z_{o}, Z_{oo}, in which ∞ represents the point at infinity in which the line $S_{o}P$ cuts the line $CZ_{o}Z_{oo}$, &c. Hence the spherical pencil $I\cdot(PF_{\prime}S_{\prime}D_{\prime})$ is harmonic.

Again, since $S_{\circ}F_{\prime}$, $S_{\circ}F_{\prime\prime}$, $S_{\circ}O$, are parallels to $NS_{\circ}$, $NS_{\circ\circ}$, NM, it follows that the arc $F_{\prime}F_{\prime\prime}$ is bisected in O; and therefore (as arc IO is a quadrant) the arc IO is cut harmonically in $F_{\prime}$, $F_{\prime\prime}$; and the spherical pencil $P \cdot (IOF_{\prime}F_{\prime\prime})$ is harmonic.

NOTATION.

When any number n of stations are to be simultaneously considered.

Let 1, 2, 3, ... , n, indicate stations on the earth's surface.
,, l_1, l_2, l_3, ... , l_n, indicate the latitudes at these stations.
,, R_1, R_2, R_3, ... , R_n, ,, the normals terminating in polar axis.
,, $\omega_{12}, \omega_{23}, \omega_{34}$, ... ,, the differences of longitude between the pairs of stations 1, 2; 2, 3; 3, 4; . . . .
Put A_{12}, A_{21}, for the azimuths of the stations 2, 1, as if observed from 1 and 2.
,, A_{23}, A_{32}, for the azimuths of the stations 3, 2, as if observed from 2 and 3.
,, ...
,, ...
,, α_{12}, α_{21}, for the angles of depression of the chord 1, 2, at the stations 1 and 2.
,, α_{23}, α_{32}, for the angles of depression of the chord 2, 3, at the stations 2 and 3.
,, ...
,, ...
,, k_{12}, k_{23}, k_{34}, for the chords 1, 2; 2, 3; 3, 4; of the spheroidal triangle 1, 2, 3.
,, $\Sigma_{12}, \Sigma_{13}, \Sigma_{23}$, for the spherical measures $\alpha_{12} + \alpha_{21}$; $\alpha_{13} + \alpha_{31}$; $\alpha_{23} + \alpha_{32}$; of the sides of the spheroidal triangle 1, 2, 3.
,, s_{12}, s_{13}, s_{23}, for the lengths of the sides 1, 2; 1, 3; 2, 3; of the spheroidal triangle 1, 2, 3.

1. For any n stations 1, 2, 3, ... $n - 1$, n, on the earth's spheroidal surface, we have the rigorously accurate equations

$$\frac{R_2}{R_{\prime}} = \frac{\sin\alpha_{12}}{\sin\alpha_{21}}\,;\quad \frac{R_3}{R_2} = \frac{\sin\alpha_{23}}{\sin\alpha_{32}}\,;\ \ldots\ldots\ldots\ldots\ \frac{R_n}{R_{n-1}} = \frac{\sin\alpha_{n-1,n}}{\sin\alpha_{n,n-1}}$$

and ∴

$$\frac{R_n}{R_{\prime}} = \frac{\sin\alpha_{12}\cdot\sin\alpha_{23}\ \ldots\ldots\ldots\ldots\ \sin\alpha_{n-1,n}}{\sin\alpha_{21}\cdot\sin\alpha_{32}\ \ldots\ldots\ldots\ldots\ \sin\alpha_{n,n-1}} \qquad (123)$$

And putting M to represent the reciprocal of the dexter of this equation, we easily find—

$$\sin^2 l_n = \frac{1}{e^2} - \left(\frac{1}{e^2} - \sin^2 l_{,}\right) \cdot M^2 \qquad (124)$$

an equation expressing the latitude of the n^{th} station in terms of the latitude of the 1st station and the sines of the angles of depression of the $n - 1$ chords joining the consecutive stations.

2. We have also the rigorously accurate relations

$$\frac{R_2 \cos l_2}{R_{,} \cos l_{,}} = \frac{\sin A_{12} \cos \alpha_{12}}{\sin A_{21} \cos \alpha_{21}}; \quad \frac{R_3 \cos l_3}{R_2 \cos l_2} = \frac{\sin A_{23} \cos \alpha_{23}}{\sin A_{32} \cos \alpha_{32}};$$

$$\cdots\cdots\cdots = \cdots\cdots\cdots\cdots\cdots; \quad \cdots\cdots\cdots = \cdots\cdots\cdots\cdots\cdots;$$

and $\therefore$

$$\frac{R_n \cos l_n}{R_{,} \cos l_{,}} = \frac{\sin A_{12} \sin A_{23} \cdots\cdots}{\sin A_{21} \sin A_{32} \cdots\cdots} \cdot \frac{\cos \alpha_{12} \cos \alpha_{23} \cdots\cdots}{\cos \alpha_{21} \cos \alpha_{32} \cdots\cdots} \qquad (125)$$

$$= \frac{\sqrt{(1 - e^2) \tan^2 l_{,} + 1}}{\sqrt{(1 - e^2) \tan^2 l_n + 1}}$$

and from this we easily find—

$$\tan^2 l_n = \left(\tan^2 l_{,} + \frac{1}{1 - e^2}\right) \cdot \left(\frac{\sin A_{21} \cdot \sin A_{32} \cdots\cdots}{\sin A_{12} \cdot \sin A_{23} \cdots\cdots}\right)^2 \cdot \left(\frac{\cos \alpha_{21} \cdot \cos \alpha_{32} \cdots\cdots}{\cos \alpha_{12} \cdot \cos \alpha_{23} \cdots\cdots}\right)^2 - \frac{1}{1 - e^2}$$

an equation expressing the latitude of the n^{th} station in terms of the latitude of the 1st station, the azimuths, and the angles of depression of the chords connecting the stations.

3. And from (123) and (125) we have—

$$\frac{\cos l_n}{\cos l_{,}} = \frac{\sin A_{12} \cdot \sin A_{23} \cdots\cdots\cdots\cdots}{\sin A_{21} \cdot \sin A_{32} \cdots\cdots\cdots\cdots} \cdot \frac{\tan \alpha_{21} \cdot \tan \alpha_{32} \cdots\cdots\cdots\cdots}{\tan \alpha_{12} \cdot \tan \alpha_{23} \cdots\cdots\cdots\cdots} \qquad (127)$$

4. Let 1, 2, 3, ... $n - 1$, n, be any *odd* number of stations on the earth's spheroidal surface, such that none of the chords (12), (23), ... $(n - 1, n)$, exceeds 100 miles in length. Then, from formula 49, it is evident we have the relations—

$$\frac{\tan (45^\circ - \frac{1}{2} l_{,})}{\tan (45^\circ - \frac{1}{2} l_3)} = \frac{\cos \frac{1}{2} (A_{12} + A_{21} + \omega_{12})}{\cos \frac{1}{2} (A_{12} + A_{21} - \omega_{12})} \div \frac{\cos \frac{1}{2} (A_{23} + A_{32} + \omega_{23})}{\cos \frac{1}{2} (A_{23} + A_{32} - \omega_{23})}$$

$$\frac{\tan\left(45^\circ-\frac{1}{2}l_3\right)}{\tan\left(45^\circ-\frac{1}{2}l_5\right)}=\frac{\cos\frac{1}{2}\left(A_{34}+A_{43}+\omega_{34}\right)}{\cos\frac{1}{2}\left(A_{34}+A_{43}-\omega_{34}\right)}\div\frac{\cos\frac{1}{2}\left(A_{45}+A_{54}+\omega_{45}\right)}{\cos\frac{1}{2}\left(A_{45}+A_{54}-\omega_{45}\right)} \qquad (128)$$

$$\cdots\cdots\cdots\cdots = \cdots\cdots\cdots\cdots \div \cdots\cdots\cdots\cdots$$

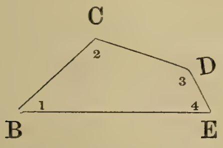
$$\frac{\tan\left(45^\circ-\frac{1}{2}l_{n-2}\right)}{\tan\left(45^\circ-\frac{1}{2}l_n\right)}=\frac{\cos\frac{1}{2}\left(\;.\;.\;.\;.\;.\;\right)}{\cos\frac{1}{2}\left(\;.\;.\;.\;.\;.\;\right)}\div\frac{\cos\frac{1}{2}\left(\;.\;.\;.\;.\;.\;\right)}{\cos\frac{1}{2}\left(\;.\;.\;.\;.\;.\;\right)}$$

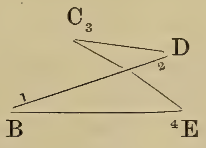
And therefore we have—

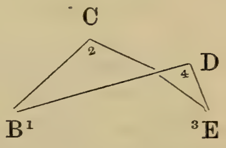
$$\frac{\tan\left(45^\circ-\frac{1}{2}l_1\right)}{\tan\left(45^\circ-\frac{1}{2}l_n\right)}=\text{the product of the dexters of these }\frac{n-1}{2}\text{ equations,}$$

an equation from which we can at once express the latitude of the n^{th} station in terms of the latitude of the 1st station and the azimuths and differences of longitudes.

Should the n^{th} station be coincident with the 1st station, we must have the dexter of (129) equal to unity. This fact will be found to be of importance in case any *even* number of stations form the vertices of a closed geodesic polygon. For instance, if there be *four* mutually visible stations such as B, C, D, E—







then numbering the stations in the orders indicated in the above diagrams, we have—

$$\frac{\cos\frac{1}{2}\left(A_{12}+A_{21}+\omega_{12}\right)}{\cos\frac{1}{2}\left(A_{12}+A_{21}-\omega_{12}\right)}\cdot\frac{\cos\frac{1}{2}\left(A_{34}+A_{43}+\omega_{34}\right)}{\cos\frac{1}{2}\left(A_{34}+A_{43}-\omega_{34}\right)}$$

$$=\frac{\cos\frac{1}{2}\left(A_{23}+A_{32}+\omega_{23}\right)}{\cos\frac{1}{2}\left(A_{23}+A_{32}-\omega_{23}\right)}\cdot\frac{\cos\frac{1}{2}\left(A_{41}+A_{14}+\omega_{41}\right)}{\cos\frac{1}{2}\left(A_{41}+A_{14}-\omega_{41}\right)}$$

corresponding to the stations taken in each of the three indicated orders. And in the case of any such even number n of stations (the first and last of which are coincident) it is obvious that if all the azimuths be known, and that all the differences of longitude with the exception of any two which are consecutive be known, then we can easily (by solving a quadratic equation) express the tangent of either of these two differences of longitude in terms of the known azimuths and differences of longitude.

5. With respect to any *three* mutually visible stations 1, 2, 3, we can easily arrive at convenient expressions for each of their latitudes in terms of their azimuths and differences of longitude. Thus—

We have (49) and (128)—

$$\tan\left(45^\circ - \tfrac{1}{2}\,l_1\right) \cdot \tan\left(45^\circ - \tfrac{1}{2}\,l_2\right) = -\frac{\cos\frac{1}{2}\left(A_{12} + A_{21} + \omega_{12}\right)}{\cos\frac{1}{2}\left(A_{12} + A_{21} - \omega_{12}\right)}$$

$$\frac{\tan\left(45^\circ - \frac{1}{2}\,l_1\right)}{\tan\left(45^\circ - \frac{1}{2}\,l_2\right)} = \frac{\cos\frac{1}{2}\left(A_{13} + A_{31} + \omega_{13}\right)}{\cos\frac{1}{2}\left(A_{13} + A_{31} - \omega_{13}\right)} \div \frac{\cos\frac{1}{2}\left(A_{32} + A_{23} + \omega_{32}\right)}{\cos\frac{1}{2}\left(A_{32} + A_{23} - \omega_{32}\right)}$$

$\therefore$

$$\left.\begin{aligned}
\tan^2\left(45^\circ - \tfrac{1}{2}\,l_1\right) &= -\frac{\cos\frac{1}{2}\left(A_{12} + A_{21} + \omega_{12}\right)}{\cos\frac{1}{2}\left(A_{12} + A_{21} - \omega_{12}\right)} \\
&\quad \cdot \frac{\cos\frac{1}{2}\left(A_{13} + A_{31} + \omega_{13}\right)}{\cos\frac{1}{2}\left(A_{13} + A_{31} - \omega_{13}\right)} \div \frac{\cos\frac{1}{2}\left(A_{23} + A_{32} + \omega_{23}\right)}{\cos\frac{1}{2}\left(A_{23} + A_{32} - \omega_{23}\right)} \\
\tan^2\left(45^\circ - \tfrac{1}{2}\,l_2\right) &= -\frac{\cos\frac{1}{2}\left(A_{23} + A_{32} + \omega_{23}\right)}{\cos\frac{1}{2}\left(A_{23} + A_{32} - \omega_{23}\right)} \\
&\quad \cdot \frac{\cos\frac{1}{2}\left(A_{21} + A_{12} + \omega_{21}\right)}{\cos\frac{1}{2}\left(A_{21} + A_{12} - \omega_{21}\right)} \div \frac{\cos\frac{1}{2}\left(A_{31} + A_{13} + \omega_{31}\right)}{\cos\frac{1}{2}\left(A_{31} + A_{13} - \omega_{31}\right)} \\
\tan^2\left(45^\circ - \tfrac{1}{2}\,l_3\right) &= -\frac{\cos\frac{1}{2}\left(A_{31} + A_{13} + \omega_{31}\right)}{\cos\frac{1}{2}\left(A_{31} + A_{13} - \omega_{31}\right)} \\
&\quad \cdot \frac{\cos\frac{1}{2}\left(A_{32} + A_{23} + \omega_{32}\right)}{\cos\frac{1}{2}\left(A_{32} + A_{23} - \omega_{32}\right)} \div \frac{\cos\frac{1}{2}\left(A_{12} + A_{21} + \omega_{21}\right)}{\cos\frac{1}{2}\left(A_{12} + A_{21} - \omega_{21}\right)}
\end{aligned}\right\} \qquad (131)$$

These equations are closely approximate to rigorous accuracy, even when the stations are from 100 to 200 miles asunder.

6. Let ①, ②, ③, be any three stations on the earth's spheroidal surface. Then if K_1, K_2, K_3, indicate the angles between the chords joining the stations which have their vertices in ①, ②, ③, respectively; and that C_1, C_2, C_3, indicate the corresponding angles of the geodesic triangle formed by the geodesic arcs connecting the stations; we have evidently

$$\left.\begin{aligned}
\cos C_1 &= \frac{\cos K_1}{\cos \alpha_{13}\ \cos \alpha_{12}} - \tan \alpha_{13} \cdot \tan \alpha_{12} \\
\cos C_2 &= \frac{\cos K_2}{\cos \alpha_{21}\ \cos \alpha_{23}} - \tan \alpha_{21} \cdot \tan \alpha_{23} \\
\cos C_3 &= \frac{\cos K_3}{\cos \alpha_{32}\ \cos \alpha_{31}} - \tan \alpha_{32} \cdot \tan \alpha_{31}
\end{aligned}\right\} \qquad (132)$$

If it were possible (and it is usually supposed so in applying LEGENDRE'S and DELAMBRE'S processes in the solution of questions pertaining to the spheroidal triangles of a trigonometrical survey) to find a sphere such that a spherical triangle described on its surface can have sides equals in length to the sides of a spheroidal triangle, and chords equal to the chords of the spheroidal triangle; then, it is obvious that by putting D_1, D_2, D_3, for the angles of this spherical triangle which correspond to the angles K_1, K_2, K_3, of the chordal triangle, we should have—

$$\left.\begin{aligned}
\cos D_1 &= \frac{\cos K_1}{\begin{array}{r}\cos\frac{1}{2}(\alpha_{13}+\alpha_{31})\cdot\cos\frac{1}{2}(\alpha_{12}+\alpha_{21})\\ -\tan\frac{1}{2}(\alpha_{13}+\alpha_{31})\cdot\tan\frac{1}{2}(\alpha_{12}+\alpha_{21})\end{array}}\\
\cos D_2 &= \frac{\cos K_2}{\begin{array}{r}\cos\frac{1}{2}(\alpha_{21}+\alpha_{12})\ \cos\frac{1}{2}(\alpha_{23}+\alpha_{32})\\ -\tan\frac{1}{2}(\alpha_{21}+\alpha_{12})\cdot\tan\frac{1}{2}(\alpha_{23}+\alpha_{32})\end{array}}\\
\cos D_3 &= \frac{\cos K_3}{\begin{array}{r}\cos\frac{1}{2}(\alpha_{32}+\alpha_{23})\ \cos\frac{1}{2}(\alpha_{31}+\alpha_{13})\\ -\tan\frac{1}{2}(\alpha_{32}+\alpha_{23})\cdot\tan\frac{1}{2}(\alpha_{31}+\alpha_{13})\end{array}}
\end{aligned}\right\}(133)$$

By comparing the values of the angles D_1, D_2, D_3, of the imaginary spherical triangle as given in the formulæ (133), with the correct values of the corresponding angles C_1, C_2, C_3, of the spheroidal triangle as given in formulæ (132), it is evident that, with due respect to the utmost accuracy required in practice, we have—

$$\left.\begin{aligned}
\cos C_1 - \cos D_1 &= \tan\tfrac{1}{2}(\alpha_{13}+\alpha_{31})\tan\tfrac{1}{2}(\alpha_{12}+\alpha_{21})\\
&\qquad -\tan\alpha_{13}\tan\alpha_{12}\\
\cos C_2 - \cos D_2 &= \tan\tfrac{1}{2}(\alpha_{21}+\alpha_{12})\tan\tfrac{1}{2}(\alpha_{23}+\alpha_{32})\\
&\qquad -\tan\alpha_{21}\tan\alpha_{23}\\
\cos C_3 - \cos D_3 &= \tan\tfrac{1}{2}(\alpha_{32}+\alpha_{23})\tan\tfrac{1}{2}(\alpha_{31}+\alpha_{13})\\
&\qquad -\tan\alpha_{32}\tan\alpha_{31}
\end{aligned}\right\}(134)$$

their logs being the same to at least 8 or 9 places of decimals.

From these it is evident that cases may occur in geodetic surveying in which one of the angles of the spherical triangle is greater than the corresponding angle of the spheroidal triangle, and that another angle of the spherical triangle is less than its corresponding angle of the spheroidal triangle.

However the differences are very small indeed. As an instance we may consider the large spheroidal triangle of article 7, page 234, of the "Account of the Principal Triangulation of Great Britain and Ireland." Here we find that at the station whose latitude is 53° 30′, the spheroidal angle exceeds the corresponding angle of the Legendre spherical triangle by about $\frac{2}{100}$ of a second; and, although such may be disregarded in actual practice, it is nevertheless obvious that the usual method of manipulating the measured angles of a spheroidal triangle (by means of Legendre's theorem, so as to have their sum give the desired spherical excess) is erroneous in principle.

NOTES.

It is easy to perceive that the principal theorems arrived at apply to any surface whatever as well as to the surface of the spheroidal earth, even when such surface is so irregular as to be inexpressible by means of an equation.

We can assume any straight line cutting the normals to the surface at the stations S_0, S_{00}, as polar axis of reference; and then, assuming any point C_0 in this polar axis as centre of reference, we can take the plane through it perpendicular to the axis as the equatorial plane of reference. Thus the figure can be constructed as already indicated in the case in which the surface is a spheroid; and we have formulæ (50), &c.

When the stations S_0, S_{00}, are so near to each other as to permit us to regard the normals as making angles with the chord such that the ratio of their sines can be regarded as equal to unity, and the traces of the normal-chordal planes as equals in length and circular measure, we have—

$$\tan \tfrac{1}{2}\, \omega = \frac{\cos \frac{1}{2}\,(l_{\prime} - l_{\prime\prime})}{\sin \frac{1}{2}\,(l_{\prime} + l_{\prime\prime})} \cdot \cot \tfrac{1}{2}\,(A_{\prime} + A_{\prime\prime})$$

$$\tan \tfrac{1}{2}\, l' \cdot \tan \tfrac{1}{2}\, l'' = -\, \frac{\cos \frac{1}{2}\,(A_{\prime} + A_{\prime\prime} + \omega)}{\cos \frac{1}{2}\,(A_{\prime} + A_{\prime\prime} - \omega)}$$

$$\frac{\sin A_{\prime}}{\sin A_{\prime\prime}} = \frac{R_{\prime\prime} \cos l_{\prime\prime}}{R_{\prime} \cos l_{\prime}}$$

and all the formulæ not implicating peculiar properties of the spheroid. If there be three stations to be simultaneously considered, the assumable position for the polar axis of reference is generally restricted, as such axis must cut the three normals to the surface drawn through the stations.

If the three normals intersect in one point, any line through this point can be assumed as polar axis. If two of the normals cut each other, and that neither of them is cut by the third, then the polar axis must pass through the point of intersection and lie in the plane of this point and the third normal. If the three normals have no point of intersection, then the polar axis must lie in the surface of a ruled quadric, &c.

And when there are four stations, then should no two of the four normals lie in one plane, there can be but two transversals drawn to cut them, and therefore but two positions for the polar axis. However, with respect to all surfaces of revolution (whose normals must all cut the axis) we can arrive at general theorems applying to any stations whatever on the surface.

For instance, we can easily demonstrate the following

THEOREM.

If ①, ⓝ, be any two stations on a surface of revolution of any kind, and $A_{1,2}$, $A_{n,n-1}$, the angles which the *true* "geodesic" joining the stations makes with the traces of the meridian planes through the stations, and that R_1, R_n, are the normals terminating in the axis, then will

$$\frac{\sin A_{1,2}}{\sin A_{n,n-1}} = \frac{R_n \cos l_n}{R_1 \cos l_1}.$$

Conceive the "geodesic" to be divided into infinitesimally small parts or elements, 1, 2; 2, 3; 3, 4; . . . . . $n - 2$, $n - 1$; $n - 1$, n.

Let A_{12}, A_{23}, A_{34}, . . . $A_{n-1,n}$ represent the azimuths of the stations

②, ③, ④, . . . ⓝ as if taken at the stations

①, ②, ③, . . . $n-1$ respectively.

Let A_{21}, A_{32}, A_{43}, . . . $A_{n,n-1}$ represent the azimuths of the stations

①, ②, ③, . . . $n-1$ as if taken at the stations

②, ③, ④, . . . ⓝ respectively.

Let R_1, R_2, ... R_n be the lengths of the normals at stations

①, ②, ... ⓝ respectively.

Then from the elements of analytic geometry, we know that the tangent lines to any infinitesimally small arc of the *first* order, which forms part of a geodesic, have their least distance apart an infinitesimally small of the *third* order; and that the ratio of the lengths of these tangents, from the points of contact to their points of least distance from each other, is that of equality. We know also that the plane of every two consecutive elements of any "geodesic" contains the normal at their point of junction; and ∴ that $\sin A_{21} = \sin A_{23}$; $\sin A_{32} = \sin A_{34}$; . . . . ; moreover, we know that the ratio of the cosines of all infinitesimally small arcs is unity. Hence we have—

$$\frac{\sin A_{12}}{\sin A_{21}} = \frac{R_2 \cos l_2}{R_1 \cos l_1}$$

$$\frac{\sin A_{23}}{\sin A_{32}} = \frac{R_3 \cos l_3}{R_2 \cos l_2}$$

$$\cdots\cdots\cdots = \cdots\cdots\cdots$$

$$\cdots\cdots\cdots = \cdots\cdots\cdots$$

And from these we at once obtain the desired proof, by equating the product of the first sides of the equations to the product of their second sides.

However, it may be proper to observe that this method of proof holds good only when none of the normals R_1, R_1, . . . R_n, is either $= 0$ or $= \infty$; and that we shall suppose this to be the case for all geodesies referred to in the present paper. We may evidently write the above relation in the form—

$$\frac{\sin A_{1,2}}{\sin A_{n,n-1}} = \frac{\text{perpendicular from } \textcircled{n} \text{ to polar axis}}{\text{perpendicular from } \textcircled{1} \text{ to polar axis}}$$

Or we may express it in words as follows:—

THEOREM.

On any surface of revolution, the sines of the angles $G_{,}$, $G_{,,}$, which the geodesic connecting two stations S_{o}, S_{oo}, makes with the meridian traces through these stations are to each other inversely as the perpendiculars from the stations to the polar axis.

For a spheroid, such as the earth's reputed surface, we can prove, in like manner, that for any two stations whatever on its surface—

$$\frac{\sin^2 A_1}{\sin^2 A_n} = \frac{\tan^2 l_1 + \frac{a^2}{b^2}}{\tan^2 l_n + \frac{a^2}{b^2}} = \frac{\tan^2 l_1 + 1{\cdot}0068314987}{\tan^2 l_n + 1{\cdot}0068314987}$$

in which A_1, $A_{,,}$ are the angles which the true "geodesic" joining the stations makes with the meridian traces through the stations, &c.

☞ The theorem expressed by formula 10, may be expressed as follows:—

The plane perpendicular to any chord of a quadric of revolution through its middle point, bisects the portion of the axis intercepted by the normals drawn through the extremities of the chord; and the straight line joining the middle of the chord to the point in which the plane cuts the axis is divided by the equatorial plane of the surface into portions whose ratio is the same as those into which it divides either normal terminating in the axis.

From this we at once perceive that—

The perpendicular bisecting any chord of a conic bisects the portions of the axes intercepted by the normals drawn through the extremities of the chord; and that the ratio of the portions of the perpendicular measured from the middle point of the chord to its intersections with the axes, is the same as the ratio of the segments of either of the normals measured from the curve to the axes.

Problem 1.

Given the latitudes $l_{,}$, $l_{,,}$ of two stations S_{o}, S_{oo} (on the earth's spheroidal surface), and their difference of longitude ω; to find the azimuths $A_{,}$, $A_{,,}$; the circular measure Σ and length s of the geodesic arc between the stations; the angles $a_{,}$, $a_{,,}$ of depression of the chord, &c.

First Method.

To find the arcs L', L'', and the azimuths $A_{,}$, $A_{,,}$ we have—

$$\cot L' = e^2 \cdot \frac{R_{,,} \sin l_{,,}}{R_{,} \cos l_{,}} + (1 - e^2) \tan l_{,}$$

$$\cot L'' = e^2 \cdot \frac{R_{,} \sin l_{,}}{R_{,,} \cos l_{,,}} + (1 - e^2) \tan l_{,,}$$

$$\cot A_{,} = \frac{\cot L'' \cos l_{,} - \sin l_{,} \cos \omega}{\sin \omega}$$

$$\cot A_{,,} = \frac{\cot L' \cos l_{,,} - \sin l_{,,} \cos \omega}{\sin \omega}$$

or having found the arcs L', L'', as above indicated, we can find the azimuths and the angles $D_{\prime}$, $D_{\prime\prime}$, by means of the formulæ—

$$\tan \tfrac{1}{2}(A_{\prime} + D_{\prime\prime}) = \frac{\cos \frac{1}{2}(L'' - l')}{\cos \frac{1}{2}(L'' + l')} \cdot \cot \tfrac{1}{2}\omega$$

$$\tan \tfrac{1}{2}(A_{\prime} - D_{\prime\prime}) = \frac{\sin \frac{1}{2}(L'' - l')}{\sin \frac{1}{2}(L'' + l')} \cdot \cot \tfrac{1}{2}\omega$$

$$\tan \tfrac{1}{2}(D_{\prime} + A_{\prime\prime}) = \frac{\cos \frac{1}{2}(l'' - L')}{\cos \frac{1}{2}(l'' + L')} \cdot \cot \tfrac{1}{2}\omega$$

$$\tan \tfrac{1}{2}(D_{\prime} - A_{\prime\prime}) = \frac{\sin \frac{1}{2}(l'' - L')}{\sin \frac{1}{2}(l'' + L')} \cdot \cot \tfrac{1}{2}\omega$$

To find $\alpha_{\prime}$, $\alpha_{\prime\prime}$, Σ, $z_{\prime}$, $z_{\prime\prime}$, and s, we may proceed as follows:— First we find $\delta_{\prime}$, $\delta_{\prime\prime}$, from

$$\delta_{\prime} = L' - l'$$

$$\delta_{\prime\prime} = l'' - L''$$

Then from the triangles $S_{\prime}ID_{\prime}$, $S_{\prime\prime}ID_{\prime\prime}$, we have, to find $IS_{\prime}$, $ID_{\prime}$, $IS_{\prime\prime}$, $ID_{\prime\prime}$—

$$\tan \tfrac{1}{2}(IS_{\prime} + ID_{\prime}) = \frac{\sin \frac{1}{2}(D_{\prime} + A_{\prime})}{\sin \frac{1}{2}(D_{\prime} - A_{\prime})} \cdot \tan \tfrac{1}{2}\delta_{\prime}$$

$$\tan \tfrac{1}{2}(IS_{\prime} - ID_{\prime}) = \frac{\cos \frac{1}{2}(D_{\prime} + A_{\prime})}{\cos \frac{1}{2}(D_{\prime} - A_{\prime})} \cdot \tan \tfrac{1}{2}\delta_{\prime}$$

$$\tan \tfrac{1}{2}(IS_{\prime\prime} + ID_{\prime\prime}) = \frac{\sin \frac{1}{2}(A_{\prime\prime} + D_{\prime\prime})}{\sin \frac{1}{2}(A_{\prime\prime} - D_{\prime\prime})} \cdot \tan \tfrac{1}{2}\delta_{\prime\prime}$$

$$\tan \tfrac{1}{2}(IS_{\prime\prime} - ID_{\prime\prime}) = \frac{\cos \frac{1}{2}(A_{\prime\prime} + D_{\prime\prime})}{\cos \frac{1}{2}(A_{\prime\prime} - D_{\prime\prime})} \cdot \tan \tfrac{1}{2}\delta_{\prime\prime}$$

Then—

$$\alpha_{\prime} = 90^\circ - IS_{\prime}$$

$$\alpha_{\prime\prime} = IS_{\prime\prime} - 90^\circ$$

$$\Sigma = \alpha_{\prime} + \alpha_{\prime\prime}$$

$$z_{\prime} = ID_{\prime\prime} - IS_{\prime}$$

$$z_{\prime\prime} = IS_{\prime\prime} - ID_{\prime}$$

$$s = z_{\prime} \cdot R_{\prime} \cdot \sin 1'' = z_{\prime\prime} \cdot R_{\prime\prime} \cdot \sin 1''$$

But we can find k and s otherwise, thus—

$$k = \frac{R_{\prime} \cos l_{\prime} \sin \omega}{\sin A_{\prime\prime} \cos \alpha_{\prime\prime}} = \frac{R_{\prime\prime} \cos l_{\prime\prime} \sin \omega}{\sin A_{\prime} \cos \alpha_{\prime}}$$

$$s = k \cdot \frac{\Sigma \cdot \sin 1''}{2 \cdot \sin \frac{1}{2}\Sigma}$$

Or having found k, in terms of the given data, from

$$k^2 = (R_{\prime} \cos l_{\prime})^2 + (R_{\prime\prime} \cos l_{\prime\prime})^2 - 2 \cdot R_{\prime} \cdot R_{\prime\prime} \cos l_{\prime} \cos l_{\prime\prime} \cos \omega + (1 - e^2)^2 \cdot (R_{\prime} \sin l_{\prime} - R_{\prime\prime} \sin l_{\prime\prime})^2$$

we can find the angles of depression $a_{,}$, $a_{,,}$ by means of (109), and then find the azimuths from

$$\sin A_{,} = \frac{R_{,,} \cos l_{,,} \cos \omega}{k \cdot \cos a_{,}}$$

$$\sin A_{,,} = \frac{R_{,} \cos l_{,} \cos \omega}{k \cdot \cos a_{,,}}$$

☞ When $A_{,}$ or $A_{,,}$ is found to be nearly 90°, it cannot be accurately obtained by means of the usual tables of logarithms; so that, in such case, it is necessary to proceed as indicated in the works on trigonometry. Thus, putting A for the angle to be found, and N for the value of the function to which sin A is equated (which is nearly equal to 1), we have—

$$\sin (45^\circ - \tfrac{1}{2} A) = \sqrt{\frac{1 - N}{2}}$$

or,

$$\tan (45^\circ - \tfrac{1}{2} A) = \sqrt{\frac{1 - N}{1 + N}}$$

from which to compute the value of the angle A.

And when, in the sequel, an angle is to be found from an expression for its sine which is nearly equal to unity; then, putting N to represent such expression, we should proceed to find the angle by these formulæ.

Otherwise.

(When the stations are not more than 40 miles asunder.)

From the spherical triangle $S_{,}PS_{,,}$ we have the formulæ—

$$\tan \tfrac{1}{2} (A_{\circ} + A_{\circ\circ}) = \frac{\cos \frac{1}{2} (l'' - l')}{\cos \frac{1}{2} (l' + l'')} \cdot \cot \tfrac{1}{2} \omega$$

$$\tan \tfrac{1}{2} (A_{\circ} - A_{\circ\circ}) = \frac{\sin \frac{1}{2} (l'' - l')}{\sin \frac{1}{2} (l'' + l')} \cdot \cot \tfrac{1}{2} \omega$$

$$\sin \nu = \frac{\sin l' \sin \omega}{\sin A_{\circ\circ}} = \frac{\sin l'' \sin \omega}{\sin A_{\circ}}$$

Then to find the azimuths we have—

$$\tan x = \frac{R_{,,} \sin l''}{R_{,} \sin l'}$$

$$\tan \tfrac{1}{2} (A_{,} - A_{,,}) = \tan \tfrac{1}{2} (A_{\circ} + A_{\circ\circ}) \tan (x - 45^\circ)$$

$$\tfrac{1}{2} (A_{,} + A_{,,}) = \tfrac{1}{2} (A_{\circ} + A_{\circ\circ})$$

To find Ω, Σ, and the angle $\triangle$, we have—

$$\Omega = A_{\circ} - A_{,} = A_{,,} - A_{\circ\circ}$$

$$\tan \tfrac{1}{2} \Sigma = \tan \tfrac{1}{2} \nu \cos \Omega, \text{ or } \Sigma = \nu \cdot \cos \Omega$$

$$\triangle = 2 \cdot \Omega \cdot \sin \tfrac{1}{2} \Sigma, \text{ or } \triangle = \Omega \cdot \Sigma \cdot \sin 1''$$

To find the length k of the geodesic chord between the stations—

$$k = \frac{R_{,} \sin l' \sin \omega}{\sin A_{,,} \cos \frac{1}{2} \Sigma} = \frac{R_{,,} \sin l'' \sin \omega}{\sin A_{,} \cos \frac{1}{2} \Sigma}$$

Then to find s, we have—

$$s = \frac{k \cdot \Sigma'' \cdot \sin 1''}{2 \cdot \sin \frac{1}{2} \Sigma}$$

And to find the angles $a_{,,}$, $a_{,}$ of depression of the chord k below the tangent planes to the earth at the stations $S_{\circ\circ}$, $S_{\circ}$, we have—

$$\tan y = \frac{R_{,}}{R_{,,}}$$

$$(a_{,,} - a_{,}) = (y - 45^{\circ}) \cdot \Sigma \cdot \sin 1''$$

$$(a_{,,} + a_{,}) = \Sigma.$$

PROBLEM 2.

Given the latitude $l_{,}$, the azimuth $A_{,}$, and the length s and circular measure Σ of the geodesic arc between the stations; to find the latitude $l_{,,}$, the azimuth $A_{,,}$, the difference of longitude ω, &c.

First Method.

To find the angle $\phi_{,}$, we have, from the spherical triangle $PS_{,}I$—

$$\tan \tfrac{1}{2} (\phi_{,} + \beta_{,}) = \frac{\cos \frac{1}{2} (l_{,} - \frac{1}{2} \Sigma)}{\sin \frac{1}{2} (l_{,} + \frac{1}{2} \Sigma)} \cdot \tan \tfrac{1}{2} A_{,}$$

$$\tan \tfrac{1}{2} (\phi_{,} - \beta_{,}) = \frac{\sin \frac{1}{2} (l_{,} - \frac{1}{2} \Sigma)}{\cos \frac{1}{2} (l_{,} + \frac{1}{2} \Sigma)} \cdot \tan \tfrac{1}{2} A_{,}$$

☞ It may be proper to observe that $\frac{1}{2} \Sigma$ is used in these formulas instead of the angle $a_{,}$ of depression of the chord; but as the difference of these will in all actual cases be less than $\frac{1}{10}$ of a second, and that the numerators vary as the denominators when $\frac{1}{2} \Sigma$ varies in value, and that any variation in $\frac{1}{2} \Sigma$ which increases or decreases $\frac{1}{2} (\phi_{,} + \beta_{,})$ will decrease or increase $\frac{1}{2} (\phi_{,} - \beta_{,})$; $\therefore$, as respects the value of

$\phi_{,} = \frac{1}{2}(\phi_{,,} + \beta_{,}) + \frac{1}{2}(\phi_{,} - \beta_{,})$, there can be no appreciable difference whether we use $\frac{1}{2}\Sigma$ or $\alpha_{,}$.

Find the chord k by means of the usual formula—

$$k = \frac{2 \cdot s \cdot \sin \frac{1}{2}\Sigma}{\Sigma \cdot \sin 1''}.$$

Then, to find the difference of longitude ω, and the angle $\phi_{,,}$ by means of the plane triangle $p_{,}C_{,}p_{,,}$, we have—

$$\tan h_{,} = R_{,} \cos l_{,}\,; \quad \tan h_{,,} = \frac{k \cdot \sin A_{,} \cos \frac{1}{2}\Sigma}{\sin \phi_{,}}$$

$$\tfrac{1}{2}(\phi_{,,} + \omega) = 90^\circ - \tfrac{1}{2}\phi_{,}$$

$$\tan \tfrac{1}{2}(\phi_{,,} - \omega) = \frac{\sin (h_{,,} - h_{,})}{\sin (h_{,,} + h_{,})} \cdot \cot \tfrac{1}{2}\phi_{,}$$

Then to find the azimuth $A_{,,}$ and latitude $l_{,,}$, we have—

$$\sin A_{,,} = \frac{\sin \phi_{,,}}{\sin \phi_{,}} \cdot \sin A_{,}$$

$$\tan \tfrac{1}{2} l'' = -\frac{\cos \frac{1}{2}(A_{,} + A_{,,} + \omega)}{\cos \frac{1}{2}(A_{,} + A_{,,} - \omega)} \cdot \cot \tfrac{1}{2} l'$$

☞ If instead of $l_{,}$, $A_{,}$, we were given $l_{,,}$, $A_{,,}$, we should first proceed to find the angle $\phi_{,,}$ by means of—

$$\tan \tfrac{1}{2}(\phi_{,,} + \beta_{,,}) = \frac{\cos \frac{1}{2}(l_{,,} - \frac{1}{2}\Sigma)}{\sin \frac{1}{2}(l_{,,} + \frac{1}{2}\Sigma)} \cdot \tan \tfrac{1}{2} A_{,,}$$

$$\tan \tfrac{1}{2}(\phi_{,,} - \beta_{,,}) = \frac{\sin \frac{1}{2}(l_{,,} - \frac{1}{2}\Sigma)}{\cos \frac{1}{2}(l_{,,} + \frac{1}{2}\Sigma)} \cdot \tan \tfrac{1}{2} A_{,,}$$

and then proceed in an analogous manner to find $\phi_{,}$, ω, $A_{,}$, and $l_{,,}$.

Otherwise (Case 1st).

Given $l_{,}$, $A_{,}$, s; to find ω, $l_{,,}$, and $A_{,,}$ (see foot-note).

To find $z_{,}$, $D_{,,}$, ω, and L'', we have—

$$z_{,} = \frac{s}{R_{,} \sin 1''}$$

$$\tan \tfrac{1}{2}(D_{,,} + \omega) = \frac{\cos \frac{1}{2}(l' - z_{,})}{\cos \frac{1}{2}(l' + z_{,})} \cdot \cot \tfrac{1}{2} A_{,}$$

$$\tan \tfrac{1}{2}(D_{,,} - \omega) = \frac{\sin \frac{1}{2}(l' - z_{,})}{\sin \frac{1}{2}(l' + z_{,})} \cdot \cot \tfrac{1}{2} A_{,}$$

$$\tan \tfrac{1}{2}(L'' - l') = \frac{\sin \frac{1}{2}(A_{,} - D_{,,})}{\sin \frac{1}{2}(A_{,} + D_{,,})} \cdot \tan \tfrac{1}{2} z_{,}$$

or,

$$\sin L'' = \frac{\sin l' \sin A_{,}}{\sin D_{,,}}.$$

Then to find $\delta_{\prime\prime}$, $l_{\prime\prime}$, and $A_{\prime\prime}$, we have—

$$\delta_{\prime\prime} = \left(\frac{e^2}{1 - e^2}\right) \cdot \sin L'' \sin \tfrac{1}{2} (L'' + l') \cdot (L'' - l')$$

$$l_{\prime\prime} = 90^\circ - (L'' + \delta_{\prime\prime})$$

$$A_{\prime\prime} - D_{\prime\prime} = \sin D_{\prime\prime} \cdot \tan \tfrac{1}{2} z_{\prime} \cdot \delta_{\prime\prime}$$

☞ This case, in which the given latitude $l_{\prime}$ is greater than the sought latitude $l_{\prime\prime}$, is made known to us by the given azimuth $A_{\prime}$ being greater than the computed angle $D_{\prime\prime}$. And as we must have (see formulæ 21) the sought azimuth $A_{\prime\prime}$ also greater than the angle $D_{\prime\prime}$ it is evident that by putting ζ to represent the excess, we have—

$$\tan \tfrac{1}{2} (A_{\prime\prime} + \omega - \zeta) = \frac{\cos \frac{1}{2} (l' - z_{\prime})}{\cos \frac{1}{2} (l' + z_{\prime})} \cdot \cot \tfrac{1}{2} A_{\prime}$$

$$\tan \tfrac{1}{2} (A_{\prime\prime} - \omega - \zeta) = \frac{\sin \frac{1}{2} (l' - z_{\prime})}{\sin \frac{1}{2} (l' + z_{\prime})} \cdot \cot \tfrac{1}{2} A_{\prime}$$

shewing that the formulæ given in the "Account of the Principal Triangulation of Great Britain and Ireland" (see pages 247, 249, 676 of that work) are erroneous in every case in which the given latitude is greater than the sought latitude.

(Case 2nd.)

Given $l_{\prime\prime}$, $A_{\prime\prime}$, s; to find ω, $l_{\prime}$, and $A_{\prime}$.

To find $z_{\prime\prime}$, $D_{\prime}$, ω, L', we have—

$$z_{\prime\prime} = \frac{s}{R_{\prime\prime} \cdot \sin 1''}$$

$$\tan \tfrac{1}{2} (D_{\prime} + \omega) = \frac{\cos \frac{1}{2} (l'' - z_{\prime\prime})}{\cos \frac{1}{2} (l'' + z_{\prime\prime})} \cdot \cot \tfrac{1}{2} A_{\prime\prime}$$

$$\tan \tfrac{1}{2} (D_{\prime} - \omega) = \frac{\sin \frac{1}{2} (l'' - z_{\prime\prime})}{\sin \frac{1}{2} (l'' + z_{\prime\prime})} \cdot \cot \tfrac{1}{2} A_{\prime\prime}$$

$$\tan \tfrac{1}{2} (l'' - L') = \frac{\sin \frac{1}{2} (D_{\prime} - A_{\prime\prime})}{\sin \frac{1}{2} (D_{\prime} + A_{\prime\prime})} \cdot \tan \tfrac{1}{2} z_{\prime\prime}$$

or,

$$\sin L' = \frac{\sin l'' \cdot \sin A_{\prime\prime}}{\sin D_{\prime}}$$

To find $\delta_{\prime}$, $l_{\prime}$, and $A_{\prime}$, we have—

$$\delta_{\prime} = \left(\frac{e^2}{1 - e^2}\right) \cdot \sin L' \cdot \sin \tfrac{1}{2} (l'' + L') \cdot (l'' - L')$$

$$l_{\prime} = 90^\circ - (L' - \delta_{\prime})$$

$$D_{\prime} - A_{\prime} = \sin D_{\prime} \cdot \tan \tfrac{1}{2} z_{\prime\prime} \cdot \delta_{\prime}$$

☞ This case, in which the given or known latitude $l_{,,}$ is less than the sought latitude $l_{,}$, will be intimated to us by the angles $A_{,,}$ and $D_{,}$; we shall have the given azimuth $A_{,,}$ less than the angle $D_{,}$. If the angle $A_{,,} = D_{,}$, then $A_{,} = D_{,}$, and $l_{,} = l_{,,}$, &c.

Otherwise.

Case 1°. When $l_{,}$, $A_{,}$, s, are given; to find $l_{,,}$, $A_{,,}$, ω.

Find $z_{,}$, ω, $D_{,,}$, as indicated in the last solution, and then find $A_{,,}$ by means of—

$$\sin A_{,,} = \frac{\cos (z_{,} - \frac{1}{2} \Sigma)}{\cos \frac{1}{2} \Sigma} \cdot \sin D_{,,}$$

And find $l_{,,}$ from—

$$\tan \tfrac{1}{2} l'' = - \frac{\cos \frac{1}{2} (A_{,} + A_{,,} + \omega)}{\cos \frac{1}{2} (A_{,} + A_{,,} - \omega)} \cdot \cot \tfrac{1}{2} l'$$

$$l_{,,} = 90^\circ - l''.$$

Case 2°. When $l_{,,}$, $A_{,,}$, s, are given; to find $l_{,}$, $A_{,}$, ω.

Find $z_{,,}$, ω, $D_{,}$, as indicated in the last solution, and then find $A_{,}$ by means of—

$$\sin A_{,} = \frac{\cos (z_{,,} - \frac{1}{2} \Sigma)}{\cos \frac{1}{2} \Sigma} \cdot \sin D_{,}$$

And find $l_{,}$ from—

$$\tan \tfrac{1}{2} l' = - \frac{\cos \frac{1}{2} (A_{,} + A_{,,} + \omega)}{\cos \frac{1}{2} (A_{,} + A_{,,} - \omega)} \cdot \cot \tfrac{1}{2} l''$$

$$l_{,} = 90^\circ - l'.$$

PROBLEM 3.

Given the latitudes $l_{,}$, $l_{,,}$, and the azimuth $A_{,}$; to find the azimuth $A_{,,}$, the difference of longitude ω, &c.

By equating the values of $\sin \alpha$, as expressed in formulæ 108, 109, we have—

$$R_{,,} \cos l_{,,} (\cos^2 l_{,} + 1) \sqrt{1 - \sin^2 \omega}$$

$$= (R_{,} + \frac{a^2}{R_{,}} - R_{,,} \cdot \frac{b^2}{a^2} \cdot \sin l_{,} \sin l_{,,}) \cos l_{,}$$

$$- (R_{,,} \cos l_{,,} \tan l_{,} \cot A_{,}) \sin \omega$$

or,
$$M \cdot \sqrt{1 - \sin^2 \omega} = L - N \cdot \sin \omega$$

in which the values of M, L, and N are known.

From this we at once obtain

$$\sin \omega = \frac{L\,N + \sqrt{M^2 (M^2 + N^2 - L^2)}}{M^2 + N^2}$$

in which the + sign only should precede the radical portion. This is evident. For since the general expression for sin ω holds when $A_{\prime} = 90°$, in which case $N = 0$; and that sin ω must be positive; therefore it is the + sign that must in such case, and in all cases, precede the radical.

We may also find ω in the following manner—

Find the arc L″ by means of formula (79), and the angle $D_{\prime\prime}$ from—

$$\sin D_{\prime\prime} = \frac{\cos l_{\prime} \sin A_{\prime}}{\sin L''}$$

and then to find ω we have—

$$\tan \tfrac{1}{2}\, \omega = \frac{\cos \frac{1}{2}\,(L'' - l')}{\cos \frac{1}{2}\,(L'' + l')} \cdot \cot \tfrac{1}{2}\,(A_{\prime} + D_{\prime\prime})$$

To find the azimuth $A_{\prime\prime}$ we then have—

$$\tan \tfrac{1}{2}\,(A_{\prime} + A_{\prime\prime}) = \frac{\cos \frac{1}{2}\,(l_{\prime} - l_{\prime\prime})}{\sin \frac{1}{2}\,(l_{\prime} + l_{\prime\prime})} \cdot \cot \tfrac{1}{2}\, \omega$$

And to find s, we have—

$$\sin z_{\prime} = \frac{\sin L'' \sin \omega}{\sin A_{\prime}}$$

$$s = z_{\prime} \cdot R_{\prime} \cdot \sin 1''$$

The other entities can be easily found as indicated by formulæ.

☞ If $l_{\prime\prime}$, $l_{\prime}$, $A_{\prime\prime}$ were given instead of $l_{\prime}$, $l_{\prime\prime}$, $A_{\prime}$; then instead of L″, $D_{\prime\prime}$, &c., in the preceding formulæ, we should have L′, $D_{\prime}$, &c.

Otherwise.

To find the azimuth $A_{\prime\prime}$, we have—

$$\sin A_{\prime\prime} = \frac{R_{\prime} \cdot \cos l_{\prime}}{R_{\prime\prime} \cdot \cos l_{\prime\prime}} \cdot \sin A_{\prime} \text{ nearly.}$$

And then to find ω, we have—

$$\tan \tfrac{1}{2}\, \omega = \frac{\cos \frac{1}{2}\,(l_{\prime} - l_{\prime\prime})}{\sin \frac{1}{2}\,(l_{\prime} + l_{\prime\prime})} \cdot \cot \tfrac{1}{2}\,(A_{\prime} + A_{\prime\prime})$$

And when instead of $A_{\prime}$, the azimuth $A_{\prime\prime}$ is given, the first of these must be replaced by

$$\sin A_{\prime} = \frac{R_{\prime\prime} \cdot \cos l_{\prime\prime}}{R_{\prime} \cdot \cos l_{\prime}} \cdot \sin A_{\prime\prime}$$

&c., &c.

PROBLEM 4.

Given the two azimuths $A_{,}$, $A_{,,}$, and one of the latitudes $l_{,}$; to find the latitude $l_{,,}$, the difference of longitude ω of the stations, &c.

To find the latitude $l_{,,}$, we have, from (53)—

$$\tan^2 l_{,,} = \frac{(1 - e^2) \tan^2 l_{,} \sin^2 A_{,,} - (\sin^2 A_{,} - \sin^2 A_{,,})}{(1 - e^2) \sin^2 A_{,}} \text{ nearly.}$$

Then to find the difference of longitude, we have—

$$\tan \tfrac{1}{2} \omega = \frac{\cos \frac{1}{2} (l_{,} - l_{,,})}{\sin \frac{1}{2} (l_{,} + l_{,,})} \cdot \cot \tfrac{1}{2} (A_{,} + A_{,,})$$

The other entities can now be found, &c.

PROBLEM 5.

Given the latitude $l_{,}$, the azimuth $A_{,}$, and the difference of longitude ω; to find the latitude $l_{,,}$, the azimuth $A_{,,}$, &c.

Find L'' by means of formula 78.

Then finding m, p, q, by means of—

$$m = \cot^2 L'' - \frac{e^4}{a^2} \cdot R^2_{,} \cdot \sin^2 l_{,}$$

$$p = \cot^2 L'' - \frac{e^6}{a^2} \cdot R^2 \cdot \sin^2 l_{,} + (1 - e^2)^2$$

$$q = 2\, e^2 (1 - e^2) \frac{R_{,}}{a} \cdot \sin l_{,}$$

the second of the formulæ 79, gives us the equation—

$$m - p \cdot \sin^2 l_{,,} = q \cdot \sin l_{,,} \sqrt{1 - e^2 \cdot \sin^2 l_{,,}}$$

And from this we immediately obtain—

$$\sin^2 l_{,,} = \frac{q^2 + 2\, m\, p + q \sqrt{q^2 + 4\, m\, (p - m\, e^2)}}{2\, (p^2 + q^2\, e^2)}$$

Now, if we conceive a case in which $l_{,}$ is of any value we wish, and that the corresponding value of $l_{,,}$ is such that $m = 0$; then it is evident $l_{,,}$, p, q, have finite values; and we perceive that in such case the + sign only must precede the radical. And it is ∴ evident that the + sign must, in all cases, precede the radical in the above general expression for $\sin^2 l_{,,}$.

Or we may proceed as follows—

From the triangle $S_{,}PD_{,,}$, we have to find L'', $z_{,}$, $D_{,,}$

$$\tan \tfrac{1}{2} (L'' + z_{,}) = \frac{\cos \frac{1}{2} (A_{,} - \omega)}{\cos \frac{1}{2} (A_{,} + \omega)} \cdot \tan \tfrac{1}{2}\, l'$$

$$\tan \tfrac{1}{2} (L'' - z_{,}) = \frac{\sin \frac{1}{2} (A_{,} - \omega)}{\sin \frac{1}{2} (A_{,} + \omega)} \cdot \tan \tfrac{1}{2} l'$$

$$\sin D_{,,} = \frac{\sin l' \cdot \sin A_{,}}{\sin L''} = \frac{\sin l' \cdot \sin \omega}{\sin z_{,}}$$

or,

$$\tan \tfrac{1}{2} (A_{,} - D_{,,}) = \frac{\sin \frac{1}{2} (L'' - l')}{\sin \frac{1}{2} (L'' + l')} \cdot \cot \tfrac{1}{2} \omega$$

Then we can find $\delta_{,,}$ by 83 or any of the formulæ 88, and the azimuth $A_{,,}$ by means of any of the formulæ 94.

Then, $l_{,,} = 90° - (L'' + \delta_{,,})$. &c., &c.

When instead of $l_{,}$, $A_{,}$, we are given $l_{,,}$, $A_{,,}$, the analogous methods of proceeding are evident.

PROBLEM 6.

Given the azimuth $A_{,}$, the latitude $l_{,,}$, and the length s and circular measure Σ of the arc between the stations; to find $A_{,,}$, $l_{,}$, ω, &c.

To find ω, $z_{,,}$, $D_{,}$, $A_{,,}$, and $l_{,}$, we have—

$$\sin \omega = \frac{s \cdot \sin \Sigma \cdot \sin A_{,}}{R_{,,} \cdot \Sigma \cdot \cos l_{,,} \cdot \sin 1''}$$

$$z_{,,} = \frac{s}{R_{,,} \cdot \sin 1''}$$

$$\sin D_{,} = \frac{\cos l_{,,} \cdot \sin \omega}{\sin z_{,,}}$$

$$\tan \tfrac{1}{2} A_{,,} = \frac{\sin \frac{1}{2} (l'' - z_{,,})}{\sin \frac{1}{2} (l'' + z_{,,})} \cdot \cot \tfrac{1}{2} (D_{,} - \omega)$$

$$\tan \tfrac{1}{2} l' = - \frac{\cos \frac{1}{2} (A_{,} + A_{,,} + \omega)}{\cos \frac{1}{2} (A_{,} + A_{,,} - \omega)} \cdot \cot \tfrac{1}{2} l''$$

If $A_{,,}$, $l_{,}$ were given instead of $A_{,}$, $l_{,,}$, the method of solution is analogous, and requires no particular elucidation.

PROBLEM 7.

Given the latitude $l_{,}$, the difference of longitude ω, and the length s and circular measure Σ of the arc between the stations; to find the azimuths $A_{,}$, $A_{,,}$, the latitude $l_{,,}$, &c.

To find $z_{,}$, $D_{,,}$, $A_{,}$, $A_{,,}$, $l_{,,}$, we have—

$$z_{,} = \frac{s}{R_{,} \cdot \sin 1''}.$$

$$\sin D_{\prime\prime} = \frac{\sin l' \sin \omega}{\sin z_{\prime}}$$

$$\tan \tfrac{1}{2} A_{\prime} = \frac{\sin \frac{1}{2}(l' - z_{\prime})}{\sin \frac{1}{2}(l' + z_{\prime})} \cdot \cot \tfrac{1}{2}(D_{\prime\prime} - \omega)$$

$$\sin A_{\prime\prime} = \frac{R_{\prime} \cdot \Sigma \cdot \cos l_{\prime} \sin \omega}{s \cdot \sin \Sigma}$$

$$\tan \tfrac{1}{2} l'' = - \frac{\cos \frac{1}{2}(A_{\prime} + A_{\prime\prime} + \omega)}{\cos \frac{1}{2}(A_{\prime} + A_{\prime\prime} - \omega)} \cdot \cot \tfrac{1}{2} l'$$

And similarly when $l_{\prime\prime}$ is given instead of $l_{\prime}$.

Problem 8.

Given the azimuth $A_{\prime}$, the difference of longitude ω, and the length s and circular measure Σ of the arc between the stations; to find the latitudes, &c.

Putting—
$$G = \frac{s \cdot \sin \Sigma'' \cdot \sin A}{\sin \omega \cdot \Sigma'' \cdot \sin 1''}$$

We easily find, from 62—
$$\sin l_{\prime\prime} = \sqrt{\frac{(a + G) \cdot (a - G)}{(a + eG) \cdot (a - eG)}}$$

And now we can find the other entities as in problems 6 and 7.

Problem 9.

Given the two latitudes $l_{\prime}$, $l_{\prime\prime}$, and the length s and circular measure Σ of the arc between the stations; to find the azimuths $A_{\prime}$, $A_{\prime\prime}$, &c.

To find L', L'', $z_{\prime}$, $z_{\prime\prime}$, we have—

$$\cot L' = e^2 \cdot \frac{R_{\prime\prime} \sin l_{\prime\prime}}{R_{\prime} \cos l_{\prime}} + (1 - e^2) \tan l_{\prime}$$

$$\cot L'' = e^2 \cdot \frac{R_{\prime} \sin l_{\prime}}{R_{\prime\prime} \cos l_{\prime\prime}} + (1 - e^2) \tan l_{\prime\prime}$$

$$z_{\prime} = \frac{s}{R_{\prime} \cdot \sin 1''}$$

$$z_{\prime\prime} = \frac{s}{R_{\prime\prime} \cdot \sin 1''}$$

Then from the spherical triangles $S_{\prime}PD_{\prime\prime}$, $S_{\prime\prime}PD_{\prime}$, we have —putting $p = \frac{1}{2}(l' + z_{\prime} + L'')$, $q = \frac{1}{2}(l'' + z_{\prime\prime} + L')$,—

$$\tan^2 \tfrac{1}{2} A_{\prime} = \frac{\sin (p - z_{\prime}) \sin (p - l')}{\sin p \sin (p - L'')}$$

$$\tan^2 \tfrac{1}{2} A_{,,} = \frac{\sin (q - z_{,,}) \sin (q - l'')}{\sin q \sin (q - L')}$$

$$\tan^2 \tfrac{1}{2} \omega = \frac{\sin (p - L'') \sin (p - l')}{\sin p \sin (p - z_{,})}$$

$$\tan^2 \tfrac{1}{2} \omega = \frac{\sin (q - L') \sin (q - l'')}{\sin q \sin (q - z_{,,})}$$

In this method of solution we have not made use of Σ. In the following method we shall not make use of s, but of Σ; and it is applicable to any two stations on the earth's spheroidal surface, as well as to mutually visible stations.

Otherwise.

Find the angles $a_{,,}$, $a_{,}$, of depression of the chord by means of—

$$\tan x = \frac{R_{,}}{R_{,,}}$$

$$\tan \tfrac{1}{2} (\alpha_{,,} - \alpha_{,}) = \tan (x - 45^\circ) \cdot \tan \tfrac{1}{2} \Sigma$$

$$\tfrac{1}{2} (\alpha_{,,} + \alpha_{,}) = \tfrac{1}{2} \Sigma$$

To find the azimuths we have the equations—

$$\cos \alpha_{,} \cos l_{,} \cos A_{,} + \cos \alpha_{,,} \cos l_{,,} \cos A_{,,} = \sin \alpha_{,} \sin l_{,} + \sin \alpha_{,,} \sin l_{,,}$$

$$\frac{1 - \cos^2 A_{,}}{1 - \cos^2 A_{,,}} = \frac{(R_{,,} \cos \alpha_{,,} \cos l_{,,})^2}{(R_{,} \cos \alpha_{,} \cos l_{,})^2}$$

By putting

$M_{,} = \cos \alpha_{,} \cos l_{,}$; $M_{,,} = \cos \alpha_{,,} \cos l_{,,}$; $Q = \sin \alpha_{,} \sin l_{,} + \sin \alpha_{,,} \sin l_{,,}$

we easily find—

$$\cos A_{,} = \frac{-Q\, R^2_{,,} + \sqrt{(Q \cdot R_{,} \cdot R_{,,})^2 - (R^2_{,} - R^2_{,,}) \cdot (M^2_{,,} \cdot R^2_{,,} - M^2_{,} \cdot R^2_{,})}}{M_{,} \cdot (R^2_{,} - R^2_{,,})}$$

$$\cos A_{,,} = \frac{Q \cdot R^2_{,} - \sqrt{(Q \cdot R_{,} \cdot R_{,,})^2 - (R^2_{,} - R^2_{,,}) \cdot (M^2_{,,} \cdot R^2_{,,} - M^2_{,} \cdot R^2_{,})}}{M_{,,} \cdot (R^2_{,} - R^2_{,,})}$$

Since $\cos A_{,}$ must be positive when the angle $A_{,}$ is acute, ∴ it is evident that in all cases it is the + sign which must precede the radical in the above expression for $\cos A_{,}$. It is evident that in the expression for $\cos A_{,,}$, it is the — sign only which should precede the radical.

☞ When $l_{,} = l_{,,}$; then $a_{,,} = a_{,}$; $R_{,} = R_{,,}$; $M_{,} = M_{,,}$; and the above expressions can be written in the forms—

$$\cos A_{,} = \frac{Q\, R_{,,} (R_{,} - R_{,,})}{M_{,} (R_{,} + R_{,,}) . (R_{,} - R_{,,})}$$

$$\cos A_{,,} = \frac{Q\,R_{,}\,(R_{,} - R_{,,})}{M_{,,}\,(R_{,} + R_{,,})\,(R_{,} - R_{,,})}$$

$$\therefore\ \cos A_{,} = \cos A_{,,} = \frac{Q}{2\,M} = \tan \tfrac{1}{2}\,\Sigma \cdot \tan l_{,}.$$

Otherwise.

To find the chord k and the angle θ which it makes with the polar axis, we have—

$$k = \frac{2\,s \cdot \sin \frac{1}{2}\,\Sigma}{\Sigma}$$

$$\cos \theta = \frac{1 - e^2}{k} \cdot (R_{,} \sin l_{,} - R_{,,} \sin l_{,,})$$

To find the sides of the plane triangle $p_{,}\,C_{\circ}\,p_{,,}$, we have—

$$C_{\circ}\,p_{,} = R_{,} \cos l_{,};\ C\,p_{,,} = R_{,,} \cos l_{,,};\ p_{,}p_{,,} = k \cdot \sin \theta.$$

And knowing the three sides of this plane triangle, we can find its angles $\phi_{,}$, $\phi_{,,}$, ω.

Then from the spherical triangles $S_{,}PI$, $S_{,,}PI$, we have the following formulæ from which to obtain the azimuths—

$$\cot \tfrac{1}{2}\,(A_{,} - \psi) = \frac{\cos \frac{1}{2}\,(\theta - l')}{\cos \frac{1}{2}\,(\theta + l')} \cdot \cot \tfrac{1}{2}\,\phi_{,};$$

$$\tan \tfrac{1}{2}\,(A_{,,} + \psi) = \frac{\cos \frac{1}{2}\,(\theta - l'')}{\cos \frac{1}{2}\,(\theta + l'')} \cdot \tan \tfrac{1}{2}\,\phi_{,,}$$

$$\cot \tfrac{1}{2}\,(A_{,} + \psi) = \frac{\sin \frac{1}{2}\,(\theta - l')}{\sin \frac{1}{2}\,(\theta + l')} \cdot \cot \tfrac{1}{2}\,\phi_{,};$$

$$\tan \tfrac{1}{2}\,(A_{,,} - \psi) = \frac{\sin \frac{1}{2}\,(\theta - l'')}{\sin \frac{1}{2}\,(\theta + l'')} \cdot \tan \tfrac{1}{2}\,\phi_{,,}$$

We can also find the sides $IS_{,}$, $IS_{,,}$, of these spherical triangles; and then we have—

$$\triangle = \psi_{,,} - \psi_{,}$$

$$\alpha_{,} = 90^{\circ} - IS_{,};\quad \alpha_{,,} = IS_{,,} - 90^{\circ}.$$

And as a test of accuracy of the work we have $\alpha_{,} + \alpha_{,,} = \Sigma$.

EXAMPLE (Problem 1).

Let $l_{,} = 38^{\circ}$; $l_{,,} = 37^{\circ}$; $\omega = 1^{\circ}{}_{,,}\ 15'{}_{,,}\ 00''$; be the given latitudes and difference of longitude of the stations.

First then, to find the values of the normals $R_{,}$, $R_{,,}$, drawn at the stations S_{o}, S_{oo}, which terminate in the polar axis, we have the well known formula

$$R_{,} = \frac{a}{\sqrt{1 - e^2 \sin^2 l_{,}}}\,;\quad R_{,,} = \frac{a}{\sqrt{1 - e^2 \sin^2 l_{,,}}}$$

and we easily obtain

$$\begin{array}{llll} \log R_{,} = 7\cdot3212526296\,; & R_{,} = 20953309\cdot5777 \text{ feet}\,; \\ \log R_{,,} = 7\cdot3212277292\,; & R_{,,} = 20952108\cdot2495 \text{ feet.} \end{array}$$

We will now proceed to find the values of the small arcs $\delta_{,}$, $\delta_{,,}$, by means of formula 80. And as $R_{,} \cos l' - R_{,,} \cos l''$ enters in both numerators and denominators of the expressions, we shall first find its value. Thus:—

$$\begin{array}{lll} \log R_{,} = 7\cdot3212526296 & \qquad & \log R_{,,} = 7\cdot321227292 \\ \cos l' \;\; = \bar{1}\cdot7893417987 & & \cos l'' \;\; = \bar{1}\cdot7794630249 \\ \hline \qquad\quad 7\cdot1105946083 & & \qquad\quad 7\cdot1006907541 \end{array}$$

$$\text{antilogs} \begin{cases} 12900145\cdot48795 \\ 12609293\cdot51225 \end{cases}$$

$$\therefore\; R_{,} \cos l' - R_{,,} \cos l'' = 290851\cdot9757$$

and $$\log (R_{,} \cos l' - R_{,,} \cos l'') = 5\cdot4636720181$$

Now to find $\delta_{,}$ we have formula 80 or—

$$\tan \delta_{,} = \frac{e^2 (R_{,} \cos l' - R_{,,} \cos l'') \sin l'}{R_{,} - e^2 (R_{,} \cos l' - R_{,,} \cos l'') \cos l'}$$

$$\begin{array}{lll} \log e^2 = \bar{3}\cdot8315591974 & \qquad & \log e^2 = \bar{3}\cdot8315591974 \\ \qquad\quad 5\cdot4636720182 & & \qquad\quad 5\cdot4636720182 \\ \sin l' = \bar{1}\cdot8965321441 & & \cos l' = \bar{1}\cdot7893419787 \\ \hline \qquad\quad 3\cdot1917633597 & & \qquad\quad 3\cdot0845731943 \end{array}$$

$$\begin{array}{rl} \text{antilog} = & 1214\cdot9913 \\ \text{but } R_{,} = & 20953309\cdot5777 \\ \therefore \text{ the value of the denominator} = & 20952094\cdot5864 \end{array}$$

$$\begin{array}{rl} \text{and its log is} & 7\cdot3212274459 \\ & 3\cdot1917633597 \\ \hline \therefore\; \log \tan \delta_{,} = & \bar{5}\;8705359138 \end{array}$$

$$\therefore\; \delta_{,} = 0^{\circ}{}_{,,}\; 00'{}_{,,}\; 15''\cdot309501$$

To find $\delta_{,,}$ we have the formula 80 or—

$$\tan \delta_{,,} = \frac{e^2 (R_{,} \cos l' - R_{,,} \cos l'') \sin l''}{R_{,,} + e^2 (R_{,} \cos l' - R_{,,} \cos l'') \cos l''}$$

$$\begin{array}{rl} \log e^2 = & \bar{3}{\cdot}8315591974 \\ & 5{\cdot}4636720182 \\ \sin l'' = & \underline{\bar{1}{\cdot}9023486165} \\ & 3{\cdot}1975798321 \end{array}$$

$$\begin{array}{rl} \log e^2 = & \bar{3}{\cdot}8315591974 \\ & 5{\cdot}4636720182 \\ \cos l'' = & \underline{\bar{1}{\cdot}7794630249} \\ & 3{\cdot}0746942397 \\ \text{antilog} = & 1187{\cdot}6658 \\ = & \underline{20952108{\cdot}2495} \end{array}$$

$$\therefore \text{ value of denominator} = 20953295{\cdot}9153$$

$$\begin{array}{rl} \text{its log} = & 7{\cdot}3212523464 \\ & \underline{3{\cdot}1975798321} \\ \therefore \log \tan \delta_{,,} = & \bar{5}{\cdot}8763274857 \end{array}$$

$$\therefore \delta_{,,} = 0^\circ{}_{,,}\ 00'{}_{,,}\ 15''{\cdot}51503$$

To find the arcs L′ and L″, we have

$$\mathrm{L}' = l' + \delta_{,} \qquad \mathrm{L}'' = l'' - \delta_{,,}$$

$$\begin{array}{rl} l' = & 52^\circ \\ \delta_{,} = & \underline{0{}_{,,}\ 00'{}_{,,}\ 15''{\cdot}30950} \\ \therefore \mathrm{L}' = & 52^\circ{}_{,,}\ 00'{}_{,,}\ 15''{\cdot}30950 \end{array} \qquad \begin{array}{rl} l'' = & 53^\circ \\ \delta_{,,} = & \underline{0{}_{,,}\ 00'{}_{,,}\ 15''{\cdot}51503} \\ \therefore \mathrm{L}'' = & 52^\circ{}_{,,}\ 59'{}_{,,}\ 44''{\cdot}48497 \end{array}$$

These values are correct to the last or fifth decimals.

To find L′ we have also the formula 79 or—

$$\cot \mathrm{L}' = (1 - e^2) \cot l' + e^2 \cdot \frac{\mathrm{R}_{,,} \cos l''}{\mathrm{R}_{,} \sin l'}$$

$$\begin{array}{rl} \log (1 - e^2) = & \bar{1}{\cdot}9970432059 \\ \cot l' = & \underline{\bar{1}{\cdot}8928098346} \\ & \bar{1}{\cdot}8898530405 \\ \text{antilog} = & 0{\cdot}7759844892 \end{array} \qquad \begin{array}{rl} \log e^2 = & \bar{3}{\cdot}8315591974 \\ \log \mathrm{R}_{,,} = & 7{\cdot}3212277292 \\ \cos l'' = & \underline{\bar{1}{\cdot}7794630249} \\ & 4{\cdot}9322499515 \end{array}$$

$$\begin{array}{rl} \log \mathrm{R}_{,} = & 7.3212526296 \\ \sin l' = & \underline{\bar{1}{\cdot}8965321441} \\ & 7{\cdot}2177847737 \\ & \underline{4{\cdot}9322499515} \\ & \bar{3}{\cdot}7144651778 \end{array}$$

$$\begin{array}{rl} \text{antilog} = & 0{\cdot}0051816154 \\ & \underline{0{\cdot}7759844892} \\ \therefore \cot \mathrm{L}' = & 0{\cdot}7811661046 \end{array}$$

$$\therefore \log \cot \mathrm{L}' = \bar{1}{\cdot}8927433907$$

$$\therefore \mathrm{L}' = 52^\circ{}_{,,}\ 00'{}_{,,}\ 15''{\cdot}3095$$

To find L″ we have formula 79 or—

$$\cot L'' = (1 - e^2)\cot l'' + e^2 \cdot \frac{R_{,}\cos l'}{R_{,,}\sin l''}$$

$$\begin{array}{rlrl}
\log(1 - e^2) = & \bar{1}{\cdot}9970432059 & & \\
\cot l'' = & \underline{\bar{1}{\cdot}8771144084} & & \\
 & \bar{1}{\cdot}8741576143 & \text{antilog} = & 0{\cdot}7484410756
\end{array}$$

$$\begin{array}{rlrl}
\log e^2 = & \bar{3}{\cdot}8315591974 & \log R_{,,} = & 7{\cdot}3212277292 \\
\log R_{,} = & 7{\cdot}3212526296 & \sin l'' = & \underline{\bar{1}{\cdot}9023486165} \\
\cos l' = & \underline{\bar{1}{\cdot}7893419787} & & 7{\cdot}2235763457 \\
 & 4{\cdot}9421538057 & & \\
 & \underline{7{\cdot}2236012457} & & \\
 & \bar{3}{\cdot}7185525600 & \text{antilog} = & 0{\cdot}0052309125'5 \\
 & & & \underline{0\ 7484410756'5} \\
 & & \therefore \text{nat} \cot L'' = & 0{\cdot}7536719882
\end{array}$$

$\therefore \log \cot L'' = \bar{1}{\cdot}8771823669$, and $L'' = 52^\circ_{,,}\ 59'_{,,}\ 44''{\cdot}4867$ the error of $0''{\cdot}0018$ being due to the insufficiency of the tables or to their inaccuracy in the 10th decimal places, &c.

Now, in each of the spherical triangles $S_{,}PD_{,,}$, $S_{,,}PD_{,}$, $S_{,}PS_{,,}$, we have the two sides and the included angle ω from which we can find the angles at their bases and also the bases.

To find the angles $A_{,}$, $D_{,,}$, and base $z_{,}$ of the triangle $S_{,}PD_{,,}$—

$$\begin{array}{rlrl}
\cot \frac{1}{2}\omega = & 11{\cdot}9622253888 & \cot \frac{1}{2}\omega = & 11{\cdot}9622253888 \\
\cos \frac{1}{2}(L'' - l') = & \underline{9{\cdot}9999836052} & \sin \frac{1}{2}(L'' - l') = & \underline{7{\cdot}9389661700} \\
 & 21{\cdot}9622089940 & & 19{\cdot}9011915588 \\
\cos \frac{1}{2}(L'' + l') = & \underline{9{\cdot}7844684133} & \sin \frac{1}{2}(L'' + l') = & \underline{9{\cdot}8994541209} \\
\therefore & & \therefore & \\
\tan \frac{1}{2}(A_{,} + D_{,,}) = & 12{\cdot}1777405807 & \tan \frac{1}{2}(A_{,} - D_{,,}) = & 10\ 0017374379
\end{array}$$

$\therefore\ \frac{1}{2}(A_{,} + D_{,,}) = 89^\circ_{,,}\ 37'_{,,}\ 10''\cdot 133745$

$\therefore\ \frac{1}{2}(A_{,} - D_{,,}) = 45^\circ_{,,}\ 06'_{,,}\ 52''\cdot 590185$

$\therefore A_{,} = 134^\circ_{,,}\ 44'_{,,}\ 02''\cdot 72393$

$D_{,,} = 44^\circ_{,,}\ 30'_{,,}\ 17''\cdot 54356$

$$\begin{array}{rlrl}
\sin l' = & 9\cdot8965321441 & \sin L'' = & 9\cdot9023239980\\
\sin\omega = & \underline{8\cdot3387529285} & \sin\omega = & \underline{8\cdot3387529285}\\
 & 18\cdot2352850726 & & 18\cdot2410769265\\
\sin D_{,,} = & \underline{9\cdot8456993857} & \sin A_{,} = & \underline{9\cdot8514912397}\\
\therefore\quad \sin z_{,} = & 8\cdot3895856869 & \therefore\quad \sin z_{,} = & 8\cdot3895856868
\end{array}$$

$$\therefore\quad z_{,} = 1^{\circ}_{,,}\ 24'_{,,}\ 18''\cdot 8798$$

To find the angles $D_{,}$, $A_{,,}$, and base $z_{,,}$ of the triangle $A_{,,}PD_{,}$—

$$\begin{array}{rlrl}
\cot\frac{1}{2}\omega = & 11\cdot9622253888 & \cot\frac{1}{2}\omega = & 11\cdot9622253888\\
\cos\frac{1}{2}(l''-L') = & \underline{9\cdot9999836034} & \sin\frac{1}{2}(l''-L') = & \underline{7\cdot9389910706}\\
 & 21\cdot9622089922 & & 19\cdot9012164594\\
\cos\frac{1}{2}(l''+L') = & \underline{9\cdot7844261226} & \sin\frac{1}{2}(l''+L') = & \underline{9\cdot8994790213}\\
\therefore & & \therefore & \\
\tan\frac{1}{2}(D_{,}+A_{,,}) = & 12\cdot1777828696 & \tan\frac{1}{2}(D_{,}-A_{,,}) = & 10\cdot0017374381
\end{array}$$

$$\therefore\quad \tfrac{1}{2}(D_{,}+A_{,,}) = 89^{\circ}_{,,}\ 37'_{,,}\ 10''\cdot 267152$$

$$\therefore\quad \tfrac{1}{2}(D_{,}-A_{,,}) = 45^{\circ}_{,,}\ 06'_{,,}\ 52''\cdot 590233$$

$$\therefore\ D_{,} = 134^{\circ}_{,,}\ 44'_{,,}\ 02''\cdot 857385$$

$$A_{,,} = 44^{\circ}_{,,}\ 30'_{,,}\ 17''\cdot 676919$$

$$\begin{array}{rlrl}
\sin l'' = & 9\cdot9023486165 & \sin L' = & 9\cdot8965573265\\
\sin\omega = & \underline{8\cdot3387529285} & \sin\omega = & \underline{8\cdot3387529285}\\
 & 18\cdot2411015450 & & 18\cdot2353102550\\
\sin D_{,} = & \underline{9\cdot8514909614} & \sin A_{,,} = & \underline{9\cdot8456996715}\\
\because\ \sin z_{,,} = & 8\cdot3896105836 & \therefore\quad \sin z_{,,} = & 8\cdot3896105835
\end{array}$$

$$\therefore\quad z_{,,} = 1^{\circ}_{,,}\ 24'_{,,}\ 19''\cdot 169884$$

To find the angles $A_{\circ}$, $A_{\circ\circ}$, and base v of the triangle $S_{,}PS_{,,}$—

$$\begin{array}{rlrl}
\cot\frac{1}{2}\omega = & 11\cdot9622253888 & \cot\frac{1}{2}\omega = & 11\cdot9622253888\\
\cos\frac{1}{2}(l''-l') = & \underline{9\cdot9999834631} & \sin\frac{1}{2}(l''-l') = & \underline{7\cdot9408418596}\\
 & 21\cdot9622088519 & & 19\cdot9030672484\\
\cos\frac{1}{2}(l''+l') = & \underline{9\cdot7844471278} & \sin\frac{1}{2}(l''+l') = & \underline{9\cdot8994666546}\\
\therefore & & \therefore & \\
\tan\frac{1}{2}(A_{\circ}+A_{\circ\circ}) = & 12\cdot1777617241 & \tan\frac{1}{2}(A_{\circ}-A_{\circ\circ}) = & 10\cdot0036005938
\end{array}$$

$$\therefore\quad \tfrac{1}{2}(A_{\circ}+A_{\circ\circ}) = 89^{\circ}_{,,}\ 37'_{,,}\ 10''\cdot 20043$$

$$\therefore\quad \tfrac{1}{2}(A_{\circ}-A_{\circ\circ}) = 45^{\circ}_{,,}\ 14'_{,,}\ 15''\cdot 02727$$

$$\therefore\quad A_{\circ} = 134^{\circ}_{,,}\ 51'_{,,}\ 25''\cdot 22770$$

$$A_{\circ\circ} = 44^{\circ}_{,,}\ 22'_{,,}\ 55''\cdot 17316$$

$$\begin{array}{rcl@{\qquad\qquad}rcl}
\sin l' & = & 9{\cdot}8965321441 & \sin l'' & = & 9{\cdot}9023486165 \\
\sin \omega & = & \underline{8{\cdot}3387529285} & \sin \omega & = & \underline{8{\cdot}3387529285} \\
 & & 18{\cdot}2352850726 & & & 18{\cdot}2411015450 \\
\sin A_{\circ\circ} & = & \underline{9{\cdot}8447496921} & \sin A_{\circ} & = & \underline{9{\cdot}8505661645} \\
\therefore\ \sin \nu & = & 8{\cdot}3905353805 & \therefore\ \sin \nu & = & 8{\cdot}3905353805
\end{array}$$

$$\therefore\ \nu = 1^{\circ}{}_{\prime\prime}\,24'{}_{\prime\prime}\,29''\cdot 956648$$

To find the portions $\nu_{\prime\prime}$, $\nu_{\prime}$, into which ν is divided by the point O.

From the spherical triangles $S_{\prime\prime}OE_{\prime\prime}$, $S_{\prime}OE_{\prime}$, we have—

$$\sin \nu_{\prime\prime} \cdot \sin O = \sin \alpha_{\prime\prime}; \quad \sin \nu_{\prime} \cdot \sin O = \sin \alpha_{\prime};$$

and from these—

$$\frac{\sin \nu_{\prime\prime}}{\sin \nu_{\prime}} = \frac{\sin \alpha_{\prime\prime}}{\sin \alpha_{\prime}} = \frac{R_{\prime}}{R_{\prime\prime}};$$

and $\therefore$ (see formulæ 27, 33, 34)—

$$\begin{array}{lcl@{\qquad}rcl}
\log R_{\prime} & = & 7{\cdot}3212526296 & \tan \tfrac{1}{2}\nu & = & \bar{2}{\cdot}0895709833 \\
\log R_{\prime\prime} & = & \underline{7{\cdot}3212277292} & \tan (x-45^{\circ}) & = & \underline{\bar{5}{\cdot}4573930282} \\
\therefore\ \tan x & = & 10{\cdot}0000249004 & \therefore\ \tan \tfrac{1}{2}(\nu_{\prime\prime}-\nu_{\prime}) & = & \bar{7}{\cdot}5469640115 \\
\therefore\ x & = & 45^{\circ}{}_{\prime\prime}\,00'{}_{\prime\prime}\,05''{\cdot}91314 & \therefore\ \tfrac{1}{2}(\nu_{\prime\prime}-\nu_{\prime}) & = & 0^{\circ}{}_{\prime\prime}\,00'{}_{\prime\prime}\,00''{\cdot}072776 \\
 & & & \text{But } \tfrac{1}{2}(\nu_{\prime\prime}+\nu_{\prime}) & = & 0^{\circ}{}_{\prime\prime}\,42'{}_{\prime\prime}\,14''{\cdot}978324
\end{array}$$

$$\therefore\ \nu_{\prime\prime} = 0^{\circ}{}_{\prime\prime}\,42'{}_{\prime\prime}\,15''{\cdot}051100$$

$$\nu_{\prime} = 0^{\circ}{}_{\prime\prime}\,42'{}_{\prime\prime}\,14''{\cdot}905548$$

To find the angles $\Omega_{\prime}$, $\Omega_{\prime\prime}$, which a plane parallel to the two normals makes with the normal chordal planes—

$$\Omega_{\prime} = A_{\circ} - A_{\prime} = 0^{\circ}{}_{\prime\prime}\,07'{}_{\prime\prime}\,22''{\cdot}50377$$

$$\Omega_{\prime\prime} = A_{\prime\prime} - A_{\circ\circ} = 0^{\circ}{}_{\prime\prime}\,07'{}_{\prime\prime}\,22''{\cdot}50377$$

$\therefore$ we have in actual practice (as has been already demonstrated) $\Omega_{\prime} = \Omega_{\prime\prime}$; and we may write Ω to represent their common value.

To find the angles $\alpha_{\prime}$, $\alpha_{\prime\prime}$, of depression of the chord below the tangent planes at the stations $S_{\circ}$, $S_{\circ\circ}$, we have—

$$\begin{array}{rcl@{\qquad}rcl}
\tan \alpha_{\prime} & = & \tan \nu_{\prime} \cdot \cos \Omega & \tan \alpha_{\prime\prime} & = & \tan \nu_{\prime\prime} \cdot \cos \Omega \\
\tan \nu_{\prime} & = & 8{\cdot}0895585138 & \tan \nu_{\prime\prime} & = & 8{\cdot}0895834524 \\
\cos \Omega & = & \underline{9{\cdot}9999990005} & \cos \Omega & = & \underline{9{\cdot}9999990005} \\
\therefore\ \tan \alpha_{\prime} & = & 8{\cdot}0895575143 & \therefore\ \tan \alpha_{\prime\prime} & = & 8{\cdot}0895824529 \\
\therefore\ \alpha_{\prime} & = & 0^{\circ}{}_{\prime\prime}\,42'{}_{\prime\prime}\,14''{\cdot}899714 & \therefore\ \alpha_{\prime\prime} & = & 0^{\circ}{}_{\prime\prime}\,42'{}_{\prime\prime}\,15''{\cdot}045266
\end{array}$$

$$\therefore\ \Sigma = \alpha_{\prime} + \alpha_{\prime\prime} = 1^{\circ}{}_{\prime\prime}\,24'{}_{\prime\prime}\,29''{\cdot}94498$$

To find the length of k the chord connecting the stations. We have—

$$\begin{array}{rcl@{\qquad\qquad}rcl}
k & = & \dfrac{R_{,,} \cos l_{,,} \sin \omega}{\sin A_{,} \cos \alpha_{,}} & k & = & \dfrac{R_{,} \cos l_{,} \sin \omega}{\sin A_{,,} \cos \alpha_{,,}} \\
\log R_{,,} & = & 7{\cdot}3212277292 & \log R_{,} & = & 7{\cdot}3212526296 \\
\cos l_{,,} & = & \bar{1}{\cdot}9023486165 & \cos l_{,} & = & \bar{1}{\cdot}8965321441 \\
\sin \omega & = & \underline{\bar{2}{\cdot}3387529285} & \sin \omega & = & \underline{\bar{2}{\cdot}3387529285} \\
 & & 5{\cdot}5623292745 & & & 5{\cdot}5565377022 \\
\sin A_{,} & = & \bar{1}{\cdot}8514912398 & \sin A_{,,} & = & \bar{1}{\cdot}8456996715 \\
\cos \alpha_{,} & = & \underline{\bar{1}{\cdot}9999672028} & \cos \alpha_{,,} & = & \underline{\bar{1}{\cdot}9999671990} \\
 & & \bar{1}{\cdot}8514584426 & & & \bar{1}{\cdot}8456668705 \\
\therefore \quad \log k & = & 5{\cdot}7108708319 & \therefore \quad \log k & = & 5{\cdot}7108708317
\end{array}$$

$$\log k = 5{\cdot}7108708318$$

$$\therefore \; k = 513890{\cdot}787$$

To find the length of the geodesic arc s connecting the stations—

$$s = \frac{k \cdot \Sigma \cdot \sin 1''}{2 \cdot \sin \frac{1}{2} \Sigma}$$

$$\begin{array}{rcl@{\qquad\qquad}rcl}
\log k & = & 5{\cdot}7108708318 & \log 2 & = & 0{\cdot}3010299957 \\
\log \Sigma & = & 3{\cdot}7050032463 & \sin \frac{1}{2} \Sigma & = & \underline{\bar{2}{\cdot}0895371846} \\
\sin 1'' & = & \underline{\bar{6}{\cdot}6855748668} & & & \bar{2}{\cdot}3095671803 \\
 & & 4{\cdot}1014489449 & & & \\
 & & \underline{\bar{2}{\cdot}3905671803} & & & \\
\therefore \quad \log s & = & 5{\cdot}7108817646 & \therefore \quad s & = & 513903{\cdot}723718 \text{ feet.}
\end{array}$$

To find the arcs OE$_{,}$, OE$_{,,}$, or $\gamma_{,}$, $\gamma_{,,}$, whose sum E$_{,}$E$_{,,}$ is the measure of the angle $\triangle$. We have—

$$\begin{array}{rcl@{\qquad\qquad}rcl}
\sin \gamma_{,} & = & \sin \nu_{,} \sin \Omega & \sin \gamma_{,,} & = & \sin \nu_{,,} \sin \Omega \\
\sin \nu_{,} & = & 8{\cdot}0895257164 & \sin \nu_{,,} & = & 8{\cdot}0895506513 \\
\sin \Omega & = & \underline{7{\cdot}3314915049} & \sin \Omega & = & \underline{7{\cdot}3314915049} \\
\therefore \quad \sin \gamma_{,} & = & 5{\cdot}4210172213 & \sin \gamma_{,,} & = & 5{\cdot}4210421562 \\
\therefore \quad \gamma_{,} & = & 0^{\circ}{}_{,,}\, 00'{}_{,,}\, 05'' \cdot 438039 & \therefore \quad \gamma_{,} & = & 0^{\circ}{}_{,,}\, 00'{}_{,,}\, 05'' \cdot 438352
\end{array}$$

$$\therefore \triangle = 0^{\circ}{}_{,,}\, 00'{}_{,,}\, 10'' \cdot 876391$$

To find the arcs $e_{,}$, $f_{,}$, whose sum $= \delta_{,}$. Since the pencil $I\,(S_{,}S_{,,}OP)$ is harmonic, we have—

$$\tan \tfrac{1}{2}\,(f_{,} - e_{,}) = \frac{\tan^2 \frac{1}{2}\, \delta_{,}}{\tan \frac{1}{2}\,(L' + l')}; \quad \tfrac{1}{2}\,(f_{,} + e_{,}) = \tfrac{1}{2}\, \delta_{,}$$

And to find the arcs $e_{\prime\prime}, f_{\prime\prime}$, whose sum $= \delta_{\prime\prime}$; we have—

$$\tan \tfrac{1}{2}(e_{\prime\prime} - f_{\prime\prime}) = \frac{\tan^2 \frac{1}{2}\delta_{\prime\prime}}{\tan \frac{1}{2}(L'' + l')};\quad \tfrac{1}{2}(e_{\prime\prime} + f_{\prime\prime}) = \tfrac{1}{2}\delta_{\prime\prime}$$

From these we easily obtain the values—

$$\begin{array}{ll} e_{\prime\prime} = 7\cdot75773 & f_{\prime\prime} = 7\cdot75729 \\ e_{\prime} = 7\cdot65453 & f_{\prime} = 7\cdot65497 \end{array}$$

In the spherical triangle $F_{\prime}PF_{\prime\prime}$, we know the values of the sides and included angle ω; and applying the usual formulæ we find—

$$\begin{array}{l} \text{angle } F_{\prime} = 134^{\circ}{}_{\prime\prime}\ 44'{}_{\prime\prime}\ 02''\cdot79079 \\ \text{angle } F_{\prime\prime} = 44^{\circ}{}_{\prime\prime}\ 30'{}_{\prime\prime}\ 17''\cdot61004 \\ \text{arc } F_{\prime}F_{\prime\prime} = 1^{\circ}{}_{\prime\prime}\ 24'{}_{\prime\prime}\ 19''\cdot02484 = \tfrac{1}{2}(z_{\prime} + z_{\prime\prime}) \\ \therefore\ F_{\prime} = \tfrac{1}{2}(A_{\prime} + D_{\prime}) \text{ to within } 0''\cdot0001 \\ \therefore\ F_{\prime\prime} = \tfrac{1}{2}(A_{\prime\prime} + D_{\prime\prime}) \text{ to within } 0''\cdot0002 \end{array}$$

We may also observe that—

$$D_{\prime} - A_{\prime} = 0''\cdot13345;\quad A_{\prime\prime} - D_{\prime\prime} = 0''\cdot13336$$

$$\therefore\ D_{\prime} - A_{\prime} = A_{\prime\prime} - D_{\prime\prime} \text{ to within } 0''\cdot0001$$

☞ In the "Account of the Principal Triangulation of Great Britain and Ireland," the following formulæ are given—

$$D_{\prime} - A_{\prime} = \tfrac{1}{4}\cdot\frac{e^2}{1 - e^2}\cdot\cos^2 l_{\prime\prime}\,\sin 2A_{\prime\prime}\cdot z_{\prime\prime}^{\,2}\cdot\sin 1''$$

$$D_{\prime\prime} - A_{\prime\prime} = \tfrac{1}{4}\cdot\frac{e^2}{1 - e^2}\cdot\cos^2 l_{\prime}\,\sin 2A_{\prime}\cdot z_{\prime}^{\,2}\cdot\sin 1''$$

In working out these expressions with respect to the present examples we have—

$$\begin{array}{rlrl} \log \frac{1}{4} = & \bar{1}\cdot3979400087 & \log \frac{1}{4} = & \bar{1}\cdot3979400087 \\ \log \frac{e^2}{1-e^2} = & \bar{3}\cdot8345159915 & \log \frac{e^2}{1-e^2} = & \bar{3}\cdot8345159915 \\ \cos^2 l_{\prime\prime} = & \bar{1}\cdot8046972330 & \cos^2 l_{\prime} = & \bar{1}\cdot7930642882 \\ \sin 2A_{\prime\prime} = & \bar{1}\cdot9999812911 & \sin 2A_{\prime} = & \bar{1}\cdot9997379520 \\ \log z_{\prime\prime}^{\,2} = & 7\cdot4081585260 & \log z_{\prime}^{\,2} = & 7\cdot4081087226 \\ \sin 1'' = & \underline{\bar{6}\cdot6855748668} & \sin 1'' = & \underline{\bar{6}\cdot6855748668} \\ \therefore \log(D_{\prime}-A_{\prime}) = & \bar{1}\cdot1308679171 & \therefore \log(A_{\prime\prime}-D_{\prime\prime}) = & \bar{1}\cdot1189418298 \end{array}$$

$$\begin{array}{l} \therefore\ D_{\prime} - A_{\prime} = 0''\cdot1352 \text{ which is too great by } 0''\cdot002 \\ \quad A_{\prime\prime} - D_{\prime\prime} = 0''\cdot1315 \text{ which is too small by } 0''\cdot002 \end{array}$$

We may also observe that in all cases in which the greater azimuth $A_{\prime}$ is less than 90°, the second of the above formulæ would intimate that $D_{\prime\prime}$ is greater than $A_{\prime\prime\prime}$, which we know to be erroneous. And when $A_{\prime} = 90^{\circ}$ it intimates that $D_{\prime\prime} = A_{\prime\prime\prime}$, which is also erroneous.

☞ In order to shew the extent to which a change in the assumed values of the earth's polar and equatorial radii can effect the results of geodetic computations, I give the following columns of results, worked out with 7 place logs.—

FOR THE LATEST CONSTANTS.	FOR CONSTANTS FORMERLY USED.
$\left\{ \begin{array}{l} a = 20926348 \\ b = 20855233 \end{array} \right\}$	$\left\{ \begin{array}{l} a = 20923713 \\ b = 20853810 \end{array} \right\}$
$A_{\circ} = 134^{\circ}{}_{\prime\prime}\ 51'{}_{\prime\prime}\ 25''\cdot225$	$A_{\circ}$ = same as before
$A_{\circ\circ} = 44{}_{\prime\prime}\ 22{}_{\prime\prime}\ 55\cdot177$	$A_{\circ\circ}$ = ” ” ”
$A_{\prime} = 134^{\circ}{}_{\prime\prime}\ 44{}_{\prime\prime}\ 03\cdot683$	$A_{\prime} = 134^{\circ}{}_{\prime\prime}\ 44'{}_{\prime\prime}\ 10''\cdot647$
$A_{\prime\prime} = 44{}_{\prime\prime}\ 30{}_{\prime\prime}\ 16\cdot718$	$A_{\prime\prime} = 44^{\circ}{}_{\prime\prime}\ 30{}_{\prime\prime}\ 09\cdot754$
$\Omega = 0{}_{\prime\prime}\ 07{}_{\prime\prime}\ 21\cdot541$	$\Omega = 0{}_{\prime\prime}\ 07{}_{\prime\prime}\ 14\cdot577$
$\nu = 1{}_{\prime\prime}\ 24{}_{\prime\prime}\ 29\cdot956$	ν = same as before
$\Sigma = 1{}_{\prime\prime}\ 24{}_{\prime\prime}\ 29\cdot945$	Σ = ” ” ”
$\alpha_{\prime} = 0{}_{\prime\prime}\ 42{}_{\prime\prime}\ 14\cdot900$	$\alpha_{\prime} = 0^{\circ}{}_{\prime\prime}\ 42'{}_{\prime\prime}\ 14\cdot901$
$\alpha_{\prime\prime} = 0{}_{\prime\prime}\ 42{}_{\prime\prime}\ 15\cdot045$	$\alpha_{\prime\prime} = 0{}_{\prime\prime}\ 42'{}_{\prime\prime}\ 15\cdot045$
$\Delta = 0{}_{\prime\prime}\ 00{}_{\prime\prime}\ 10\cdot852$	$\Delta = 0{}_{\prime\prime}\ 00{}_{\prime\prime}\ 10\cdot681$
$s = 513905\cdot8$ feet	$s = 513847\cdot7$ feet

The increase in $A_{\prime}$ is equal to the decrease in $A_{\prime\prime}$, and the whole amount $6''\cdot9$ of such increase or decrease is owing to the change in the ratio of a to b, and not to their absolute magnitudes. This shews that if the assumed value $\frac{a}{b}$ be not suitable to the locality of the survey, there must of necessity be discrepancies between the azimuths as found by direct observation and computations, in closing work carried on by means of two series of stations. We see also that the values of s differ by about 58 feet in an arc of 97 miles, owing to the change in the values of a and b.

EXAMPLE (Problem 2).

Case 1.

Given the latitude $l_{\prime} = 38^{\circ}$; the azimuth $A_{\prime} = 134^{\circ}{}_{\prime\prime}\ 44'{}_{\prime\prime}\ 02''\cdot72393$; and the length of the geodesic arc $s = 513903\cdot7237$ feet; to find the difference of longitude ω, the latitude $l_{\prime\prime}$ the azimuth $A_{\prime\prime}$, &c.

To find $z_{,}$ we have (from the "Account of the Principal Triangulation of Great Britain and Ireland") the formula—

$$\log z_{,} = \log\left(\frac{s}{R_{,} \sin 1''}\right) + 0\cdot0004862 \times \sin^2(\triangle l') \cdot \sin^2 l'$$

in which $(\triangle l')$ represents any close approximate to the difference of the given and unknown latitudes, so as to have the first three or four decimal places in the expression log $(\sin^2 \triangle l')$ correct.

In the present example we know that $\triangle l' = 1^\circ$ nearly, and $\therefore$ to find $z_{,}$—

$$\begin{array}{rlrl}
\log(0\cdot0004862) = & \bar{4}\cdot6868 & \log R_{,} = & 7\cdot3212526296 \\
\sin^2(\triangle l') = & 6\cdot4837 & \sin 1'' = & \bar{6}\cdot6855748668 \\
\cline{4-4}
\sin^2 l' = & \bar{1}\cdot7931 & & 2\cdot0068274964 \\
 & & \log s = & 5\cdot7108817646 \\
\cline{4-4}
\text{antilog} = & 919\cdot6 & & 3\cdot7040542682 \\
 & & & 919\cdot6 \\
\cline{4-4}
 & & \therefore \log z_{,} = & 3\cdot7040543601
\end{array}$$

$$\therefore z_{,} = 1^\circ{}_{,,}\ 24'{}_{,,}\ 18''\cdot8798$$

Were we to use the more simple formulæ—

$$z_{,} = \frac{s}{R_{,} \cdot \sin 1''}$$

we evidently have—

$$\log z_{,} = 3\cdot7040542682$$

$$\therefore \quad z_{,} = 5058''\cdot8785 = 1^\circ{}_{,,}\ 24'{}_{,,}\ 18''\cdot8785,$$

which is too small by about $0''\cdot001$ only. And since the $0''\cdot001$ part of one second represents not more than an error of $\frac{1}{10}$ of a foot in the whole length of the arc $s = 97$ miles; $\therefore$ it is evident that in all cases we can safely find $z_{,}$ by means of this formula.

Now knowing $A_{,,}$, l', $z_{,}$, in the spherical triangle $S_{,}PD_{,,}$, we can find the angles ω, $D_{,,}$, and the side L'' by the usual forms—

$$\tan \tfrac{1}{2}(D_{,,} + \omega) = \frac{\cos \frac{1}{2}(l' - z_{,})}{\cos \frac{1}{2}(l' + z_{,})} \cot \tfrac{1}{2} A,$$

$$\tan \tfrac{1}{2}(D_{,,} - \omega) = \frac{\sin \frac{1}{2}(l' - z_{,})}{\sin \frac{1}{2}(l' + z_{,})} \cot \tfrac{1}{2} A,$$

$$\begin{array}{rlrl}
\cot\frac{1}{2}A_{,}= & 9\cdot6200681684 & \cot\frac{1}{2}A_{,}= & 9\cdot6200681684\\
\cos\frac{1}{2}(l'-z_{,})= & \underline{9\cdot9562174764} & \sin\frac{1}{2}(l'-z_{,})= & \underline{9\cdot6307496490}\\
 & 19\cdot5762856448 & & 19\cdot2508178174\\
\cos\frac{1}{2}(l'+z_{,})= & \underline{9\cdot9510220423} & \sin\frac{1}{2}(l'+z_{,})= & \underline{9\cdot6525942988}\\
\therefore\tan\frac{1}{2}(D_{\prime\prime}+\omega)= & 9\cdot6252636025 & \therefore\tan\frac{1}{2}(D_{\prime\prime}-\omega)= & 9\cdot5982235186
\end{array}$$

$$\frac{1}{2}(D_{\prime\prime}+\omega) = 22^{\circ}{}_{\prime\prime}\ 52'{}_{\prime\prime}\ 38''\cdot7711$$

$$\therefore\ \frac{1}{2}(D_{\prime\prime}-\omega) = 21^{\circ}{}_{\prime\prime}\ 37'{}_{\prime\prime}\ 38''\cdot7719$$

$$\therefore\ D_{\prime\prime} = 44^{\circ}{}_{\prime\prime}\ 30'{}_{\prime\prime}\ 17''\cdot5430$$

$$\omega = 1^{\circ}{}_{\prime\prime}\ 14'{}_{\prime\prime}\ 59''\cdot9992$$

☞ This case, in which the given latitude is *greater* than the sought latitude, is made known to us by $A_{,}$ being greater than the angle $D_{\prime\prime}$.

To find L″—

$$\begin{array}{rlrl}
\sin z_{,} = & 8\cdot3895856868 & \sin l' = & 9\cdot8965321441\\
\sin A_{,} = & \underline{9\cdot8514912398} & \sin A_{,} = & \underline{9\cdot8514912398}\\
 & 18\cdot2410769266 & & 19\cdot7480233839\\
\sin\omega = & \underline{8\cdot3387529285} & \sin D_{\prime\prime} = & \underline{9\cdot8456993857}\\
\therefore\ \sin L'' = & 9\cdot9023239981 & \therefore\ \sin L'' = & 9\cdot9023239982
\end{array}$$

$$\because\ L'' = 52^{\circ}{}_{\prime\prime}\ 59'{}_{\prime\prime}\ 44''\cdot4850$$

or to find L″ we may use the formula—

$$\tan\frac{1}{2}(L''-l') = \frac{\sin\frac{1}{2}(A_{,}-D_{\prime\prime})}{\sin\frac{1}{2}(A_{,}+D_{\prime\prime})}\cdot\tan\frac{1}{2}z_{,}$$

To find $\delta_{\prime\prime}$ we have the approximate formula 84—

$$\delta_{\prime\prime} = \frac{e^2}{1-e^2}\cdot\sin L''\sin\frac{1}{2}(L''+l')\cdot(L''-l')$$

or the more closely approximate formula 83—

$$\sin\delta_{\prime\prime} = \frac{2\cdot e^2\cdot\sin\frac{1}{2}(L''+l')\sin\frac{1}{2}(L''-l')\sin L''}{(1-e^2)-2\cdot e^2\cdot\sin\frac{1}{2}(L''+l')\sin\frac{1}{2}(L''-l')\cos L''}$$

$$\begin{array}{rl}
\log\frac{e^2}{1-e^2} = & \bar{3}\cdot8345160\\
\sin L'' = & \bar{1}\cdot9023240\\
\sin\frac{1}{2}(L''+l') = & \bar{1}\cdot8994540\\
\log(L''-l') = & \underline{3\cdot5544268}\\
\therefore\ \log\delta_{\prime\prime} = & 1\cdot1907208\\
\delta_{\prime\prime} = & 0^{\circ}{}_{\prime\prime}\ 00'{}_{\prime\prime}\ 15''\cdot5139
\end{array}$$

$$\begin{array}{rl@{\qquad}rl}
\log 2 = & 0\cdot3010300 & & \\
\log e^2 = & \bar{3}\cdot8315592 & & \\
\sin \tfrac{1}{2}(\mathrm{L}'' + l') = & \bar{1}\cdot8994540 & & \\
\sin \tfrac{1}{2}(\mathrm{L}'' - l') = & \underline{\bar{3}\cdot9389661} & & \\
 & \bar{5}\cdot9710093 \;\; . \;\; . \;\; . \;\; . \;\; . & & \bar{5}\cdot9710093 \\
\cos \mathrm{L}'' = & \underline{\bar{1}\cdot7795064} & \sin \mathrm{L}'' = & \underline{\bar{1}\cdot9023240} \\
 & \bar{5}\cdot7505157 & & \bar{5}\cdot8733333 \\
\text{antilog} = & 0\cdot000056300 & & \underline{\bar{1}\cdot9970186} \\
1 - e^2 = & \underline{0\cdot993214854} & \therefore\ \sin \delta_{,,} = & \bar{5}\cdot8763147 \\
 & 0\cdot993158554 & \therefore\ \delta_{,,} = & 0°_{,,}\ 00'_{,,}\ 15''\cdot5146 \\
\text{its log} = & \bar{1}\cdot9970186 & &
\end{array}$$

Then to find l'', and $l_{,,}$, we have—

$$l'' = \mathrm{L}'' + \delta_{,,}; \quad l_{,,} = 90° - l''$$

$\therefore$ By first value of $\delta_{,,}$ we find $l_{,,} = 37°_{,,}\ 00'_{,,}\ 00''\cdot0019$
„ second „ „ , $l_{,,} = 37°_{,,}\ 00'_{,,}\ 00''\cdot0004$

To find $\mathrm{A}_{,,}$, we have—

$$\mathrm{A}_{,,} - \mathrm{D}_{,,} = \sin \mathrm{D}_{,,} \tan \tfrac{1}{2} z_{,} \cdot \delta_{,,}$$

$$\begin{array}{rl@{\qquad}rl}
\sin \mathrm{D}_{,,} = & \bar{1}\cdot8456994 & \therefore\ \mathrm{A}_{,,} - \mathrm{D}_{,,} = & 0°_{,,}\ 00'_{,,}\ 00''\cdot13336 \\
\tan \tfrac{1}{2} z_{,} = & \bar{2}\cdot0886210 & \text{but } \mathrm{D}_{,,} = & \underline{44°_{,,}\ 30'_{,,}\ 17''\cdot5430} \\
\log \delta_{,,} = & \underline{1\cdot1907207} & \therefore\ \mathrm{A}_{,,} = & 44°_{,,}\ 30'_{,,}\ 17''\cdot6764 \\
\therefore\ \log(\mathrm{A}_{,,} - \mathrm{D}_{,,}) = & \bar{1}\cdot1250411 & &
\end{array}$$

☞ In the "Account of the Principal Triangulation of Great Britain and Ireland" (see pages 247, 249, 676, of that work) there is given what is considered the most accurate method of solving this problem. The values of $z_{,}$, ω, $\mathrm{D}_{,,}$, are there found as in the present paper, but the azimuth $\mathrm{A}_{,,}$ and latitude $l_{,,}$ are determined otherwise: thus—

To find $\mathrm{A}_{,,}$ the erroneous formula 96 is used, which gives $\zeta = \mathrm{A}_{,,} - \mathrm{D}_{,,} = 0''\cdot1315$ instead of $0''\cdot1334$.

Then to find $l_{,,}$ the following formula is given—

$$l_{,} - l_{,,} = \frac{s}{\rho \cdot \sin 1''} \cdot \frac{\sin \frac{1}{2}(\mathrm{A}_{,} - \mathrm{A}_{,,} + \zeta)}{\sin \frac{1}{2}(\mathrm{A}_{,} + \mathrm{A}_{,,} + \zeta)} \cdot \left\{ 1 + \frac{z^2_{,}}{12} \cdot \cos^2 \tfrac{1}{2}(\mathrm{A}_{,} - \mathrm{A}_{,,}) \sin^2 1 \right\}$$

in which ρ is the radius of curvature for the meridian for the mean between the known and unknown latitudes, and in which—

$$\tfrac{1}{2}(A_{\prime} - A_{\prime\prime} + \zeta) = \tfrac{1}{2}(A_{\prime} - D_{\prime\prime})$$
$$\tfrac{1}{2}(A_{\prime} + A_{\prime\prime} + \zeta) = \tfrac{1}{2}(A_{\prime} + D_{\prime\prime}).$$

The value of $l_{\prime} - l_{\prime\prime}$ as computed from the above is—

$$l_{\prime} - l_{\prime\prime} = 3600''\cdot0057 = 1^\circ_{\prime\prime}\,00'_{\prime\prime}\,00''\cdot0057$$
$$\therefore\ l_{\prime\prime} = 36^\circ_{\prime\prime}\,59'_{\prime\prime}\,59''\cdot9943,$$

which is nearly $0''\cdot006$ in error, when by the method followed in this paper the error amounts only to about $0''\cdot0004$.

It may perhaps be proper to observe that in the example under consideration we have in reality—

$$\tfrac{1}{2}(A_{\prime} + A_{\prime\prime} - \zeta) = \tfrac{1}{2}(A_{\prime} + D_{\prime\prime})$$

so that the fact of the expression for $l_{\prime} - l_{\prime\prime}$, being written as above shews that its author considered $A_{\prime\prime}$ to be less than $D_{\prime\prime}$: however, we know that $A_{\prime\prime}$ must be greater than $D_{\prime\prime}$.

EXAMPLE (Problem 2).

Case 2.

Given the latitude $l_{\prime\prime} = 37^\circ$; the azimuth $A_{\prime\prime} = 44^\circ_{\prime\prime}\,30'_{\prime\prime}\,17''\cdot67692$; and the length of the geodesic arc $s = 513903\cdot7237$ feet: to find ω, $l_{\prime}$, and $A_{\prime}$, &c.

To find the arc $z_{\prime\prime}$, we have—

$$\log z_{\prime\prime} = \log \frac{s}{R_{\prime\prime} \sin 1} + 0\cdot0004862 \times \sin^2(\triangle l'') \sin^2 l''$$

in which $\triangle l''$ is the nearest approximate which we can easily find to the difference of the known and unknown latitudes. In the present case we know that $\triangle l''$ is nearly 1°.

$$\begin{aligned} \log(0\cdot0004862) &= \bar{4}\cdot6868 \\ \log \sin^2(\triangle l'') &= 6\cdot4837 \\ \sin^2 l'' &= \underline{\bar{1}\cdot8047} \\ & \quad 2\cdot9752 \\ \text{antilog} &= 944\cdot5 \end{aligned}$$

$$\begin{aligned} \log R_{\prime\prime} &= 7\cdot3212277292 \\ \sin 1'' &= \underline{\bar{6}\cdot6855748668} \\ & \quad 2\cdot0068025960 \\ \log s &= \underline{5\cdot7108817646} \\ & \quad 3\cdot7040791686 \\ & \qquad\qquad\quad \underline{944} \\ \therefore\ \log z_{\prime\prime} &= 3\cdot7040792630 \end{aligned}$$

$$\therefore\ z_{\prime\prime} = 5059''\cdot16988 = 1^\circ_{\prime\prime}\,24'_{\prime\prime}\,19''\cdot16988$$

Were we to use the simpler formula—

$$\log z_{\prime\prime} = \log \frac{s}{R_{\prime\prime} \sin 1''};$$

then, obviously, we have—

$$\log z_{\prime\prime} = 3\cdot7040792, \quad \text{and} \therefore \quad z_{\prime\prime} = 1^\circ_{\prime\prime}\ 24'_{\prime\prime}\ 19''\cdot1687$$

which is $0''\cdot0011$ too small.

To find $D_{\prime}$ and ω, we have—

$$\tan \tfrac{1}{2}(D_{\prime} + \omega) = \frac{\cos \frac{1}{2}(l'' - z_{\prime\prime})}{\cos \frac{1}{2}(l'' + z_{\prime\prime})} \cdot \cot \tfrac{1}{2} A_{\prime\prime}$$

$$\tan \tfrac{1}{2}(D_{\prime} - \omega) = \frac{\sin \frac{1}{2}(l'' - z_{\prime\prime})}{\sin \frac{1}{2}(l'' + z_{\prime\prime})} \cdot \cot \tfrac{1}{2} A_{\prime\prime}$$

$$\begin{aligned}
\cot \tfrac{1}{2} A_{\prime\prime} &= 10\cdot3881059553 \\
\cos \tfrac{1}{2}(l'' - z_{\prime\prime}) &= \underline{\ \ 9\cdot9544060605} \\
& \quad 20\cdot3425120158 \\
\cos \tfrac{1}{2}(l'' + z_{\prime\prime}) & \quad \underline{\ \ 9\cdot9490947477} \\
\therefore \tan \tfrac{1}{2}(D_{\prime} + \omega) &= 10\cdot3934172681
\end{aligned}$$

$$\begin{aligned}
\cot \tfrac{1}{2} A_{\prime\prime} &= 10\cdot3881059553 \\
\sin \tfrac{1}{2}(l'' - z_{\prime\prime}) &= \underline{\ \ 9\cdot6386781718} \\
& \quad 20\cdot0267841271 \\
\sin \tfrac{1}{2}(l'' + z_{\prime\prime}) &= \underline{\ \ 9\cdot6600485181} \\
\therefore \tan \tfrac{1}{2}(D_{\prime} - \omega) &= 10\cdot3667356090
\end{aligned}$$

$$\therefore \tfrac{1}{2}(D_{\prime} + \omega) = 67^\circ_{\prime\prime}\ 59'_{\prime\prime}\ 31''\cdot4286$$

$$\therefore \tfrac{1}{2}(D_{\prime} - \omega) = 66^\circ_{\prime\prime}\ 44'_{\prime\prime}\ 31''\cdot4287$$

$$\begin{aligned}
\therefore D_{\prime} &= 134^\circ_{\prime\prime}\ 44'_{\prime\prime}\ 02'\cdot8573 \\
\omega &= \quad 1^\circ_{\prime\prime}\ 15'_{\prime\prime}\ 00''\cdot0001
\end{aligned}$$

☞ This case in which the given latitude is *less* than the sought latitude, is made known to us by the given azimuth $A_{\prime\prime}$ being less than the computed angle $D_{\prime}$.

To find L',—

$$\begin{aligned}
\sin z_{\prime\prime} &= 8\cdot3896105836 & \sin l'' &= 9\cdot9023486165 \\
\sin A_{\prime\prime} &= \underline{9\cdot8456996715} & \sin A_{\prime\prime} &= \underline{9\cdot8456996715} \\
& \quad 18\cdot2353102551 & & \quad 19\cdot7480482880 \\
\sin \omega &= \underline{8\cdot3387529285} & \sin D_{\prime} &= \underline{9\cdot8514909614} \\
\therefore \sin L' &= 9\cdot8965573266 & \therefore \sin L' &= 9\cdot8965573266
\end{aligned}$$

$$\therefore L' = 52^\circ_{\prime\prime}\ 00'_{\prime\prime}\ 15''\cdot3097$$

To find L′ we can also use the formula—

$$\tan \tfrac{1}{2}\,(l'' - \mathrm{L}') = \frac{\sin \frac{1}{2}\,(\mathrm{D}_{,} - \mathrm{A}_{,,})}{\sin \frac{1}{2}\,(\mathrm{D}_{,} + \mathrm{A}_{,,})} \cdot \tan \tfrac{1}{2}\, z_{,,}$$

To find $\delta_{,}$, we have—

$$\begin{array}{rcll}
\log \dfrac{e^2}{1 - e^2} & = & \bar{3}\cdot83451 & \\
\sin \mathrm{L}' & = & \bar{1}\cdot89655 & \therefore\ \delta_{,} = 0^{\circ}{}_{,,}\ 00'{}_{,,}\ 15''\cdot3098 \\
\sin \frac{1}{2}\,(l'' + \mathrm{L}') & = & \bar{1}\cdot89946 & \therefore\ l' = \mathrm{L}' - \delta_{,} = 51^{\circ}{}_{,,}\ 59'{}_{,,}\ 59''\cdot9999 \\
\log\,(l'' - \mathrm{L}') & = & \underline{3\cdot55445} & \\
\therefore\ \log \delta_{,} & = & 1\cdot18497 & \therefore\ l_{,} = 38^{\circ}{}_{,,}\ 00'{}_{,,}\ 00''\ 0001
\end{array}$$

To find $\mathrm{A}_{,}$, we have—

$$\mathrm{D}_{,} - \mathrm{A}_{,} = \sin \mathrm{D}_{,} \tan \tfrac{1}{2}\, z_{,,} \cdot \delta_{,,}$$

$$\begin{array}{rcll}
\sin \mathrm{D}_{,} & = & \bar{1}\cdot85149 & \\
\tan \frac{1}{2}\, z_{,} & = & \bar{2}\cdot08865 & \therefore\ \mathrm{D}_{,} - \mathrm{A}_{,} = 0^{\circ}{}_{,,}\ 00'{}_{,,}\ 00''\cdot1334 \\
\log \delta_{,,} & = & \underline{1\cdot18497} & \text{But } \mathrm{D}_{,} = \underline{134_{,,}\ \ 44_{,,}\ \ 02\ \cdot8573} \\
\therefore\ \log\,(\mathrm{D}_{,} - \mathrm{A}_{,}) & = & \bar{1}\cdot12511 & \therefore\ \mathrm{A}_{,} = 134^{\circ}{}_{,,}\ 44'{}_{,,}\ 02''\cdot7239
\end{array}$$

☞ In the "Account of the Principal Triangulation of Great Britain and Ireland" the formula from which to find $l_{,}$ is—

$$l_{,} - l_{,,} = \frac{s}{\rho \cdot \sin 1''} \cdot \frac{\sin \frac{1}{2}\,(\mathrm{D}_{,} - \mathrm{A}_{,,})}{\sin \frac{1}{2}\,(\mathrm{D}_{,} + \mathrm{A}_{,,})} \cdot \left\{ 1 + \frac{z^2_{,,}}{12} \cdot \cos^2 \tfrac{1}{2}\,(\mathrm{A}_{,} - \mathrm{A}_{,,}) \sin^2 1'' \right\}$$

and the resulting value of $l_{,} - l_{,,} = 1^{\circ}{}_{,,}\ 00'{}_{,,}\ 00''\cdot0059$
$\therefore\ l_{,} = 38^{\circ}{}_{,,}\ 00'{}_{,,}\ 00''\cdot0059$ which is too great by 0″·006, while by the method in this paper the error is only 0″·0001.

In the treatise on "Geodesy" in Spon's Dictionary of Engineering, the unknown latitudes in the first and second cases of the problem are determined by means of the formulæ—

$$l_{,} - l_{,,} = \left\{ - \frac{s \cdot \cos \mathrm{A}_{,}}{\mathrm{R}_{,} \sin 1''} + \frac{s^2 \cdot \sin^2 \mathrm{A}_{,} \tan l_{,}}{2 \cdot \mathrm{R}^2_{,} \cdot \sin 1''} \right\} (1 + e^2 \cdot \cos^2 l_{,})$$

$$l_{,} - l_{,,} = \left\{ + \frac{s \cdot \cos \mathrm{A}_{,,}}{\mathrm{R}_{,,} \cdot \sin 1''} - \frac{s^2 \cdot \sin^2 \mathrm{A}_{,,} \tan l_{,,}}{2 \cdot \mathrm{R}^2_{,,} \cdot \sin 1''} \right\} (1 + e^2 \cdot \cos^2 l_{,,})$$

from which we find $l_{,} - l_{,,} = 3600\ 091$
and $l_{,} - l_{,,} = 3600{\cdot}632$; giving an error of $0''{\cdot}1$ in the first case, and an error of $0''{\cdot}6$ in the second case.

In Chambers' "Practical Mathematics" the formulæ differ from the above in having the factors $(1 + e^2 \cdot \cos l_{,})$, $(1 + e^2 \cdot \cos^2 l_{,,})$, replaced by $(1 + 2\epsilon \cdot \cos^2 l_{,,})$ and $(1 + 2\epsilon \cdot \cos^2 l_{,})$ which are greater; and ∴ obviously the results must be the more erroneous.

Their method of finding the difference of longitude is by means of the formula

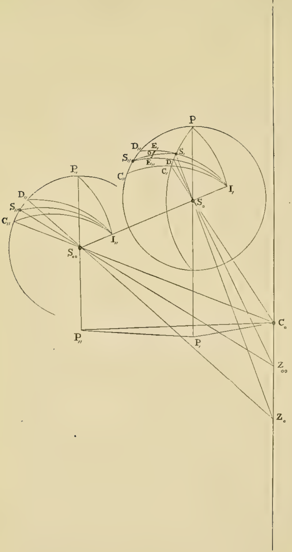
$$\omega = \frac{s \cdot \sin A_{,}}{R_{,} \cdot \sin 1'' \cdot \cos l_{,,}} = \frac{s \cdot \sin A_2}{R_{,,} \cdot \sin 1'' \cdot \cos l_{,}}$$

$$= z_{,} \cdot \frac{\sin A_{,}}{\cos l_{,,}} = z_{,,} \cdot \frac{\sin A_{,,}}{\cos l_{,}}$$

from which we obtain the values

$$\omega = 4499''{\cdot}838 = 4500''{\cdot}355$$

having a difference $= 0''{\cdot}517$.


P
$D_{\prime\prime}$
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ART. II.—*Notes on the Radiometer.*

BY R. L. J. ELLERY, ESQ.

[Read 11th May, 1877.]

ART. III.—*On the Improvement of the Port of Melbourne.*

BY T. E. RAWLINSON, C.E.

[Read before the Royal Society of Victoria, 8th June, 1876.]

In resuming the subject of a paper read before the members last session on proposed works for the improvement of the Port of Melbourne, I purpose replying, as far as possible, to questions asked and objections raised at the time and since to certain features of the proposed scheme.

These questions and objections appear to resolve themselves into the following:—

1st. The data on which I assume the width of 1000 feet as necessary for the proposed new channel and basin.

2nd. The oft repeated allegation that the River Yarra has debouched at various times at several places between St. Kilda and the present entrance at Williamstown.

3rd. That the estimated total cost is far in excess of our present means.

In replying to the first I must remind members that I stated the width assumed was based on certain generalisations, and subject to modification if necessary on receipt of accurate data as to the amount of flood discharges down the Yarra; but although to this extent empirical, it was in a large measure based on a knowledge of the extensive discharge of flood waters over the St. Kilda-road, between the Prince's Bridge approach, and the Immigration Barracks Hill, additional to the heavy discharge through the Prince's Bridge and the Dry Arch south of it. In addition to this evidence there was the 200 feet span of Church-street Bridge flooded to a great height, through which the water tore in a torrent, destroying the sheet piling and roadway underneath; while at Johnston-street Bridge, with an opening of 175 feet, the water rose to a great height and was equally mischievous, owing to its great velocity and consequent destructive energy. The sectional area of the torrent at this place was between 4000 and 5000 feet, whilst between this bridge and Melbourne the volume of the Yarra was considerably augmented by numerous small streams and creeks flowing into it, adding, at least, from 800 to 1000 feet additional of sectional area of flood water.

Since the date of my paper I have noticed that Mr. Gordon in one of his reports estimates that an additional flood channel, of about 4000 feet sectional area, in addition to the present river, is required for the passage of flood waters below Melbourne to the Bay—making a total of about 8000 feet of area; but in the face of all the facts known of the great volume of the waterflow through Prince's Bridge, I do not think such sectional area equal to the work to be done.

The discharge in heavy floods through Prince's Bridge and the Dry Arch is a pitch or fall of water rather than a flow, whilst over the St. Kilda-road causeway the water rushed as over a weir head, the velocity in each case being necessarily very great.

In Flinders-street the water stood upwards of ten feet deep, and spread in a sheet southwards to the foot of Emerald Hill, and although extending over so large a surface, it flowed with considerable velocity even when the flat was comparatively unobstructed; but now, with solid embanked causeways and extensive piles of buildings covering the low ground, the waters of any future flood will of necessity be confined in narrower bounds, and rise to a greater height, in order to escape to the Bay.

For the above reasons I do not think the width given for the proposed new channel (1000 feet) excessive for the outflowing water when the above conditions are fully considered; but although 1000 feet width be adopted for a flood channel, it is unnecessary for the present to excavate the full breadth and depth for that purpose only, as the work may be deferred until the space is required for dock extension, or the materials wanted for reclamation of new land.

For carrying away flood waters, the channel, if taken out to 1000 feet wide, and to the depth of ordinary high-water mark, and the ship channel taken out for its full depth of 20 feet at low water, and 400 feet wide at the top, would give a sectional area of about 10,000 feet, the mean velocity and area of which would be more nearly approaching the required capacity for discharging the excess of the waters requiring passage, without unduly impeding the free flow and consequent backing-up of the flood waters which a narrower channel would cause.

In reply to the allegation that the Yarra has at various times debouched by different outlets between St. Kilda and Williamstown into the Bay, I fail to see any grounds for such assertions, for the statement is almost too absurd for refutation, that because there is a slight depression in made ground it must at some time or other have been a water course.

The arguments are based on a fallacy, and cannot in my opinion be justified by analogy or by reason; and, were it not for the repetition of these views from time to time, I would not again recur to them, having in the previous paper dealt with the question, but it is perhaps better to risk a slight repetition than uncertainty or obscurity on this point.

Before the low-lying lands around Emerald Hill were formed, the Yarra must have entered the Bay about the site of Prince's Bridge, and as the land made by precipitation from its waters, by silt and by drift, the embouchure would gradually be forced along in the direction of its present channel, and the singular formation of the river at Humbug Reach is one of the strongest possible evidences of such growth.

It is quite possible and probable that in times of flood, such as in the year 1863, the surcharged waters overflowing their banks would pass away over the low flats in a direct line for the Bay, but this is quite a different matter to the bold assertions made, that such courses are the old filled-in beds of the Yarra, or that the Yarra in its normal condition ever flowed in any other channel than its present one.

That views such as are enunciated in this and the preceding paper are correct, and proven as far as such things can be proven, are amply illustrated by analogy with similar causes and results of both the past and the present.

In this country, as elsewhere, we have ample evidence that from remote ages climatic agencies have been much the same as in modern times, and that storms of thousands of years ago prevailed from the same quarter of the heavens as in the present day.

The extinct volcanoes of the West give evidence of this fact in the deposition of ashes, scoriæ, and tufa on what must have been the leeward of the Hill then, as it is the leeward now, during bad weather.

Tower Hill, near Warrnambool, is a case in point, where the greatest preponderance of volcanic ash and tufa lies towards the south-east, in the direction where such deposits would be made in the present day under the influence of the prevailing gales during stormy weather.

That such deposits have not been casual outbursts is evidenced in sinking well-shafts through the strata for water. In one case, after passing through alternations of this strata, a bottom was reached between 60 and 70 feet from the surface, showing an ancient turf and grass surface.

The make of the land around Emerald Hill, by deposit of silt and gravel brought down the rivers and the literal drift along the shores, is not only illustrated by similar action within the brief period of our occupancy of Port Phillip, but by analogy with the examples of make of the low flat country of Gippsland, terminating against the sea in the Ninety-Mile Beach.

The importance and extent of the agencies in operation causing these deposits can be better comprehended when it is remembered that the whole of the ravines and gullies of the Yarra basin, as well as those on the Gippsland slopes of the Dividing Range, have been eroded by rains and melted snows, and the materials washed down to form the lower flat country.

The geological evidence of these facts may be termed as almost absolute and complete.

The objections made to the large amount of the estimated cost for the whole work of port and harbor formation are equally untenable with those raised against the theory of the river formation, when it is borne in mind that the gross estimated sum is for a scheme of works extending over many years, and the whole cost of which will be more than recouped by the vastly increased value given to the reclaimed lands, a large portion of which at the present time is of little, if any, value. The works proposed, whether as a whole or only in part, will be actual creation of a large amount of valuable property in addition to the conservation of the harbor and improvement of the port, leaving it free for ever.

Up to the present I have been unable to obtain information as to the expenditure on the ports of London, Liverpool, or other places; but, from personal knowledge of the character and extent of the two named, and the nature of the works, I have no hesitation in stating that the cost cannot have been less than from 15 to 20 millions each, whilst in the case of Liverpool nearly the whole of the outlay has been made within the last 150 years, and nearly one-half within my own recollection.

For Melbourne the question of Harbor Improvements is now becoming one of vital importance, for in a few years, if nothing is done, its harbor will be a thing of the past, owing to the rapid silting-up which is now going on.

To object to the large sum named for a whole scheme of harbor works, is scarcely a fair objection as put; because the sum named, although a very large one, is but prospective, and its rate of expenditure dependent on the future extension of the trade of the port; and because the amount, if expended, is for objects equal in importance and value to any ever accomplished in any age or country for extent, usefulness, or economy; and further, the capital named for expenditure is nominal only, seeing that the whole amount is refunded from the increased value of the reclaimed lands, giving to the country a surplus of value beyond the nominal capital named, in addition to which we would have an acreage of water space, quay wall, and quay room equal to many of the large ports of the world, free of debt, and which may be open to the navies and commerce of the world free of all charges beyond those for lights and pilotage.

The modified scheme which I now submit as being adapted to our immediate requirements presents the same advantages—proportional in their extent with the original scheme for the whole—as before submitted, without in any way interfering with the ultimate carrying out of the entire work.

REDUCED ESTIMATE OF EXPENDITURE.

Excavation, 11,296,395 cubic yards, at 10d. ...		£470,183	2	6
Coring the harbor quays with rubble-stone, 8500 lineal yards at £50 ...		425,000	0	0
Quay wall to channel and Queen's Wharf, 8000 yards at £100 ...		800,000	0	0
		£1,695,183	2	6
Fender Piling and miscellanea ...		304,816	17	6
Gross Total ...		£2,000,000	0	0
Materials available for the reclamation of land, equal to ...	1167 acres.			
Deduct for quays ...	427 „			
	740 „ at £5000	£3,700,000	0	0
Surplus Value ...		£1,700,000	0	0

Immediate gain to the port—

Harbor ...	... ...	500 acres.
New channel ...	...	200 ,,
River basin ...	...	50 ,,
Total	...	750 acres.

Length of quay wall	...	8,000 lineal yards.
Area of quay space	...	427 acres.

In the above estimate and statement the injuries accruing from delay and the advantages to be derived from immediate action are so great that I now leave the facts to speak for themselves. In this paper, as in the original one, the cost of all works are estimated at outside prices, and the benefits understated.

Before closing I may be permitted to point out how the proposed harbor works, whilst materially affecting the question of harbor defences as originally submitted by the Royal Engineer Officers who have considered the question, owing to the material change of conditions in the Bay, in the event of these or similar works being undertaken, may be converted into strong and almost impregnable fortifications for the defence of the port, and render the possibility of shelling Melbourne and Williamstown from the Bay improbable, without first silencing the batteries—a thing which ought to be impossible.

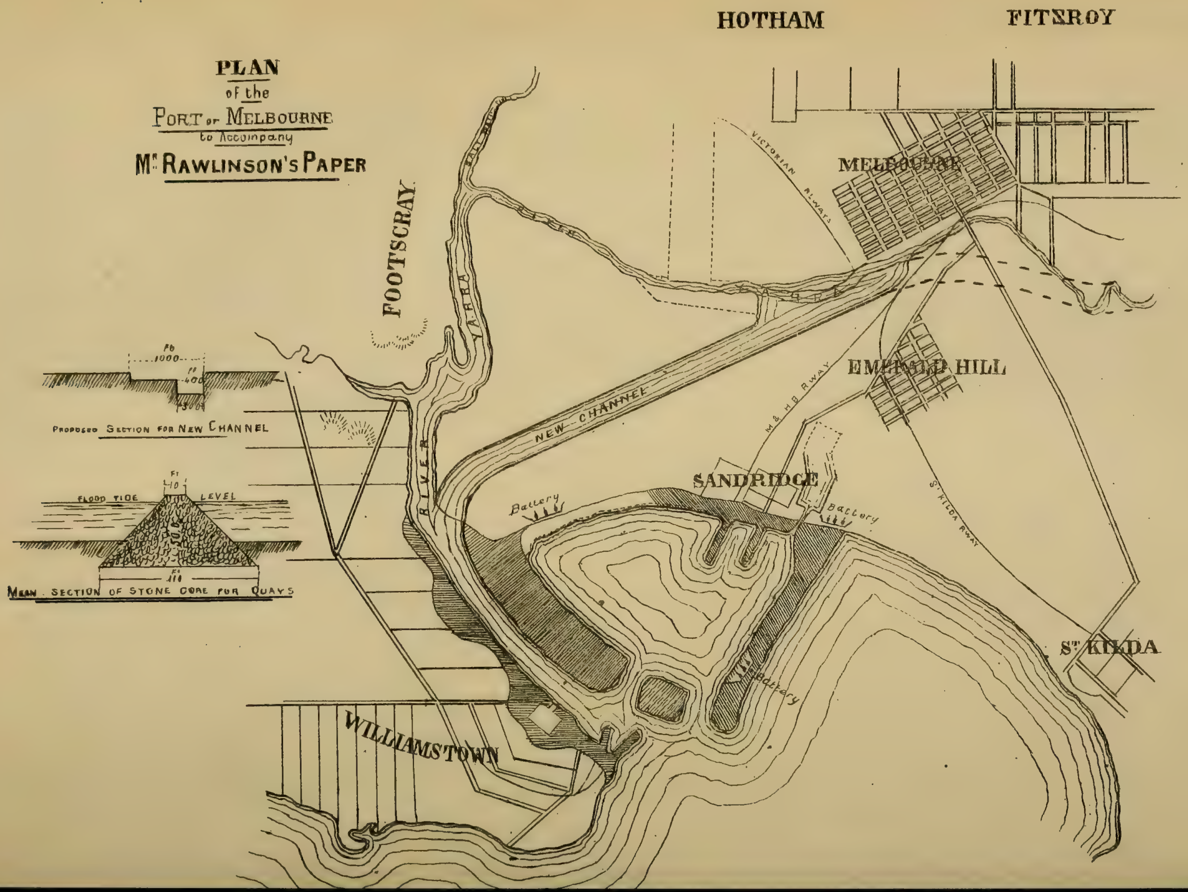
At the end of the south pier a site is shown for a battery in position of Moncrief guns, which construction, with stone facing to above high-water, may from that point have earthwork defences, sodded in the usual way; and such guns as described, with a horizontal fire, would sweep a range of not less than five miles, being themselves unassailable, except to chance shots or an uncertain, plunging, or vertical fire.

The magazines for these guns need to be of no great size, because along the causeway a light tramway could be constructed, under shelter of a covered way, for the purpose of conveying ammunition from land magazines as required.

In case of the quay battery being injured from any cause, it would be untenable for an enemy without first silencing the land batteries which cover it from Sandridge and the river entrance.

I do not presume on these matters to speak with authority, but rather as indicating the points which are available for harbor defence, and how they may be utilised.

The rates previously given for the cost of the work so much exceed those paid for similar work that I have been



PLAN of the PORT OF MELBOURNE to Accompany MR RAWLINSON'S PAPER

induced to bring them down more in consonance with actual prices now current, but even with this reduction the margin of excess is very large.

ADDENDA.—The dotted lines on Plan show where a diversion of the Yarra from the Botanic Gardens to the junction of the new channel may be made with great advantage to serve in times of flood, and also afford very great facilities in increased station ground and quay and dock room abutting on Flinders-street; but as this portion of the subject was not directly connected with the Port improvements, as generally understood, it was omitted from the body of the original paper. T. E. R.

7th February, 1878.

ART. IV.—*Comparison of the Melbourne and Paris Reflecting Telescopes.*

BY R. L. J. ELLERY, ESQ.

[Communicated 8th June, 1876.]

ART. V.—*On Various Forms of Electrometer.*

BY R. L. J. ELLERY, ESQ.

[Communicated 10th July, 1876.]

ART. VI.—*On the Absence of Sun Spots during the Year.*

BY R. L. J. ELLERY, ESQ.

[Communicated 25th September, 1876.]

ART. VII.—*Notes on a Chronographic Apparatus, with Huyghen's Parabolic Pendulum.*

BY R. L. J. ELLERY, ESQ.

[Read 25th September, 1876.]

ABOUT three years ago, at a meeting of the physical section of this Society, I gave a brief *résumé* of the various methods that had been tried for obtaining *uniform rotation*, more especially for astronomical and physical instruments; and I pointed out that as the desired result had been only approached, but in no case obtained, it was a subject worthy of the consideration of the section, and it consequently formed the matter for discussion at a subsequent meeting.

It may be as well to state here that all the most successful attempts to solve this mechanical problem involved the use of the fly, the rotating or conical pendulum, and reciprocating pendulum, either alone or in combination.

The governor of a steam-engine is an apparatus the object of which is to secure uniform rotation, and is usually simply a double conical pendulum; but we know that as the time of rotation of a conical pendulum varies very considerably with the distance the pendulum's bobs are from the axis of rotation, this arrangement alone cannot possibly secure the desired effect, while it usually serves to govern the supply of steam sufficiently to obtain enough uniformity of motion for the practical purposes of a steam-engine. It is, however to the case of the astronomical or physical chronograph, where absolute uniformity is the most to be desired, and indeed a necessity, that I shall have principally to refer; and I shall therefore limit my observations to this higher requirement.

Although the conical pendulum is sometimes used for governing chronographic instruments, it does not, for the reason stated above, afford good results; if however it were possible to secure a constant driving force and resistance, and therefore a constant arc, it would no doubt be perfect; but we know it is impossible to attain these conditions.

In my experiments I have found that a simple free conical pendulum, with a "bob" very heavy in proportion to its length, gives results very near to uniformity if the train be moderately good.

In order to secure a nearly uniform arc with the conical pendulum many devices have been adopted, most of which depend upon having an excess of driving power and the variable excess used up by friction which is brought into play by the pendulum itself as its arc increases beyond a certain limit; but as giving the pendulum any work of this kind to do leaves it no longer free, it becomes simply a "make shift," and can only approach uniformity within larger limits than should be nowadays admissible.

The most successful "governors" of this class hitherto constructed appear to be those where the motion of the mechanism is rendered approximately uniform by the fly, and then finally controlled by a reciprocating pendulum, as in "Bond's Spring Governor," or "Cook's Governor," where a driven train of wheels is governed by a fly, but pulled up every half-second by a vibrating pendulum; the pulling-up being made as gradual as possible by means of a light spring or weight inserted between the fly and the pendulum, allowing the former to continue revolving with increasing resistance until the latter allows its wheel to escape and so free the fly. These are practically the best forms of chronographic governors in general use, but as there is a periodic error of half a second inherent in them they are really imperfect.

There is a form of governor which almost secures uniform rotation, namely the vibrating spring; and the more rapid the vibrations are the more nearly perfect is the result. Some chronographs have been made on this plan, and are known as Hipps' Chronographs. They consist of a driven train and registering barrel, governed by a flat, straight steel spring, whose end just touches the ends of the teeth of a wheel, but which by a little rotatory force in the wheel can be pushed or bent so as to allow the teeth to pass it one after another; the rate at which the wheel rotates being governed by the natural time of vibration of the spring, which is constant at the same temperature, and the rotation of the train is therefore uniform, except for the small periodic error of which the time of the spring's vibration is the measure. In practice, however, I believe the escape-wheel sometimes slips or runs. The noise, too, caused by the vibration of the spring is almost intolerable, and one of the American observers at the late transit of Venus told me he had to dig a big hole in the ground, place the apparatus in it, and cover it over before he could bear the din.

Siemens proposed a "governor" where the control was afforded by the varying friction of a fluid in a rotating parabolic cup. This, although theoretically excellent, does not appear to have given satisfactory results in practice.

After this brief glance at the methods already adopted or proposed for obtaining uniform rotation, I will now return to the more special subjects of these notes.

At the subsequent meeting of our Section A the question of uniform rotation was discussed, and Mr. Kernot suggested Huyghens' Parabolic Pendulum as a governor, and submitted a plan for its construction. Now, Huyghens' pendulum was invented 200 years ago, and is theoretically a perfect governor; but with the exception of a rough imitation of the principle in a steam-engine governor I could not find that it had ever been used or even tried. I determined, however, to adopt Mr. Kernot's suggestion, and try this governor. At first the results gave me no encouragement, and I almost determined to give it up, more especially as I imagined that there must be some almost insuperable practical difficulty in the way to account for so old and theoretically perfect a "governor" never having been adopted. However, by a little perseverance and alteration of form of pendulum, I arrived at better results, and eventually succeeded in getting a pendulum constructed which is almost practically perfect, and the performance of which has withstood far more trying tests than it would be subjected to in practice. Huyghens' Parabolic Pendulum therefore has in my hands given the closest approximation to uniform rotation ever yet, I believe, obtained; and that with a mechanism so simple and easily constructed as to put all the more elaborate but less effective forms in the shade.

While in England last year I read a paper to the Royal Astronomical Society on "Some Experiments with Huyghens' Parabolic Pendulum," but was not able to show one in operation. I can now do so, and that is my excuse for bringing it under your notice this evening. In the paper referred to I gave the principle of construction I had adopted, and the conditions I had found necessary to secure success. It is nevertheless, I think, desirable to give a brief description of the pendulum in this place, more especially as I have the whole apparatus in working order before you.

This chronograph apparatus is not very different from the ordinary forms, and is styled a "barrel chronograph," because the registration takes place on paper covering a barrel which, by reason of the perfect governance of the pendulum, revolves precisely once in a minute, while a syphon pen, actuated by an electro magnet, makes a mark on the paper every second, as the current from a galvanic battery is transmitted by a miniature key operated by the mechanism of a clock or chronometer.

The syphon pen really marks a continuous line, which is interrupted every second by a small "offset" or "tooth" and constitues the "mark;" and an "offset" is left out once in every complete revolution of the barrel, every minute in fact, at the same time the little carriage carrying the pen and magnet is continually progressing in the direction of the length of the barrel, at the rate of about one-tenth of an inch per minute, converting the continuous line into a spiral on the cylinder.

I described a chronograph to this Society about 13 or 14 years ago, and as the principle in this is much the same as in the one then described, and very similar to other barrel chronographs—such as Bond's, Hipps', &c.—it will not be necessary to refer to any details except the pendulum, which in this case is the only new or peculiar arrangement.

"Let A A *(Fig.* 1.) be a vertical axis of rotation, which can be driven by clockwork acting at the top or bottom of the axis; from this axis a pendulum (P) is suspended in such a way that when it hangs vertically the string (S) lies wrapped over a curved surface, which forms part and parcel of the vertical axis. This curve is the evolute of a parabola, whose distance from vertex to focus is half the length of the required pendulum (when vertical). Now, let the axis revolve, and the pendulum will fly out from its vertical position, more or less, according to its weight and the driving power; the arc described by the pendulum, as it increases its distance from the vertical, will be a parabola, by reason of the string gradually unwrapping from the evolute (E). Now, from the properties of the parabola, it follows that the vertical distance between the centre of rotation of the pendulum (P) and the intersection of the string (S) with the axis of rotation of the pendulum will remain constant; and therefore that the length of the pendulum remains constant at whatever arc it may rotate.

"To practically secure these conditions it is necessary, first that the evolute shall be properly and precisely made; and secondly, that it shall be so adjusted that the axis of the evolute and involute shall be coincident with the axis of rotation.

"The pendulums I had constructed are *half-seconds,* that is, rotating once in a second. They are suspended in a hard gun-metal frame (Fig. 2), pivoted at the top and bottom, the lower pivot resting on an end jewel, the upper pivot supported by a strong cast-iron bracket, and it is driven by a contrate wheel in the clock train, engaging into a pinion in the lower end of a frame. The frame is open (as shown in *Fig.* 1) to allow of the middle part of the axis of rotation being clear for the evolute and the pendulum string or rod. The evolute is fixed at M, and is capable of adjustment at right-angles to the axis of rotation by a screw (Q), the proper position of the curve in the other direction being practically secured by careful workmanship, more especially in the construction of the evolute itself.

"The pendulum consists of a spherical bob, weighing about two and a half pounds, on a steel rod about one-tenth of an inch thick, and suspended by a long and *exceedingly thin* steel spring secured to the top of the evolute at N.

"The regulation of the length of the pendulum is done in the ordinary way with a nut at the bottom of the steel rod.

"The governor thus made with ordinary care and workmanship is by far the best of any of which I have had experience, and has furnished results better, I believe, than any others used with chronographs; at the same time it is simple and inexpensive."*

It is very necessary that the suspension-spring should be of the thinnest steel possible, and I have found what is known as French clock pendulum-spring to answer very well. The adjustment of the evolute is a somewhat tedious operation, but can be accomplished with great precision with care. To get its proper position, if the time of rotation increases with an increase of arc—in other words, if it revolves slower for increase of arc—the axis of the evolute is beyond the axis of rotation (reckoning from the pendulum side of the axis), and it is too near if it revolves more rapidly for increase of arc. Of course for each alteration of the position of the evolute a considerable alteration of the length of the pendulum becomes necessary, and this somewhat complicates the adjustment; but with a barrel chronograph this is easily overcome by alternately increasing and diminishing the arc of the pendulum by adding to and subtracting from the driving weight.

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* Extract from Monthly Notices of the Royal Astronomical Society; page 72, Vol. XXXVI.

Fig. 1.

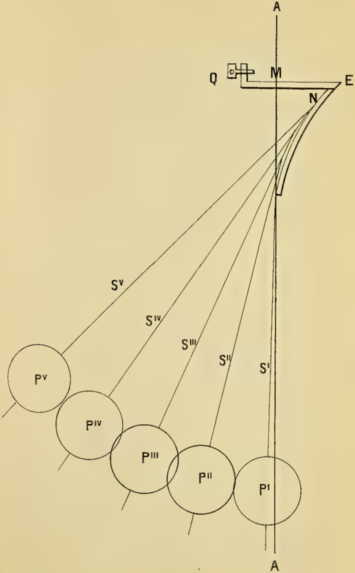

Fig. 2.

ART. VIII.—*Notes on the Longitude of the Melbourne Observatory.*

BY E. J. WHITE, ESQ.

[Read before the Royal Society of Victoria, 25th September, 1876.]

THE Melbourne Observatory having been selected by the American and German parties charged with the observation of the last transit of Venus in these parts of the world as a principal station of reference for the determination of the longitudes of their stations, it becomes a matter of some importance to investigate the authority on which the longitude of the Melbourne Observatory itself depends.

The longitude of Melbourne Observatory was originally determined from that of Williamstown by means of triangulation. The longitude of Williamstown Observatory was found by means of moon culminations observed in the years 1860, 1861, and 1862; of these 142 were compared with corresponding observations at Greenwich and the Cape of Good Hope, from which 9h. 39m. 38·8s. was computed and adopted as the longitude east of Greenwich; the triangulation showed that the Melbourne Observatory was 16·00s. to the east of Williamstown, so that 9h. 39m. 54·8s. was adopted for the former. In the year 1874 we were requested by the German Commissioners entrusted with the management of the transit of Venus expeditions to observe all the moon culminations that were visible in Melbourne during the months of October, November, and December, in 1874, and January of the next year. This was done, and we succeeded in observing 29 culminations of the first limb, and 20 of the second limb. On finally reducing these observations lately, it became a matter of interest to see how this independent determination of our longitude would agree with the one derived from Williamstown. Sir George Airy, the Astronomer Royal, having recently obligingly furnished us with the observations of the moon taken during the same period at Greenwich, it became possible to easily determine this agreement without directly computing the longitude. This was done in the following manner:—The Greenwich list contains the Nautical Almanac errors of the moon's right ascension, as found from actual observation at Greenwich; the errors of the Nautical Almanac were also computed from the Melbourne observations, using our adopted longitude; if, now, the Melbourne errors for the same dates come out the same as the Greenwich errors, it may be inferred that our adopted longitude is correct, or any difference that may be found could be converted into a correction of our adopted longitude. On comparing the Greenwich and Melbourne lists it was found that on fifteen days the moon had been observed at both places, and on interpolating the Greenwich errors, to make them correspond to the time of the Melbourne errors, and taking their mean, it was found that the mean error of the Nautical Almanac was + 0·58s. from the Greenwich observations, and +0·57s. from the Melbourne ones. These results are so nearly identical as to show that our adopted longitude is quite as accurate as can be possibly obtained from the method of moon culminations. A distinguished American mathematician, Professor Peirce, of Harvard University, from theoretical considerations, estimated one second of time as the utmost limit of accuracy to be obtained by this method. Professor Hall, however, of the Washington Observatory, has recently discussed the longitude of his Observatory, as determined by means of the Atlantic cable, transportation of chronometers, and moon observations; and assuming the telegaphic result to be the correct one, he finds a difference of rather more than two seconds to exist between the moon and electric determination, while the chronometric and electric results are nearly identical. Now, if we convert the above difference between the errors of the moon's place, as found at Greenwich and Melbourne into a correction of the latter's longitude, it will amount to only three-tenths of a second; combining this with a weight proportional to the number of observations from which it is derived, it would indicate an increase to our adopted longitude of only three-hundredths of a second of time. Having thus reached the limit of accuracy of which the method of moon culminations is capable, any other determination of our longitude would have to be made either by transmission of large numbers of chronometers—a very expensive and troublesome process—or by means of the electric telegraph. In conclusion, I will state that I consider the longitude of Melbourne to be as well determined as that of any other place in the Southern hemisphere, and better than that of any other place in Australasia. The only other places in Australia where long-continued observations of moon culminations have been made for finding the longitude are Parramatta and Sydney; at both of these places, however, very inferior instruments were used. For the latter place, however, a fine transit circle, of greater power than the Melbourne one, has been lately constructed, and is now daily expected to arrive from England; and as the difference of longitude between Melbourne and Sydney has been accurately measured by means of the telegraph, it will be easy to compare its longitude results with our own. At the Adelaide Observatory no special observations for longitude have as yet been taken. There, also, the Government is just about to order a transit circle, the telescope of which will be somewhat larger than our own; and as the difference of longitude has also been telegraphically determined, its results will be immediately comparable with our own. The acquisition of two such fine instruments by the neighbouring Observatories is a matter for congratulation, and will enable them in future to take their share of the immense work to be done in the Southern hemisphere, an undue proportion of which has lately fallen to Melbourne.

ART. IX.—*Notes on Iron Arches.*

BY W. C. KERNOT, M.A., C.E.

[Read 25th September, 1875.]

THE application of iron, and especially of wrought iron, to bridge-building is deservedly ranked as one of the most notable of those innovations in civil engineering practice that have been made in modern times. It has enabled us to cross chasms of enormous width and depth, and to erect safe and commodious structures in situations and under circumstances which would in many cases totally preclude the employment of the materials known to the bridge-builders of an earlier date. So long as stone and brick were the only available materials, the engineer was confined in his choice to small spans, and to sites where a thoroughly sound foundation was easily attainable. The largest stone arch ever constructed, as far as I can ascertain, is considerably less than 250 feet span, while iron structures on the arch or girder principle of double, and on the suspension principle of three times, this span are by no means uncommon, and we are yet far from approaching the limit of the maximum possible span in this material. Moreover, iron bridges can be employed with perfectly satisfactory results in sites where, from lack of headway, defective foundation, or other local peculiarity, a stone or brick structure would be quite out of the question; and the selection of lines of communication is thus greatly facilitated, and their length and cost consequently diminished.

The most usual form in which iron is employed for bridge purposes is the beam or girder, consisting of two parallel flanges united by a vertical web, consisting either of a continuous plate or of a series of diagonal bars. The average cross-section of such a girder is shown in Fig. 1. In a girder supported at each end the upper flange is in compression, like a pillar; the lower flange is in tension, like a chain—indeed, in some girders the lower flange actually consists of a chain; while the web is in a somewhat complex state of stress, being compressed in an oblique direction, and extended in another oblique direction at right-angles to the first. In girders with parallel flanges, subject to distributed loads of the usual kind, the compression and tension of the flanges attain maximum values at the centre of the span, and diminish toward the ends, while the web stresses are but small at mid-span, and increase towards the supports. Hence the cross-sections of a theoretically perfect girder, at the centre and the end, would be of the forms represented by Figs. 2 and 3 respectively.

Occasionally girders are made of varying depth, as shown in Fig. 4, the bottom flange being retained straight, while the top one is curved; and if this curve be properly designed in view of the special distribution of load anticipated, the following results will be secured:—

1. The tension on the lower flange will be uniform throughout.
2. The compression on the upper flange will be nearly uniform throughout, increasing slightly towards the ends.
3. The stresses on the web will vanish, and the web may consequently be dispensed with.

We have now left but two flanges, one curved and the other straight, like a bow and its string, and these two flanges will together contain rather less metal than an ordinary parallel girder of equal depth and strength.

In the girder as thus modified, the compression of the upper or curved flange at the end of the girder may be resolved into two forces—one vertical, which is balanced by the upward reaction of the support, and one horizontal, which is antagonised by the tension of the lower flange.

Let us now suppose the lower flange to be removed, thus reducing the amount of material employed, in the case of wrought iron, by nearly one-half, and we shall find the upper or curved flange alone to be fully competent to endure the load, provided that the supports or abutments be so constructed as to resist the horizontal as well as the vertical resolved parts of the compression at the ends of the remaining flange.

We have now gradually transformed our structure from an ordinary parallel girder with two flanges and a web into an iron arch, and in so doing we have reduced the amount of material theoretically requisite by almost exactly one-half. From this it follows that as far as material is concerned an arch is a far more economical means of supporting an unvarying load than a girder whenever a good abutment is available capable of resisting a horizontal thrust as well as a vertical pressure.

In working this form of bridge out in practice we are, however, met by certain difficulties, in order to overcome which we are obliged to relinquish a part of the economic advantage which theory indicates.

1. The arch will be exposed to variations of temperature, which may amount to as much as 100° Fahrenheit in a Victorian climate, and which will cause considerable variations of dimension through alternate expansion and contraction of the metal. These changes of dimension, though perfectly harmless in the case of girders free to elongate horizontally, may lead to very serious if not dangerous results in the case of arches placed between immovable abutments; and it is imperatively necessary to take such precautions as shall prevent injury to the structure under extreme variations of temperature.

The most thorough method of meeting this requirement is to divide the arch rib into two parts at the crown, and connect these two parts together, and the ends of the arch to the abutment by joints possessing the character and performing the functions of hinges (see Fig. 5). The arch as thus modified will rise slightly when the temperature increases, and fall slightly when the temperature diminishes, and the change of temperature will be powerless to produce any sensible variation in the stress to which the material is subject.

Sometimes the arch rib is made with hinges at the ends only, and the elasticity or spring of the iron itself is depended upon in lieu of the central hinge, and by properly proportioning the transverse dimensions of the rib it is possible to ensure that within a given range of temperature the metal shall not be strained to any dangerous extent. An arch of this second kind will be manifestly less economical in material than one of the first, seeing that it is required to endure considerable stresses due to variations of temperature over and above those due to the load supported. Nevertheless there are certain practical considerations—such as simplicity of construction, facility of erection, &c.—which may be reasonably held in some cases to justify its use in preference to the more theoretically perfect form previously described.

2. A second difficulty arises when in addition to the unvarying or dead load, consisting of the weight of the structure itself, we desire the arch to support a varying, or as it is often termed a live, load, such as the weight of a crowd of people, a mob of cattle, or a railway train in motion. So long as the load is a perfectly unvarying one, no matter how irregularly it may be distributed, it is possible to adopt a form of arch which will be perfectly suited to the load to be carried, but with a varying load, occuping the same position and affecting the structure in the same way for no two successive instants, such adaptation is manifestly impossible. Hence the rib will be subjected to a cross-bending action, and be required to act to a considerable extent as a beam as well as to perform its proper functions as an arch; and this cross-bending action will be severe in small structures in which the live load is equal to or greater than the unvarying or dead load, but will become unimportant in gigantic works in which the live load becomes but an insignificant fraction of the total weight carried. Thus it will be seen that while in large structures we may reasonably expect to realise nearly the whole of the theoretical economic advantage of the arch over the girder, in small ones the additional metal necessary in order to provide for the extra stresses due to the varying distribution of the moving or live load will greatly diminish, if not altogether annul, the superior economy of the arch as compared with its competitor.

I may here parenthetically remark that there is one class of structures in which we might at first expect to realise the full theoretic gain even in the smallest examples. I refer to bridges for the sole purpose of carrying water-pipes, or channels for water supply or canal purposes. Further reflection will, however, show that this is not the case, for if arches be employed it will be necessary to have a distinct trough or tube, separate from but supported by the ribs, whereas if the girder principle be adopted the girders themselves may be made to assume the form of a trough or tube, thus dispensing with any separate structure to contain the water; and in this way the balance will be turned against the arch in the question of economy of material.

Let us now endeavour briefly to analyse the stresses endured by the material of an arched rib under varying conditions of temperature, load, &c.

We will first assume that the arch as originally designed is of a form adapted to the dead or unvarying load to be borne, which form in the usual case of a uniformly distributed load is a parabola having its axis vertical; and it may further be remarked that a circular curve will usually be found not to deviate in any important degree from the parabola, and is, from a practical point of view, decidedly preferable. Let us also assume that the rib is hinged at the crown as well as the springing. Let W represent the total weight of the structure, which may usually be taken as uniformly distributed over the whole length of the rib, b the span and h the rise of the arch; then the compression of the rib will be $\frac{Wl}{8h}$ at the crown, and at every other point $\frac{Wl \sec. \theta,}{8h}$ when θ is the angle made by a tangent to the rib at the point in question with a horizontal line, and this compression will be uniformly distributed over the whole cross-section of the rib in every case. In other words, there will be no approach to a cross-bending action on any part of the rib, even though the temperature should vary or the abutments yield slightly to the thrust of the arch. If an additional load of W, uniformly distributed, be placed upon the bridge, these compressions will become $\frac{(W+W') l}{8h}$ and $\frac{(W+W') l \sec. \theta}{8h}$ respectively, and the perfect freedom from cross-bending before mentioned will still be maintained. If, however, the live load, instead of being uniformly distributed over the whole span, cover a part of it only, a cross-bending action will come into play, which will attain its maximum when half the bridge is loaded, and which will be unimportant or severe according as the live load is small or large compared with the weight of the structure. The tendency of this cross-bending action will be to increase the radius of curvature in the loaded side of the arch, as in Fig. 6, and reduce it in the unloaded; and the compression endured by the material of the rib will no longer be uniformly distributed, but will be greatly increased on the upper side of the loaded and the under side of the unloaded half of the rib. Hence, bearing in mind that either half of the arch may be the loaded portion, it is evident—1st. That the amount of metal in the rib must be increased. 2nd. That the best section for the rib is like a girder section consisting of two massive flanges united by a comparatively slight web. 3rd. That the rib should be made as deep as practical considerations will allow. The formulæ to be employed in computing the actual stresses in this case are too complex to be introduced here; they do not, of course, contain any terms representing change of temperature.

Let us now consider the behaviour of a rib hinged at the springing but continuous at the crown. When a load is imposed the metal will be compressed longitudinally, the rib will shorten, its crown will sink, and its radius of curvation increase (see dotted lines in Fig. 7), and any yielding of the abutments will tend to augment this result. The alteration in the radius of curvature implies a cross-bending action tending to increase the compression on the upper part of the rib, and to diminish it on the lower part, and this action will be present no matter how accurately the original form of the arch may have been adapted to the load to be carried. Let us now suppose the temperature to diminish. The crown of the arch will fall still further, the cross-bending action will be intensified, and the increasing inequality in the distribution of stress will produce a corresponding diminution in the available strength of the structure; the colder it becomes the more liable the bridge is to give way, and when fracture does ensue it will commence by the crushing of the upper part of the rib. We will next assume the temperature to increase. The crown of the arch will rise, its radius of curvation will be reduced, and the cross-bending action and consequent inequality of stress will diminish and ultimately vanish, and the arch will be stronger—*i.e.*, it will be able safely to bear a greater load than before; and under these conditions the formulæ quoted in the preceding case will apply to this also. A further increase of temperature will cause a further rise of the crown, and a further reduction of the radius of curvation, involving a cross-bending action in an opposite direction to that originally present, and a consequent inequality of stress and diminution in the power of the structure to endure a load. Thus the bridge will be best able to bear its load at a certain calculable temperature somewhat higher than that at which it was first put together, and its strength will fall off as this temperature is departed from in either direction. Hence we draw the inference that it is desirable to complete the erection of such an arch at a comparatively low temperature, in order that it may attain its maximum strength at or near the mean temperature to which it will be exposed. The engineer of the great St. Louis Bridge over the Mississippi enveloped the arch ribs in a kind of gigantic poultice of ice, in order to effect the final junction at a temperature sufficiently low.

The effect of a live load extending over a portion of the span will be the same as in the preceding case, the maximum effect being produced when the bridge is half-loaded and half-unloaded; the extra stresses due to the partial distribution of the live load being, of course, cumulative upon those due to temperature.* The most appropriate section for the rib will, as before, be a girder section; but we cannot say, as in the preceding case, that the deeper the rib the better, for great depth in the rib, while it will reduce the extra stresses due to partial loading, will increase those due to temperature, and a compromise will have to be made avoiding each extreme.

Having thus briefly detailed the considerations to be borne in mind when designing an iron arch, I will conclude by supplying a few particulars relative to a structure of the kind referred to, erected some time since by my friend Mr. T. E. Rawlinson, C.E., and which is, as far as I am aware, the only wrought-iron arched bridge in this colony.

This bridge is situated at Heidelberg on the River Yarra, and consists of a central opening originally occupied by a laminated wooden arch of 100 feet clear span and 17 feet rise and two lateral openings of smaller size. About three years ago the laminated arches gave way through decay of the timber; and Mr. Rawlinson, to whom the work of reconstruction had been entrusted, requested me to determine by computation the stresses on the proposed structure.

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* This is not mathematically correct, but is practically so for arches of the proportions commonly adopted by engineers.

It was in this way that my attention was first directed to this subject, and it is in compliance with a request made by him that I bring the subject before you to-night.

Figs. 8 and 9 respectively show a half-elevation and half-cross-section of the bridge to a scale of eight feet to one inch. The span, as before stated, is 100 feet in the clear, and the rise of the soffit of the arch twelve feet. The section of each flange of each of the two arched ribs is about twenty-four square inches at the crown, and increases slightly to the springing; and the web varies from $\frac{1}{4}$ inch thick at the crown to $\frac{1}{2}$ inch at the springing. The arches are continuous at the crown, but are probably capable of a very slight hinge action at the springing. Assuming them to be hinged at the springing, the following results have been obtained by calculation:—

1. Maximum compression of the metal, bridge half-loaded with load of 84 lbs. per square foot, at a temperature 40° below that at which it was erected =7180 lbs. per square inch.

2. When the load extends over the whole span the cross-bending stress vanishes at a temperature of about 16° Fahrenheit above that at which it was erected.

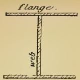
3. With a load extending half-way across, as in Fig. 6, the minimum stress occurs at a temperature 13° Fahrenheit above that at which the bridge was erected.

4. Ordinary plate girders to carry the same load would have contained from 30 to 40 per cent. more material than the iron arches.

The spandrils and roadway are constructed of timber as shown, and possess no doubt some stiffness and power of resisting the effect of irregular loads. In the previous calculations, however, no account was taken of this fact, it being considered unwise to rely upon two such different materials as wood and iron acting to any considerable degree in concert. The arch was therefore made strong enough to endure all irregular stresses without assistance from the spandrils.

In Fig. 10 a detailed section of one arched rib is given, and a portion of the lateral bracing connecting the two ribs together at intervals is shown.

Fig. 1.



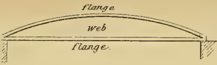


Fig. 4.

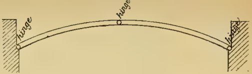


Fig. 5.

Fig. 2.

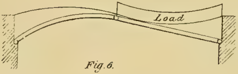


Fig. 6.

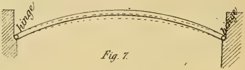


Fig. 7.

Fig 3

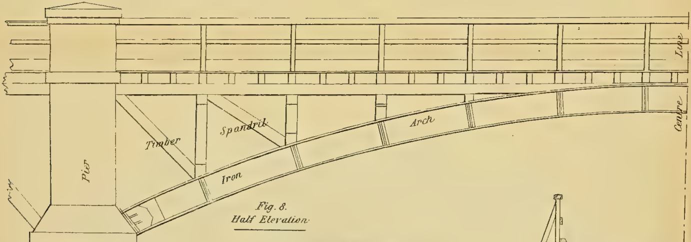
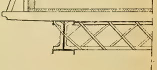


Fig. 8.
Half Elevation



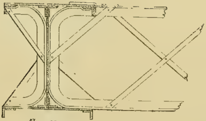


Fig. 10
Section of Wrought Iron Arch.

Fig. 9.
Half Cross Section.

Art. X.—*Notes on Some Observations of Atmospheric Electricity.*

By R. L. J. Ellery, Esq.

[Read before the Royal Society of Victoria, 16th November, 1876.]

Some years ago I described to you an apparatus which I had arranged for obtaining a continuous record of the electrical condition of the atmosphere at the Melbourne Observatory, which was a modification of the exquisite electrometers devised by Sir William Thompson. This apparatus was in operation for several years with most satisfactory results, and a valuable series of records were obtained. It was found, however, almost impossible to maintain the instrument in perfect working condition in some states of the atmosphere, through the subtle nature of the force dealt with and the difficulty of maintaining the requisite insulation of all parts of the apparatus. In consequence of this, the working of the instrument had to be frequently interrupted for improvements in the methods of insulation and of collecting the electricity from the air; and, I regret to say, eventually stopped altogether until a more efficient plan for insulation could be obtained.

It is, however, with respect to the results of some observations with this instrument that I now wish to say a few words; but I will at first briefly refer to the generally accepted theory of the distribution of electricity over the earth's surface.

As a rule, the potential of the earth's surface is negative relative to that of the air above it. Exceptions to this, however, sometimes occur. Generally speaking, I have found in quiet and fine weather that if the air has a certain electric potential, say six feet from the ground, a contour of an equi-potential line traced over the ground, buildings, trees, &c., will be approximately six feet from the surface of such portions of the earth's surface; the line will, however, usually approach the summit of a building, hill, or tree, to something less than six feet; and as the potentials of higher strata are contoured this difference decreases, so that at a few hundred feet the equi-potential lines will probably be found to be parallel to the earth's surface. This is only the case in very serene weather, for in wind, rain, fog, or dust, the case is very different, and nothing more variable than the electric condition of the air can well be conceived, and widely different potentials of the air the same height from the ground in two different places but little removed from one another will be constantly found; and even in the most serene days, when no clouds are seen, no disturbance apparent, sudden and inexplicable variations sometimes occur.

The passing of clouds constantly alters the electric condition of the air on the earth's surface; and indeed all the induction and other phenomena which one can exhibit at the lecture table with an electric machine are in almost incessant operation in the earth's atmospheric envelope. In observing the electric condition of the air we adopt Sir William Thompson's method, and select a certain stratum of air, say six or eight feet from the ground and four to six feet from the walls of any building or other object projecting above the surface of the ground, and the collecting point is always maintained in this position; the measurement given by the apparatus being the difference of potential between the surface of the earth and the air at the selected point. If the air is at the same potential as the earth the instrument will indicate zero, if it be at a higher potential it will indicate above zero, and below if at a lower; the latter state of things may be considered as abnormal. The unit of measurement adopted is the difference of potential between the two poles of a galvanic battery cell, so that the statement that the electric potential of the air at six feet above the ground was equal to 300 Daniell's elements means that the difference of potentials between the air and the surface of the ground was equivalent to that between the two poles of a Daniell's battery composed of 300 cells.

The photographic curves obtained with our electrometer have not yet been tabulated, but some facts have already been deduced, of which the following perhaps are the most interesting:—

In calm and serene weather a regular diurnal maximum and minimum are very marked, the highest part of the curve taking place about 7 a.m. and the lowest about 2 p.m. A second maximum about 9.30 p.m., and a second minimum about 1 a.m., are also indicated.

Hot winds are always accompanied by strong negative tension, and more especially so if dust is present in the air, when sparks can often be got from the collector. The usual turning of the wind from north to south-west is always accompanied for a short period by a high positive tension. In squally weather, rapid and large variations from low negative to high positive generally occur; and during continuous rain strong negative tension is frequently present, which gradually gives place to an increasing positive one some little time before the rain ceases. In very heavy rains, however, the air seems to be reduced to zero, or the same potential as the earth's surface.

It has also been noticed that, if after continuous rain it clears up, the setting-in of rain again is usually preceded by a gradually increasing negative tension. Fogs are always accompanied by a high positive condition.

In the course of some experiments on a very fine day, for the purpose of ascertaining the best position for placing the collector of our electrometer, the following notable results were obtained:—The electric condition of the air being normal (positive potential), when an insulated conductor connected with the electrometer was rapidly raised from the surface of the ground to the height of about 20 feet, a large and rapid increase of positive electricity was shown; and when the conductor was as rapidly lowered, a corresponding diminution was observed. If the conductor was moved rapidly from south to north, keeping it at as nearly the same height from the ground as possible, a strong positive indication was noted, while moving it from north to south the reverse took place. Moving it from east to west gave strong positive, while moving it from west to east gave a strong negative indication.

In repeating these experiments a few days ago in a hot wind, when the air had a strong negative potential, the following results were obtained:—

Raising the conductor gave a strong negative indication, and lowering it a strong positive.

Moving the conductor from south to north gave a strong negative, and from north to south a strong positive indication. Moving the conductor from east to west gave also a strong negative, while moving from west to east gave a strong positive indication.

These results are exactly opposite to those obtained in the first experiments, and can no doubt be accounted for by the negative potential of the air which prevailed at the time.

It must be remarked that in these experiments the indications of the electrometer *took place during the motion of the conductor,* and that immediately the conductor was at rest in its new position the reading of the electrometer became normal for the position the conductor was then in.

To give an idea of the extent of these indications, I may state that with an electrometer where one Daniell's cell will deflect five divisions, the following average readings were obtained :—

		Scale reading.
Zero 125.	Raising the Conductor 18 feet ...	50
	Lowering ,, ,, ...	250
	Moving N. to S. ,, ,, ...	150
	,, S. to N. ,, ,, ...	40
	,, E. to W. ,, ,, ...	60
	,, W. to E. ,, ,, ...	160

I obtained some very interesting results some years ago from observations made on the summit of Mount Macedon while a terrific thunderstorm was passing over Melbourne and the surrounding level country.

Over the mountain it was quite clear, fine, and calm, while the plains below were hidden from view by a dense stratum of low-lying cloud, in and through which incessant lightning could be seen, while occasionally the low and distant roll of thunder could be faintly heard.

The electrometer was placed in a tent at the bottom of the tower used for trigonometrical observation, and was connected with the collector (burning fungus) on the tower 50 feet high. The potential of the air was slightly positive and quiet; but simultaneous with every flash of lightning the electrometer became violently but momentarily depressed with negative electricity, and instantly returning to its normal positive indication, suggesting the occurrence of a sudden electric vacuum with each flash of lightning.

These then are some of the most prominent facts deduced from our observations of atmospheric electricity up to the present time. They are interesting so far as they go, but are scarcely sufficient in the present state of our knowledge of the subject for tracing the relations which exist between the electric condition of the earth's surface and other atmospheric phenomena, although we may hope as our observations are extended (for I propose to resume them) this will be eventually accomplished. Not the least interesting or valuable point for investigation in this subject is the effect the various electric conditions of the air have on the human or animal economy, both in health and disease; for I am convinced from what I have already observed that it plays a most important part in this direction, and I intend at some future time to make a communication to the Society on this branch of the subject.

ART. XI.—*Amorphous Phosphorus.*

BY PROFESSOR ANDREW.

[Read before the Royal Society of Victoria, 16th November, 1876.]

IN 1873 I noticed on the surface of a quantity of chocolate-coloured, amorphous phosphorus, a quantity of clear, syrupy liquid, having a strong acid reaction. It appeared to contain phosphorous acid, but there are probably other oxygen compounds of phosphorus present. The liquid was poured off, and the residue washed and put away until the beginning of this year, when I found that as much more of a similar liquid had collected in the bottle (specimen produced). Mr. Ford tells me that he has noticed the same thing, and that Professor Smith, of Sydney, had also observed it, and was in the habit of giving it for analysis to students as a substance containing phosphorous acid. It is possible that the formation of the fluid may be due to the residue of ordinary phosphorus which the bisulphide of carbon used in its preparation has failed to remove, or to instability of the amorphous phosphorus causing a gradual return to its original state under certain conditions. This can only be ascertained by repeated experiments. I would invite the attention of members to the subject, which is of considerable practical importance now that the substance is so much used by itself in the manufacture of safety matches. (The sample was left for the use of any members who wished to examine it.) H. M. A.

Art. XII.—*Account of the Telegraphic Determination of the Difference of Longitude between Melbourne and Hobart Town in the Year* 1875.

By E. J. White, Esq.

[Read before the Royal Society of Victora, 14th December, 1876.]

The late transit of Venus having been successfully observed at Hobart Town by the American party under the command of Professor Harkness, it became a matter of necessity to obtain the longitude of the observing station. Instead of an absolute determination with reference to the meridian of Greenwich, which would have required months, or even years, for its successful execution, Professor Harkness resolved to obtain it differentially from Melbourne, the two places being connected by means of the land lines and submarine cable of the electric telegraph; and for the purpose of arranging a scheme for carrying out this intention he visited Melbourne towards the latter end of November, 1874. Having settled upon a plan of operation with Mr. Ellery, and having obtained the consent and promise of hearty co-operation of Mr. Warren, the managing engineer of the Tasmanian Cable Company, and Messrs. James and Payter, the Melbourne managers of the electric telegraph, he returned to Tasmania, and immediately after he had observed the transit of Venus a few unsuccessful attempts were made to send the signals direct, with automatic repeaters, between Melbourne and Hobart Town. Soon after this, Professor Harkness had to accompany the "Swatara" during her cruise in the South Pacific, to collect the different parties of American observers in that part of the world, and further attempts were deferred till his return. Advantage was taken of the interval to improve the repeating apparatus, and on his return at the end of January the signals were transmitted without any difficulty.

At Hobart Town the observations were taken by Professor Harkness, who employed a portable transit instrument of 2½ inches clear aperture and 30 inches focal length, with a magnifying power of 60 diameters. The transit was reversed each night near the middle of the observations. Three clock stars and two azimuth stars were observed in each position of the axis, and from the complete set of ten stars equations of condition were formed, the solution of which by the method of least squares gave the most probable values of the collimation, azimuth, and clock errors, the level error having been previously found by means of the striding level. The positions of the azimuth stars are taken from the Melbourne General Catalogue for 1870, and those of the clock stars from a specially prepared list. The places of these latter stars differ slightly from their places as given in the *English Nautical Almanac;* the resulting clock errors are, however, generally within one-hundredth of a second of what the latter places would produce.

At Melbourne I observed with the transit circle, which has an aperture of 5 inches and a focal length of 6 feet; the eye piece used has a magnifying power of 167 diameters. This instrument does not admit of reversal, but the collimation error is found according to Bessel's method, with two collimators. The level error is obtained by means of reflection from a surface of quicksilver, and the azimuth error is found from the transits of circumpolar stars in the ordinary way, one star being generally observed above the pole and another below.

At both places self-recording chronographs were employed; that of Professor Harkness was a barrel one, regulated by a vibrating spring. The timepiece which marked the seconds on the chronograph sheets, and which transmitted the signals through the telegraph lines to Melbourne, was a box chronometer, No. 1520, by T. S. & J. D. Negus, of New York, the going of which quite justifies the fame enjoyed by those celebrated makers. The Melbourne clock was the famous Frodsham, No 991, which continues to perform as well as it did some years ago, when its going was declared to be the most remarkable for accuracy on record. It is attached to a chronograph by Siemens and Halske, of Berlin, which registers on a fillet of paper, the motion of which is governed by means of a Froude's fly.

The usual practice was to commence observing a set of stars soon after sunset; and as soon as the telegraph lines were clear from their ordinary work, the Hobart Town clock was made to transmit its time to the Melbourne chronograph, on which the Frodsham clock marked its seconds at the same time. After this the Frodsham clock sent its time to the Hobart Town chronograph, where it was registered simultaneously with the Negus chronometer. Now, taking the results as recorded on the Melbourne chronograph, and correcting them for the clock errors as determined from the star observations, the difference between the times will represent the difference of longitude *minus* the time of transmission, *plus* the difference of personal equation of the observers. On the other hand, the Hobart Town results will exibit the difference of longitude, *plus* the time of transmission, *plus* the difference of personal equation. On taking, then, half the sum of the two quantities, we shall get the difference of longitude freed from the transmission time, but still affected with personal equation. And half the difference of the quantities will give the time of transmission. The effect of personal equation could be eliminated by the observers exchanging their stations; but as that would have been attended with great inconvenience, the difference of personal equation was directly obtained on several occasions during Professor Harkness's visit to Melbourne. The method adopted for this purpose was for both observers to determine the error of the Melbourne transit clock on the same evening, selecting the stars in such a way that the mean epoch of each observer would be so nearly alike as to give the personal equation free from the influence of the rate of the clock. The following is an abstract of the results:—

COMPUTATION OF THE PERSONAL EQUATION.

Date, Melbourne Mean Time, 1874 & 1875.	Observer.	N. of Stars.	Mean of Times of the Corrected Transits.	Observed Clock Corrections.	Adopted Correction for Clock Rate.	H − W Reduced to the same Epoch.	Weight.	Product.
d. h. m.			h. m.	s.	per diem. s.	s.		
Nov. 17 9 6	H	5	0 51	− 30·921	− 0·26	+ ·125	55	6·875
9 25	W	5	1 10	31·049				
Feb. 23 8 15	H	7	6 27	32·113	+ 0·36	+ ·171	77	13·167
8 34	W	7	6 46	32·280				
26 8 43	H	6	7 7	31·078	+ 0·30	+ ·171	66	11·286
8 35	W	6	6 59	31·251				
27 8 20	H	6	6 47	30·718	+ 0·29	+ ·242	60	14·520
8 25	W	5	6 52	30·959				
							258	45·848

Adopted Personal Equation H − W + ·178

COMPUTATION OF THE DIFFERENCE OF LONGITUDE.

Date, 1875.	Difference of Longitude.		Double the time of Transmission.	Number of Clock Stars Observed.		Weight.
	Hobart Town Register.	Melbourne Register.		H	M	
	m. s.	m. s.	s.			
Jan. 30	9 25·996	9 25·762	0·234	7	8	392
Feb. 1	26·084	25·900	·184	6	6	315
2	25·720	25·551	·169	0	8	0
4	26·193	26·000	·193	6	6	315
5	25·935	25·609	·326	7	0	0
6	25·774	25·423	·351	6	9	378
7	25·820	25·577	·243	6	8	360

The weights are proportional to the quantity found by multiplying the number of stars observed by one observer by the number observed by the other, and dividing the product by their sum. On February 2nd no stars were observed at Hobart Town, and on February 5th no stars could be observed at Melbourne, so the difference of longitude marked in the columns has been found by carrying on the rates of the chronometer and clock respectively; as the combination weights, however, are nothing, they will not influence the final results. The transmission times, however, are independent of the rate of the clock, except for the few minutes intervening between the receipt of the set of signals; these nights, therefore, have equal weights for this purpose with the others. Carrying out the combination we get 9m. 25·841s., from this is to be subtracted 0·178s. for personal equation; we then get for the final difference of longitude 9m. 25·66s. ± ·06s., and for the mean time of transmission we get 0·121s. Taking the length of the land lines and cable at 420 miles, this would represent a speed of only 3360 miles per second; the actual speed, however, must have been considerably greater than this, for the above quantity, 0·121s., includes also the armature time of the relays and repeating apparatus. From some measures made of the speed of the current on the land lines during the determination of the difference of longitude between Melbourne and Sydney in

1868 we found the velocity on the land line to be 15,400 miles per second.

Professor Harkness's temporary Observatory in Hobart Town was situated in the Barrack-square in latitude 42° 53′ 24·6″ south, and by applying the above difference to 9h. 39m. 54·8s., the longitude of Melbourne, we get 9h. 49m. 20·46s. for the longitude of his station, which is marked by a pier, which the Tasmanian authorities have promised to preserve. Mr. Ellery has written to the Surveyor-General at Hobart Town for the situation of this pier, with reference to Fort Mulgrave, from which the longitude of the city has been hitherto reckoned; but as no reply has been as yet received, I cannot say how this new determination of longitude will agree with the old one. As a final result we have then—

Pier in Barrack-square.

Latitude	42°	53′	24·6″	South.
Longitude	147	20	6·9	East of Greenwich.

NOTE.—Since the above was written a letter has been received from Prof. Harkness, giving the results of his triangulation in Hobart Town, according to which, adopting the above position of the Pier in Barrack-square, the positions of the following places will be as under:—

	Lat.	Long.
Flagstaff at Prince of Wales Battery (Fort Mulgrave)	42° 53′ 22·3″	147° 20′ 36·3″
Flagstaff at Queen's Battery	42 52 44·0	147 20 38·8
Centre of front of St. David's Cathedral ...	42 53 6·9	147 20 10·2

1876.

PROCEEDINGS.

ROYAL SOCIETY OF VICTORIA.

ANNUAL MEETING.

Held in the Library of the Society, Monday, March 13th, 1876.

George Foord, F.C.S., Vice-President, in the chair.

The election of office-bearers for 1876 took place, with the following results :—

President : R. L. J. Ellery, F.R.S., &c.

Vice-Presidents : G. Foord and E. J. White.

Hon. Treasurer : Percy de J. Grut.

Hon. Secretary : F. J. Pirani.

Hon. Librarian : Dr. James E. Neild.

Members of Council : J. Bosisto, W. C. Kernot, T. E. Rawlinson, H. K. Rusden, G. H. F. Ulrich, Professors H. M. Andrew and E. J. Nanson.

Annual Report and Balance-sheet for 1876 were read and adopted, as follows :—

Report of the Council of the Royal Society for the year 1876.

"Your Council has the honour to report that the papers and notes read, instruments exhibited, &c., since last Annual Meeting are as follow :—

"On the 11th of May Mr. Gardiner gave an abstract of a paper by him on 'Geodetic Surveying,' and Mr. Ellery exhibited a 'Radiometer.'

"On the 8th of June Mr. Rawlinson read a continuation of his paper 'On the Improvement of the Harbour of Melbourne ;' and Mr. Ellery gave an account of the Great Paris Telescope.

"On the 10th of July Mr. Ellery exhibited a form of Thomson's Quadrant Electrometer ; Mr. Foord exhibited a Gas-pressure Gauge ; Mr. Pirani exhibited a Lecture Apparatus for Measuring the Mechanical Equivalent of Heat; and also a two-fluid Barometer, invented by Mr. H. Venables; and Mr. Arnold exhibited some preparations of Compressed Leather.

"On the 25th of September Mr. Ellery read a note on 'The small number of Sun Spots visible during 1876,' and also a paper on 'The Chronograph;' Mr. White read a paper 'On the Determination of the Longitude of the Melbourne Observatory,' and Mr. Kernot read a paper 'On Iron Arches.'

"On the 16th November Mr. Ellery read an 'Account of some Experiments on Atmospheric Electricity,' and Mr. Andrew read a note 'On Amorphous Phosphorus.'

"On the 14th of December Mr. Pirani exhibited a Holtz's Electric Machine, and Mr. White read a paper 'On the Telegraphic Determination of the Difference of Longitude between Melbourne and Hobart Town.'

"[Of the above the following papers have been printed in pamphlet form :—'Geodetic Surveying,' by Mr. Gardiner; 'The Improvement of the Harbour of Melbourne,' by Mr. Rawlinson; 'The Determination of the Longitude of the Melbourne Observatory,' and 'The Telegraphic Determination of the Difference of Longitude between Melbourne and Hobart Town,' by Mr. White; and 'Experiments on Atmospheric Electricity,' by Mr. Ellery.]

"Vol. XII., containing the papers read during the years 1874 and 1875, has been published.

"The Government have again liberally continued the grant of £200 in aid of our fuuds. Debentures to the amount of £40 have been paid off during the past year, and the amount of unclaimed interest has been reduced by £12 12s. The balance in hand amounts to £323 18s. 3d., and of this amount your Council considers it advisable that a large portion should be spent in paying off debentures and executing necessary repairs to the building.

"Some proposed alterations in the Laws will be submitted to you at the next annual meeting."

The Hon. Treasurer in Account with the Royal Society of Victoria.

Dr.

To Balance from last Balance-sheet				£340	7	5
,, Government Grant, 1876-7				200	0	0
,, Interest on Fixed Deposits				4	0	0
,, Surveyor's Society's Rent, 1875				4	4	0
,, Sale of Transactions				0	5	0
,, Entrance Fees				14	14	0
,, Subscriptions :—						
1874. 1 ordinary	£2	2	0			
1875. 6 ordinary	12	12	0			
,, 2 ½ ordinary	2	2	0			
,, 1 country...	1	1	0			
1876. 53 ordinary	111	6	0			
,, 3 ½ ordinary	3	3	0			
,, 7 country...	7	7	0			
1877. 1 ordinary	2	2	0			
				141	15	0
				£705	5	5

Cr.

By Printing and Stationery				£176	8	0
,, Bookbinding				7	19	6
,, Freight and Charges on Books				14	16	0
,, Conversazione				10	12	0
,, Interest on Debentures...				41	7	3
,, 8 Debentures paid				40	0	0
,, Improvements and Repairs				17	5	6
,, Insurance				2	17	6
,, Rates				3	6	8
,, Gas and Fuel				5	3	5
,, Hall Keeper and Clerical Assistant				25	0	0
,, Collector's Commission				16	11	2
,, Postages and Petty Cash				20	0	2
				£381	7	2
,, Balance in Bank :—						
Fixed Deposit	£200	0	0			
Current Account	123	18	3			
				323	18	3
				£705	5	5

Compared with the Vouchers, Bank Pass Book, and Cash Book, and found correct.

THOMAS E. RAWLINSON } *Auditors.*
HENRY MOORS }

2nd March, 1877.

P. DE JERSEY GRUT, *Hon. Treasurer.*

STATEMENT OF ASSETS AND LIABILITIES.

ASSETS.				LIABILITIES.			
Balance in Bank	£323	18	3	Publishing Fund	£216	11	2
Estimated Value of outstanding Subscriptions	25	0	0	63 Debentures	315	0	0
Rent due	20	5	6	Interest unclaimed	14	5	7
Hall, Library, Furniture, &c., insured for	2300	0	0	Accounts outstanding	10	0	0
					£555	16	9
				Balance	2113	7	0
	£2669	3	9		£2669	3	9

On the motion of Dr. Neild, seconded by Mr. Hunt, it was resolved that in Law VII. the word Thursday be substituted for Monday; and the meeting adjourned.

Read and confirmed.

Robert L. J. Ellery, *Chairman.*

Ordinary Meeting,

Held in the Library, Thursday, May 11*th*, 1876.

The President in the Chair.

The following gentlemen were nominated for election at the next meeting:—F. Goldstraw, M.A., proposed by H. M. Andrew, seconded by F. J. Pirani; W. C. Watts, proposed by A. K. Smith, seconded by T. E. Rawlinson.

Mr. Martin Gardiner gave an abstract of a paper by him on "Geodetic Surveying."

Mr. Ellery then exhibited a radiometer, and gave a short account of the different theories which had been propounded to explain its action. Discussion ensued.

(Signed) R. L. J. Ellery, *Chairman.*

Ordinary Meeting,

Held in the Library of the Society, June 8*th*, 1876.

The President in the Chair.

Messrs. Goldstraw and Watts, nominated at the last meeting, were duly elected ordinary members of the Society.

Mr. Rawlinson read a paper on the improvement of the Harbour of Melbourne. Discussion ensued in which Mr. Rawlinson's plan was generally commended.

Mr. Ellery gave an account of the Great Paris Telescope which had recently been erected, and compared its construction with that of the Melbourne instrument. A general discussion took place.

The thanks of the Society was given to the Rev. L. Fison for a work on "Consanguinity" which he had presented to the Society.

(Signed) R. L. J. Ellery, *Chairman.*

Ordinary Meeting,

Held in the Library of the Society, July 18*th*, 1876.

The President in the Chair.

The President stated that a donation of a photograph of an Engraving of Sir Isaac Newton had been received from Mr. Noone. A vote of thanks was accorded to Mr. Noone for it.

The President having resigned the chair to Mr. White, Vice-President, read a note on "The small number of Sun Spots visible during the year 1876."

The President read a paper on "The Chronograph," with especial reference to a parabolic governor which he had successfully adapted to the instrument. He also exhibited a form of governor recently invented by Mr. Cooke. Discussion ensued.

The President resumed the chair, and Mr. White read a paper on "Determination of the Longitude of the Melbourne Observatory." Discussion ensued, in course of which the President stated that in consequence of the proprietors of submarine cables objecting to strong currents being sent through them, there was at present no prospect of obtaining determination of our longitude by means of the electric telegraph.

Mr. Kernot read a paper on "Iron Arches," with reference to the iron arched bridge at Heidelberg, recently erected by Mr. Rawlinson. Discussion ensued.

(Signed) R. L. J. Ellery, *Chairman.*

ORDINARY MEETING,

Held in the Library of the Society, November 16*th*, 1877.

The President in the Chair.

Mr. F. C. Klemm was elected an ordinary member of the Society.

The President having vacated the chair (which was taken by Mr. White), he read a paper on "Some Experiments in Atmospheric Electricity." Discussion ensued.

The President resumed the chair, when Professor H. M. Andrew gave an account of a hitherto undescribed peculiarity of "Amorphous Phosphorus." The phosphorus was of the chocolate-coloured variety. Exteriorly there had accumulated during two or three years a layer of syrupy fluid which contained phosphorous acid. The same phenomenon had been observed by Mr. Foord and Professor Smith; the latter had found the fluid to be a mixture of phosphorus and hypophosphorous acid. There was some probability that any action of this sort might be dangerous if occuring in the amorphous phosphorus used in safety matches. Discussion ensued, in course of which one or two facts were mentioned which went to show that safety matches were not so free from danger as was commonly supposed.

Mr. White's paper on "The Recent Telegraphic Determination of the Longitude of Hobart Town" was postponed till next meeting.

(Signed) R. L. J. Ellery, *Chairman.*

ORDINARY MEETING,

Held in the Library of the Society, December 14*th*, 1876.

The President in the chair.

The following gentlemen were nominated for election at next meeting:—Mr. J. Bywater Humphreys, proposed by E. Howitt, seconded by H. K. Rusden; Dr. P. Moloney, proposed by E. Howitt, seconded by Mr. Ellery; Rev. A. Paul, proposed by Mr. Foord, seconded by Mr. Ellery.

The President read the list of retiring office-bearers, as follows:—President, Mr. R. L. J. Ellery; Vice-Presidents, Mr. G. Foord and Mr. E. J. White; Hon. Treasurer, Mr. Percy de J. Grut; Hon. Librarian, Dr. J. E. Neild; Hon. Secretary, Mr. F. J. Pirani; Members of Council, Messrs. A. C. Allan, E. Howitt, S. W. M'Gowan, F. Poolman, J. T. Rudall; Members of Council who retain office being—Professors Andrews and Nanson, J. Bosisto, W. C. Kernot, T. E. Rawlinson, H. K. Rusden, G. H. F. Ulrich.

Mr. Pirani described and exhibited a small Holtz's electric machine.

Mr. White read a paper on the "Telegraphic Determination of the Difference of Longitude between Melbourne and Hobart Town."

Mr. Pirani's paper on "Force" was postponed.

(Signed) R. L. J. ELLERY, *Chairman.*

MEMBERS

OF

The Royal Society of Victoria.

(Names marked thus (*) are those of Life Members.)

Alcock, Peter C., Esq., 41 Swanston-street
Allan, Alex. C., Esq., Crown Lands Office
Andrew, Professor H. M., Wesley College

Barker, Edward, Esq., M.D., F.R.C.S., Latrobe-street East
*Barkly, His Excellency Sir Henry, K.C.B., Cape Colony
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Royal Society of Victoria.

ANNIVERSARY ADDRESS

OF

The President,

MR. R. L. J. ELLERY, F.R.S., F.R.A.S., Government Astronomer.

(Delivered to the Members of the Royal Society, at their Annual *Conversazione*, held on Thursday, July 26th, 1877.)

YOUR EXCELLENCY AND GENTLEMEN OF THE ROYAL SOCIETY,

Since I had the honour of addressing you at the annual *Conversazione* in August last year, we have entered upon our twentieth session, and I think I may at the outset congratulate you on the past year's work, and the present aspect of the affairs of the Society, concerning which, however, according to ordinary custom, I shall speak more in detail presently.

The numerical strength of the Royal Society fluctuates very little from year to year. For a long time our losses by removal and secession equalled and sometimes exceeded our gain by new members; but during the last few years our roll shows unmistakable signs of a small but steady increase, which includes the names of many of the most intelligent and scientifically industrious young men of our community, all of whom will no doubt eventually become, as many have already, active members and regular contributors to our Transactions.

Our financial position is on the whole satisfactory; this, however, is in a great measure due to the continuance of the grant voted to us by Parliament, which you will remember was withheld from the Society for several years, when we were compelled to stop the publication of our Transactions, which we should still be quite unable to continue without this aid. The loan we raised some years ago by debentures, chiefly amongst our members, has now been reduced to £315, and it is intended to reduce this still further during the current year.

It has been necessary to effect some repairs in the building, but the Council has not been in a position to undertake the cementing of the exterior; the grounds have been somewhat improved by the growth of the trees, and by the more regular attention bestowed upon them. I cannot, however, on the whole, congratulate you on the appearance of the premises generally; for although both the building and fencing are in a fair state of repair, there is much to be hoped for æsthetically. In the original design of this building a central hall surrounded by chambers for offices, laboratories, and meeting-rooms, was provided for, and, in its entirety, would have constituted a fine and handsome building. Unfortunately, however, the central hall only was built, and has since stood alone in its solitary ugliness, while some years ago its interior was divided into several chambers to meet the requirements of the Society, which would no doubt have been better done by carrying out the original design, had the Society's financial position admitted of it.

The Council has had the desirability of improving the appearance of the exterior of the building continually before it, and still nurses the hope that it will eventually be able to carry out the original design, which contemplated the domiciliation of other scientific bodies besides that of the

Royal Society. In view of this proposition, then, some little time since, when the Medical Society had the question of building a house of meeting under consideration, overtures were made by your Council to their committee with the view, if possible, of affording accommodation on these premises to that society; and it was thought not improbable that this and other kindred societies might be similarly domiciled in this building, which might then become known as the Institute of Scientific Societies, or under some such name. I regret, however, to state that the proposition does not seem to have been favourably entertained, although if adopted it could not have failed to have been beneficial to both societies, enabling them together to have instituted most convenient arrangements, which alone neither can well secure.

Our library is rapidly increasing by donations from kindred societies in all parts of the world with which we are in communication, and some scheme by which these valuable books may be easily found on the bookshelves and be made immediately available to our members, is now imperatively demanded, and is under the consideration of your Council.

The state of publication of our Transactions is satisfactory; all the contributions, except those at our last meeting, have been printed in pamphlet form and distributed to the members, and another volume, in which these are included, is now in the press. Some of our earlier volumes are nearly out of print, and we are thus unable to supply societies that exchange with us with full sets of the Transactions. A question has therefore been raised whether we should not reprint these volumes either in full or partly in abstract, and as many of you are aware a Sub-Committee has been appointed to report on the matter.

Referring now to the work of the session, we find that the Society has held eight ordinary meetings since our last annual gathering, at which papers and communications of great interest and scientific value have been contributed; but as the Council have been able to print and distribute these amongst the members immediately after the meetings at which they were presented, it will be unnecessary for me to refer to them in detail here. It cannot but be remarked, however, that while these contributions have been more than sufficient to occupy our ordinary meetings, the names of the contributors are limited, and, as is too often the case in scientific societies, most of the work is done by a few. We have on our roll now many young members whose recreations, if not their general occupations, are such as should enable them to become active and useful in the Society, and it is greatly to be desired that they should add their names to the comparatively small list of working members.

The attendance of members at our ordinary meetings has been much greater than in former years, and I think we may safely conclude that interest in those branches of knowledge and inquiry which come within the scope of the Society has considerably increased.

In my last address I expressed a belief that the functions of the Society might be beneficially extended so as to embrace, besides the reception of papers and communications, the delivery of brief special lectures for the demonstration of new and interesting facts in physical and other sciences. I regret to say that up to the present time your Council has been unable to mature any scheme for accomplishing this. I hope, however, that something in this direction may be done before entering on our next session.

I have on former occasions of this kind alluded cursorily to the progress made within the colony in our various departments devoted to scientific and technological research and teaching, and other cognate matters of more than passing interest. In my last address, however, I unfortunately omitted to do so; but I think you will grant me your indulgence for a few moments on this occasion, while I briefly review the year's work in these directions.

As an excuse for referring to Astronomical Work first, I may plead both alphabetical precedence as well as the fact that I am more intimately acquainted with what has been done in this direction than in many others. While our Observatory has been, as usual, fully occupied with its allotted work in Astronomy, Meteorology, and other physical investigations, there is nothing of very prominent interest in its last year's history, but nevertheless there are one or two facts worthy of record.

You will remember that while our great reflector has been kept at work ever since its erection in 1869, no results of this work, except in a few cases of immediate interest, have been given to the world, and a feeling has gained ground that nothing was being done with it, except for simple idle star-gazing. The fact is, we have accumulated a very large mass of observations, descriptions, and drawings—the work of the three several observers to whom the use of the telescope has been entrusted; but these, for several reasons, have not hitherto been published. I am glad, however, to say that their publication is now in progress, and in a forward state.

Lithographic copies of most of the drawings of the nebulæ observed with the telescope have now been made on stone, and I have no doubt will soon be published, with a full description of what has been done with this giant instrument. The work for the most part consists of a revision of the nebulæ observed by Sir John Herschel at the Cape of Good Hope from 1835 to 1837 with his great reflector, and a comparison of the changes that have taken place in the interval of forty years will prove interesting and furnish ample food for speculation, even if it does not add considerably to our definite knowledge of these mysterious occupants of space.

The exact distance of the sun from the earth is yet an unsolved problem, and although a large addition to our knowledge upon this subject has been anticipated from the very successful observations of the late Transit of Venus, I am sorry to say the results are not yet arrived at. Success having attended so many of the numerous observing parties, the necessary calculations have assumed almost stupendous proportions, and it yet remains doubtful how much longer the final results will be delayed. Another favourable opportunity for determining the solar parallax is now about to occur in the opposition of the planet Mars, which takes place on September 5th, on which occasion its distance from the earth will be almost a minimum. You will remember that all our methods of determining the sun's distance depend on the determination first of the distance of any planet from the earth, when Kepler's famous law (that the distances of the planets from the sun are proportional to the times in which they complete their revolution about the sun) furnishes the rest of the required data; so that in the transit of Venus what is actually determined is the distance of Venus from us, and hence by Kepler's third law the distance of the sun; and the observation consists in the measurement of the displacement of the planet upon the sun's disc as seen from various parts of the earth's surface. In the case of Mars, its displacement with regard to certain selected fixed stars near it, is measured at widely different points of the earth's surface, instead of with respect to the solar disc, as in the case of Venus. The opportunity which is now about to occur will not be lost sight of, and arrangements have already been made by which the co-operation of various Observatories in both hemispheres will be secured, and we have already commenced operations at Melbourne in conjunction with Greenwich and Washington.

The comet discovered by D'Arrest in 1851, which has a period of about five and a half years, and which is one of the most interesting of the comets of short period because of the enormous disturbances it experiences from the planet Jupiter, was observed during the month of June. This is a very difficult object to observe, owing to its excessive faintness; so that during its perihelion passage in 1864 it could not be seen at all. In the present instance it was found with little difficulty, owing to the excellent ephemeris which had been sent to me by M. Leveau, of the Paris Observatory.

The question of the existence of a planet between the sun and Mercury has been revived during the past year, and M. Leverrier announced the probability of the supposed planet transiting the sun's disc about the 22nd of March last. Most of the Observatories throughout the world were requested to keep a strict watch for its appearance on the 21st, 22nd, and 23rd, and this was, I believe, generally done, but with a negative result, no appearance of a planetary transit being observed anywhere. We had very favourable weather here, and could not have failed to see it had it crossed the sun during our daylight. The existence of an intra-Mercurial planet is therefore a problem yet to be solved.

At our last gathering I spoke of the progress that had been made in Meteorology in Europe and America by the adoption of a widely co-operative system, and I stated that I had taken steps to bring about an analogous system in Australia; and at the ordinary meeting in May last, in a paper I read on the present state of meteorology, I detailed the outcome of this effort. It will therefore be only necessary now to tell you in the briefest manner what has been and is being done. The hearty co-operation of the astronomers and meteorologists, as well as of the telegraphic departments, of New South Wales and South Australia has been secured, and weather telegrams in cypher are exchanged daily from ten to twelve a.m. between Adelaide, Melbourne, and Sydney. These telegrams are utilised in a different way at each place; for while in Sydney they are used for constructing a weather chart, in Melbourne a weather bulletin is lithographed at the Observatory, which, as a rule, is posted at various places about Melbourne before one p.m., and gives a synopsis of the state of the weather and sea at nine a.m. along our coast-line from Cape Borda to Cape Howe, and north as far as Brisbane, as well as a general idea of the weather in the settled interior of Australia. Afternoon telegrams are also received from a certain number of stations, from which a synopsis is deduced and published in the morning papers. I think there can scarcely be two opinions as to the utility of this method as compared to the partial and somewhat indiscriminate meteorological reports hitherto issued, and I have reason to believe that a large portion of the public already appreciate the value of the innovation.

Concerning Botanical Science, which is so ably represented in this colony by our eminent fellow-member, Baron Von Mueller, I have also a few words to say. The investigations and labours of our Government Botanist have made considerable progress, and during the last year or two he has very largely increased the literature of the science, more especially with respect to Australia, by the publication of several important works, and by the continuance of his serials on Australian plants. The *Phytographia Australis* has now nearly reached the completion of its tenth volume, and it must be a matter for regret to most Australian students of botany that this valuable work was not written in the

English language. Baron Von Mueller is now engaged on an exceedingly valuable work on the plants of New Guinea, the first volume of which is nearly completed, and its publication is anxiously looked for by all who are interested in botanical science. A very useful volume for students in this science has also been issued by the Government Botanist, entitled *Introduction to Botanic Teachings in the Schools of Victoria through Native Plants.* This work is largely and carefully illustrated; and while intended for schools, in reality constitutes a valuable work for the advanced student. Amongst his other literary labours it is much to be hoped that Baron Von Mueller will soon be able to complete his long contemplated *Atlas of the Eucalypti*, for which he has already a very fine series of drawings prepared. I must not overlook another most useful work that has appeared, entitled *Select Plants Readily Available for Industrial Culture in Victoria*—a most important work, which cannot fail to be appreciated as it becomes more generally known. It is to be hoped that the Government Botanist will soon be able to resume the phytochemical researches which he commenced some years ago, and which gave promise of results not only of the highest scientific interest, but also of immediate commercial value.

Our National Museum, under the direction of Professor M'Coy, has also its year's history; and I am glad to state that it continues to increase in specimens illustrative of all the branches of natural science, whilst the systematic naming and classification by the director continually advances. Professor M'Coy, however, informs me that want of room, owing to the western half of the building not being yet built, renders the labour of maintaining the collections in good order, and properly classifying them, each year greater than before. It must be apparent to all who take an interest in our Museum of Natural History that there is not sufficient room for the proper display of our fine collection, and the time has obviously arrived when the building should be completed, and the director enabled to give the colony the benefit of the complete classification which he desires to exhibit, and which will include classification according to geographical distribution. The freshness and good state of preservation exhibited by the specimens is remarkable compared with many collections I have seen in large cities, and I have no doubt this fact may be traced to the absence at the University site of those corrosive products of combustion which prevail in the more densely populated parts of a city, and which are so destructive to collections of this kind. In glancing over some statistics of the Museum, we find that last year 98,000 people visited it, and that it contains over 37,000 specimens. Professor M'Coy, the director of the Museum, is proceeding with the publication of the *Decades of the Zoology and Palæontology of Victoria,* with figures and descriptions from specimens in the Museum. The fourth decade of the palæontology has been issued during the year, and the fifth is nearly ready. Further decades may shortly be expected, and numerous beautiful plates of the snakes, fishes, and insects of the colony are already prepared for them.

One of our most useful institutions, though perhaps not the best known, is our Technological School and Museum, and comparatively few people know how much good work is being done, and what a wealth of knowledge in technology and the arts is being acquired there for the future advancement of our community and colony. During the last year the progress of the Industrial and Technological Museum has been very satisfactory, both as regards the number of persons availing themselves of the classes and lectures on technical subjects, and the increase of the various scientific and economic collections, their systematic arrangement and proper display. Over 3000 specimens have been added, and amongst the most recent and interesting are the collections received from America through the instrumentality of Mr. George Collins Levey. These collections comprise—1. Ingenious mechanical contrivances; 2. Manufactured metals, and the ores, fuels, fluxes, &c., used in smelting; 3. Rare and interesting rocks and minerals from the United States; 4. A collection of the rocks and minerals of the Dominion of Canada, presented through Mr. Levey by Mr. A. R. C. Selwyn; 5. A collection of seeds, amongst which the varieties of wheat are to be specially noted (advantage has been taken to distribute, through the Department of Agriculture, a portion of each sample to growers in the country); 6. Manufactured fibres, American cottons, &c. The publications of the year have been limited to a catalogue of the Timbers of Victoria, which contains all available information as to the value of our trees or timbers in manufacture. The walls and pillars in the museum are being made to answer the purpose of catalogues by being covered with copious instructive notes to assist the interested visitor in his studies.

Since the abolition of the department of the Geological Survey by the Government, the geological work of the colony has been carried on under the auspices of the Mining Department; and if one can judge from the maps and reports, and more especially from the admirable report of progress recently issued by the Secretary for Mines, there can be no doubt that this branch of science, at least in so far as it bears upon the development of the resources of the country, is by no means neglected. The formation of the geological maps of the colony, commenced by Mr. Selwyn, is still going on; nearly 5000 square miles have been embraced in sketch-maps on a scale of 2 in. to the mile, and other maps are in progress; the most important among which may be mentioned as those of the goldfields at Stawell, Creswick, and on the Mitchell River, Gippsland. Among the valuable publications of this department, you will be pleased to learn that there are most interesting and exhaustive reports by our fellow-members, Professor M'Coy, on *Fossil Specimens;* Mr. Cosmo Newbery, *On the Analysis of Assays, and the Examination of Minerals;* and Mr. William Nicholas, on *Some Characteristics of Auriferous Quartz Reefs or Veins.*

The rapid denudation of our Forests, and almost reckless destruction of our indigenous timber, has from time to time been strongly and warningly commented upon by scientific men and by the public press of the colony; but as the want of useful timber does not immediately stare the community in the face, it is allowed to pass. If any of you have ever seen, as I have too often done, the gigantic timber trees lying rotting in some of the ranges near Melbourne, where they have been felled by saw-mill proprietors, but never used, and in many cases magnificent trees with inferior trees felled by rival proprietors across them to prevent their being readily removed to the mills of those who felled them, you will at once admit that the term "reckless destruction" is not too strong. The necessary clearing away of timber for agriculture is rapidly altering the face of the country, and will doubtless alter the climate, most probably for the worse; but the indiscriminate denudation of our mountain forests will certainly tend to reduce the precipitation of water on our soil, which already we often eagerly hope for and sometimes pray for.

Some legislation in this direction has been attempted, and it devolves on the Department of Agriculture to put what power Parliament has given it into force. This department, which is in charge of Mr. Wallis, the Secretary for Agriculture, is doing all in its power to stem the tide of mischief, and to re-forest, as far as possible, our stripped mountain sides, not only with indigenous, but exotic timber and other useful trees. The State nurseries at Mount Macedon are making wonderful progress; valuable trees to replace the indigenous giants which have been so indiscriminately felled are now covering a large part of the summit of that mountain. Thousands of plants are yearly raised in the nursery for this purpose and for distribution over the country to local bodies. It is a noteworthy fact that numbers of the European and American timber trees are being successfully grown here, and many of them make more rapid growth than they were ever known to do in the countries to which they belong. It is intended also to sow many of our wrecked forest areas broadcast with the seeds of indigenous trees, notably the ironbark, and this process will also be tried on some of the treeless plains to the north—of course, after some preparation of the ground and adoption of some means for protecting the young trees as they come up.

The establishment of Colleges and Schools of Agriculture and Forestry would be a step in the right direction, the value of which to posterity, if not to our own generation, cannot be well over-estimated in a country in which the ruling policy is to fix the people on the soil. Already the Agricultural Department has taken preliminary steps for the establishment of a college in embryo at South Dookie, in the north-east district. It is intended to confine the operations this year to conducting experiments on plants likely to be adapted for cultivation in Victoria, and the establishment of an Agricultural Chemical Laboratory in connection with the institution. It is only expected to establish the college by degrees, but I am sure the success of the Secretary for Agriculture in this and his other efforts will be sincerely hoped for by every member of this Society.

If we glance back over the past year's history of scientific research and progress, we find but little of more than ordinary interest to arrest our attention. There are, however, one or two instances which may worthily claim our attention for a few moments.

In my last address I referred to that interesting little instrument the Radiometer, and to Mr. Crooke's discoveries of the action of light and heat on bodies in vacuo; and one of the instruments was exhibited and described by Mr. Foord, who also made some remarks on the experiments he had made with it, and the principles involved in its peculiar action. The behaviour of light bodies freely suspended in vacuo, under the influence of heat and light, seemed at first inexplicable according to known laws, and the question arose whether Mr. Crooke's experiments did not point to the existence of a new force. Our best physicists, however, suggested that the whole phenomena might be satisfactorily explained as pertaining to the action of radiant heat in a partial vacuum. Mr. Crooke has now, by the continuance of his investigation, conclusively proved this to be the case, and also finds that if instead of an ordinary vacuum the most perfect one attainable is secured, the action of the Radiometer is largely weakened, and indeed ceases altogether.

Few sciences have made such strides in a utilitarian direction as that of Electricity, more especially in reference to Telegraphy. We had scarcely been able to realise the fact that two different messages could be sent simultaneously on a single wire in opposite directions, as in the duplex system of telegraphy, than we hear of a quadruplex and multiple system being in actual operation, the latter embracing the power of sending two or more messages each way simultaneously on a single line, provided a synchronous movement or identical revolution of similar portions of the apparatus at the two stations can be secured, a thing not very difficult to accomplish. From the skill required to work the ordinary duplex system successfully, it is still doubtful whether it will come into general use, and the complications of the ordinary multiple system will, I imagine, keep it rather in the category of telegraphic curiosities, which are already numerous, than permit of its practical application to commercial telegraphy.

These remarks, however, do not apply with so much force to the last achievement in Telegraphic Electricity, the Harmonic Telegraph or Telephone; and as the discoveries in this direction bear signs of promise, a few words on the subject may not be out of place here. It has long been known that the number of electric impulses that can be sent along a conductor in a given time under proper conditions appear to be, comparatively speaking, almost unlimited; at all events, as numerous as are necessary to produce almost every sound audible to the human ear. It has been shown, for instance, that if the electric contacts, and hence impulses, are given by the vibrations of tuning forks or musical reeds at the sending station of a telegraph line, tuning forks or reeds of a similar pitch can be set in action at the distant station, and that a full series of musical notes can thus be transmitted from one station to another. Some electricians have lately put this into practice, notably Mr. Reiss, of Friedrichsdorf, in Germany; Mr. Elisha Gray, of Chicago; Professor Bell, of Boston; and M. Paul la Cour, of Copenhagen. To make what I have to say clear, I must call your memory to the fact that musical notes or sounds to which the human ear is sensible consist of vibrations varying from eight up to about 36,000 per second; if they are below eight they simply constitute a number of separate noises, but if more than eight they form a tone; beyond 36,000 per second they become insensible to the human ear, but, there is reason to believe, not to the auditory systems of some animals and insects. Also that there are certain characteristics in sound—for instance, *pitch* or *tone* is governed by the number of vibrations per second, and simply relates to the highness or lowness of the sound; then we have *intensity*, by virtue of which sounds are loud or soft; and again, there is the *timbre*, or, as it is sometimes termed, *quality*, of sound, instances of which may be given by the difference in tone between the vibrating string of the piano and the vibrating reed of the clarionet or oboe. Musical sounds produced by an instrument such as a flute or violin consist of variations in pitch and intensity, while the organ can be made to produce variations in quality also by the help of the various stops; and the human voice eminently encompasses all these characteristics. Now the telephonic apparatus of Reiss, Gray, and La Cour, so far as they have yet gone, simply transmit sounds which vary only in pitch, although Mr. Gray appears to have succeeded in transmitting notes of varying intensity—that is, loud or soft, at will—for he has been able to convey a musical tune along a telegraph line so as to be identified at the distant end. The practical triumph of Gray's telephone appears in the fact that he has been able to send four simultaneous messages telephonically along a single wire, while four others were received on the same wire — a double quadruple system, as it were. This is accomplished in the following manner:—We have seen that notes can be transmitted by means of reeds or tuning forks, so we will suppose a set of such instruments arranged at both ends of a telegraph wire. Now if a reed with the pitch of G natural be set in vibration at the sending station, no other reed but the G natural will vibrate at the receiving station, and it will continue to hum this note as long as the current is passing, but ceases immediately the sender opens his key and stops the current. The sound can thus be broken up into long or short notes, crotchets and quavers as it were, to represent the dots and dashes of the Morse code, which can be as easily read by the telegraphist as the short and long taps of the Morse sounders. Now, if with another operator, key, and reed on the same wire, we send B natural, it will set the reed of the same pitch humming at the receiving end, and not interfere with the G natural reed, which will continue humming its own note. A second operator reads the B natural message, and a third and fourth any third or fourth reed that may sound. In this way several messages can be sent simultaneously by as many operators, and read by as many readers, while the principles of the duplex system provide for sending an equal number of messages at the same time and on the same wire in opposite directions.

Professor Bell's telephone, so far as I can gather, must partially embrace the third characteristic of sound, that is, the *timbre,* so that the human voice can be intelligibly transmitted through a telegraphic wire for short distances; and although it appears that the received sound of the voice is weak and not always distinct, the simple fact that the quality of the sound can be transmitted with its pitch and intensity is a most remarkable one, and we shall look forward with great interest to the future development of both this and Mr. Gray's method of telephonic communication. The details of the apparatus of Professor Bell are not generally known yet, but the principle involved is much the same as in the others, although the method differs. The sound of the human voice is projected into a kind of funnel-shaped chamber, closed by a membrane which is set in vibration in consonance with the vocal sounds. Attached to the membrane is a small permanent magnet, which vibrates with it opposite the poles of an electro-magnet, through which a constant current from a galvanic battery flows; the induction brought about by the vibration of the magnet so affects the battery current that the composite characteristics of the sound are manifested on the receiving apparatus, which, so far as one can judge from the descriptions given, consists of an electro-magnet within an iron box, the armature of which is a loose iron plate covering the box, and which is set in vibration, approximately reproducing the sound of the voice speaking against the membrane at the sending station.

I have, I am afraid, already tried your patience too long, but, before concluding, I wish to urge our younger members to greater activity in the society; there is plenty of work to do, and broad fields of untouched ground for research. The discoveries I have just spoken of come principally from our American cousins, who have done more than any other nation for electric telegraphy, which even yet presents to us an almost boundless field for research and useful discovery; and why should not some of it fall to Australians? Discovery and useful results of scientific work are not got except by persistent and grooved application. The very ground over which one must travel before he gets upon "pastures new" with any hope of success has already become long and weary; but those who steadily keep in one path not only arrive on the new ground first, but have the best chance of seeing any that has been left unturned on the way.

TRANSACTIONS.

ART. I.—*On Force.*

BY F. J. PIRANI, M.A.

[Read 12th April, 1877.]

THE nature of our conception of Force and of Force itself, if there be any such thing, have been the matter of frequent discussion; but the various questions raised cannot be said to have received answers which are universally accepted as satisfactory.

Why does a stone fall to the ground if unsupported? It is stated in explanation of this phenomenon that the stone is attracted by the earth, or that the earth exerts a force upon it. What do we mean in the first place by saying that a force is exerted upon the stone; and secondly, by saying that that force is exerted by the earth? Had we said that the motion of the stone was due to a force exerted by John Smith, the meaning of such a statement is plain enough—that a certain state or act of John Smith's mind, such as we call an effort, pull, or force, preceded and was the cause of the motion. Do we mean, then, in the former case, that a similar state of consciousness, a similar effort or pull, was antecedent to the motion of the stone? and if so, do we imagine the earth to be a being capable of exerting such pulls? As a matter of analytical convenience it is doubtless extremely useful to imagine inanimate bodies as exerting efforts to move each other about, similar to the forces which each man knows that he exerts himself, and which he believes to be exerted by other human beings; but do they really do so? I follow the system of philosophy which Mr. G. H. Lewes is now expounding, so far at all events as to reply that we have no means of ascertaining whether they really do or not; that the idea of forces supposed to be exerted by inanimate bodies is a metempirical concept, indispensable perhaps for purposes of calculation, but resembling subsidiary unknowns introduced in the course of solving a mathematical problem, which disappear in the final result.

The effects of which the forces are supposed to be the causes are all we are concerned with, and whether the earth really exerts a pull on the stone or not is a question which neither common sense nor science can solve, nor, in my opinion, need desire to solve; let the metaphysician undertake the impossible and unprofitable task if he will.

The answers I have given to the above questions concerning Force would probably be accepted by all disciples of the modern Experience school of philosophy, but many able investigators of nature and powerful reasoners have not been content with the bounds which it sets to the kingdom of knowledge. Thus Sir John Herschel has said—and his dictum is quoted with approval in a very clever and eloquent article by the late Mr. Martineau (*Contemporary Review,* March, 1876), which has important bearings on the question at issue:—

"It is our own immediate consciousness of effort when we exert force to put matter in motion, or to oppose and neutralise force, which gives us this internal conviction of *power* and *causation* so far as it refers to the material world, and compels us to believe that whenever we see material objects put in motion from a state of rest, or deflected from their rectilinear paths, and changed in their velocities if already in motion, it is a consequence of such an *effort somehow* exerted, though not accompanied with *our* consciousness."

Mr. Martineau also quotes Du Bois-Reymond, a philosopher of a very different way of thinking, who says:—

"Power, regarded as the cause of motion, is nothing but a more recondite product of the irresistible tendency to personify which is impressed upon us. What do we gain by saying that it is reciprocal Attraction whereby two particles of matter approach each other? Not the shadow of any insight into the nature of the process."

And Mr. Martineau is forced to admit that Du Bois-Reymond is justified in his criticism if the human mind has nothing to do but to become an accomplished *Natur forscher;* which is, I presume, the only aim of the human mind which Physical Science is concerned with.

The question under discussion may be not unprofitably illustrated by an analogy from the undulatory theory of light. As that theory is commonly taught in the textbooks, it supposes that at each point of space through which light is being propagated there goes on a backward and forward motion of particles analogous to the vibrations of a pianoforte-wire, and to students, nay, even to expert physicists, it is doubtless a great assistance to have the hypothesis stated in that concrete and specific form. But the truth of the undulatory theory is only established by the agreement of its results with those of experiments, and the same results could be obtained from a much more general hypothesis than that usually made. It is only necessary to suppose that, as Clerk Maxwell says (*Electricity and Magnetism*, Vol. II., p. 407), the disturbance which constitutes light is of the nature of a vector (*i.e.*, a quantity having both magnitude and direction) perpendicular to the ray; and all the beautiful theorems whose truth has been so abundantly confirmed by experiment and observation, could still be deduced if we supposed that the vector disturbance is a strain, a rotation, a magnetisation, or electrification of particles, instead of supposing the particles to have motions of translation.

Still it would be inconvenient, if not impossible, especially for purposes of instruction, to abandon the ordinary specific hypothesis. In the same manner should the hypothesis of forces exerted by inanimate bodies be maintained, as though not necessarily true, still very convenient, and invariably leading to true results. It is often said that if all calculated results of an hypothesis agree with experiment, that hypothesis must itself be true. The statement is not correct. The most that we are warranted in believing is that all other calculated results will also be found to be experimentally true, and this is especially the case when the hypothesis is one like that of Forces, which from its very nature cannot and could not under any conceivable circumstances be directly subjected to an experimental test. Surely it is more hopeless to attempt to verify the existence of the earth's attraction than it is to endeavour to see the vibrations of the ether.

Professor Tait, in a lecture delivered before the British Association last year, has attacked the existence of Force in a different manner; and although I agree so far with his conclusions as to believe that the existence of material forces is not and cannot be proved, I do not believe the reasoning by which he arrives at that conclusion is valid. He not only believes that Force is proved not to have real objective existence, but that that peculiar and abstruse quality is proved to be possessed by Matter and by Energy. One of the premises from which he is led to his conclusions is that Matter and Energy are unalterable in quantity, while Force is not so. True enough; but consider the other premise—that those qualities or entities whose total quantity is unalterable, and those only, do really exist.

By anything having real objective existence, Professor Tait explains that he means that it exists altogether independently of the senses and brain processes, by which we are informed of its presence. Whether anything does exist in this independence, I do not know; nor do I believe that any one else does or can. But without going into the controversy between Realism and Idealism, I simply ask whence does Professor Tait obtain his axiom connecting absolute reality and indestructibility? What higher claim has it to credence than any of the axioms criticised by Mill, in his chapter on Fallacies of Simple Inspection, such as "Circular motion is the most perfect," "Things which we cannot think of together cannot coexist," "Things which we cannot help thinking of together must coexist," "Whatever can be thought of apart exists apart," and so on?

Moreover, if the negative portion of the axiom be accepted, although Matter—that is Mass—is proved to exist, Time, Distance, Motion, are degraded to the rank of nonentities along with Force.

But how is the mass of a body defined and measured? By the effect which a certain *force* acting on the body for a certain *time* would produce. And how is energy defined and measured? As power of doing work—that is, of overcoming a given *force* through a certain *distance.* Surely I cannot be accused of presumption in criticising the conclusions of a thinker of Professor Tait's high standard when he tells us that that which is defined in terms of, and measured by means of, that which does not exist, has itself independent real existence.

As probably most of you have read the lecture referred to, it is unnecessary for me to say anything about the most valuable part of it—Professor Tait's exposition of the loose and ambiguous way in which the term Force is often used even by those who should know better. For this he should have earned the gratitude of all lovers of that accuracy in scientific language without which accuracy of thought is almost unattainable.

ART. II.—*Some Experiments in Propulsion.*

By S. R. DEVERELL, ESQ.

[Read 12th April, 1877.]

THE following are particulars of some experiments made at Torbay (England) in February last, by Mr. B. Tower, of Newcastle-on-Tyne, respecting the application of the power represented in the movement of a ship on waves. The experiments were made in the presence of Mr. W. Froude, F.R.S., and Mr. H. Brunel. The vessel in question was a miniature ship of six (6) feet in length, and was lent for the purpose from the Admiralty Works at Torquay. The apparatus used was similar in plan to that of a model exhibited at the Exhibition in Melbourne in 1873, with the exception that a strong metallic spring was employed instead of a pneumatic one. The tension on the spring was such that when the vessel was horizontally placed in smooth water the loaded working beams of the machinery were also horizontal. The relative motions of the load were limited to one dimension only—viz., in a plane at right-angles to the plane of the deck. These relative movements were imparted to a ratchet-wheel, causing it to revolve continuously in one direction. The shaft of the ratchet actuated a large wheel and pinion, and the continuous rotation of the pinion was ultimately conveyed to the screw shaft by an indiarubber band accumulator, which stored up the power transmitted to the screw.

As the vessel was decked, and had only been lent for the trial, the machinery had to be placed above deck, and owing to this it could not be loaded to its full power: a load of only seven pounds being placed on it. This was a serious disadvantage, as, had the machinery been below, a load three times as great would have been placed on it, the power developed being increased in the same proportion. Notwithstanding this, the results completely verified the calculations which had been made respecting the operations of the machinery, the screw on an average making forty (40) revolutions per minute, the vessel attaining a speed of 3½ knots against a head sea and wind. The maximum effect was observed to take place when the play of the load was isochronous with the period of the waves; whenever this occurred the machine worked with great vigour, the screw sometimes making as many as 180 revolutions per minute. It should be remembered, however, that this great speed of rotation of the screw is not the best suited for propulsion, on account of the creation of what is known as negative slip of the screw. Indeed the difficulty throughout in the experiments which have been made is not in obtaining sufficient power, but rather in controlling the excess of it. The wind on the occasion under notice was off shore, the waves therefore very small, about four feet long, and a few inches only in height, with a period of six seconds. The reason why the period of the waves is so important an element in the effect produced, is that the efficacy of the principle depends mainly on the velocity of the movements, not their magnitude, as shown in the fact that the model in question worked vigorously with the movement of only an inch, repeated however ten times per minute. In point of theory the action of the apparatus involves some very abstruse points; indeed it had proved not a little perplexing to those who had witnessed it. Mr. Froude, at the first meeting of the Institution of Naval Architects, 1874, referred to the principles involved in the action of the machine as a very obscure subject; and again, at the British Association at Bristol, September, 1875, he spoke of it as a most complex proposition which he and others had at first only dimly seen through. Mr. G. Rendel also, the distinguished engineer and originator of the "Staunch" class of gun-boats, and the partner of Sir Wm. Armstrong, has referred to the principle of the machine as (to repeat Mr. Rendel's words) a very curious and beautiful idea, and that it has been well worked out; as a scientific principle, he adds, he considers it perfect. Similarly at the April session of the Institution of Naval Architects, Lord Hampton, the President of the Institution, spoke of it as one of the most important, but at the same time most difficult, of projects. It need hardly be added that the development of a principle so little understood as is admitted in these opinions is necessarily a work of slow progress, when every step in the demonstration nearly exhausts for a long time individual means.

The dynamical effect exhibited by the model during the experiments as accurately taken at the time, was at the rate of 1½ horse-power per ton of working load. With regard to this vigour of action, which occasioned some surprise at the time, it may be remarked that the load acquired such a proportion of the large moving force of the water displaced by the ship as the mass of the load bears to the mass of the ship. Thus if 100 tons be employed in a vessel of 1000, the machine acquires 1-10th of the whole moving force of the water displaced ; this being indirectly abstracted, as Mr. Tower well expresses it, from the vast store of energy passing beneath the feet. In other words, every ton becomes imbued with the force with which the same weight of water —*i.e.*, of thirty-five cubic feet— is moving at the time : in the case of a load of 100 tons consequently representing the energy of 3500 cubic feet of water moving with the speed of the wave motion. The considerable effect of this may perhaps be apparent (though the applications are quite dissimilar) by observing the effect of even a sluggish stream in turning a water-mill.

The experiments briefly detailed above have been since repeated in different forms with the same results, and have been admitted to have shown the correctness of the method employed, whatever may be the theory of its action, in applying the energy stored in the movements of the sea. As some doubt was expressed at the British Association (Bristol, 1875) as to the ability of the machine to drive a ship against a head sea, Mr. Froude (who was at the time President of the Mechanical Section) stated that he had himself witnessed the model in Torbay driving itself against and through a head sea which, in comparison with the size of the model, was mountainous. As this refers to a point of importance, the testimony of so distinguished an authority may, I think, be regarded as definitive on the matter. A proposition to which value has been attached is that, given the same bulk and weight, the power developed under ordinary circumstances compares favourably with that of a steam-engine, and under exceptional states of the sea it is very much greater. I think I may say that the very carefully repeated experiments of Mr. Tower do not leave room for doubt on this head. In any case it would appear that, apart from auxiliary propulsion, a useful source of power for many minor purposes at sea exists. As regards pumping, it may be remembered that the power referred to is mostly greatest in those emergencies when it is most required—viz., when a vessel is at the mercy of the elements, and when fires cannot be maintained.

Art. III.—*The Present State of Meteorology.*

By R. L. J. Ellery, Esq., F.R.S., F.R.A.S.

[Read 10th May, 1877.]

The desirability of increasing our knowledge concerning the weather, and more especially with the view of securing some amount of prescience on meteorology, is, I believe, generally admitted; and few will for a moment question the propriety of expending labour, pains, and money, if thereby the more important changes of weather could be predicted with certainty a few days in advance, or if reasonable premonition of climatic vicissitudes—such as rains, droughts, excessive heat, or cold—could be deduced from the discussion of past and present meteorological observations. Assuming this much, then, I purpose to refer briefly to what has been and is being done towards these ends, and with what probability of success and usefulness to the world.

Although the systematic meteorological observations and investigations of the physical laws dominating the changes and movements of the earth's atmosphere have occupied the attention of physicists and observers in past times, it is only within the last few years, comparatively speaking, that the subject has been grappled with comprehensively and scientifically. The tentative essay at prediction and forecast on scientific principles which has been made in Europe and America are matters almost of to-day, and must be considered as yet only "feeling its way." It is true we have had from time to time, from Murphy downwards, weather systems propounded, weather predictions a year in advance, and almanacs printed with a prediction allotted to each day; a lucky coincidence or two enlists the belief of the ignorant for a time, but that great teacher, experience, eventually relegates all these spurious systems to the limbo of fools. The truly scientific meteorologist knows the difficulty of the matter, and how little has yet been made light which will enable him to predict with confidence what weather will prevail in any one locality a few hours ahead, and will at once admit his inability to deal with the facts of meteorology as he would with those of any of the physical sciences.

Attempts have also been made, upon scientific grounds, to deduce from a discussion of seasonal mean temperature the probable characteristics of coming seasons; to ascertain if there be a periodicity in climatic vicissitudes, as well as to generalise in other ways from past experience. As an instance of these attempts, I may refer to the very clever and exhaustive paper by my friend and co-labourer, Mr. H. C. Russell, of Sydney, given to the Royal Society of New South Wales, entitled "Meteorological Periodicity;" but while this paper is one of the most valuable extant for reference on the subject of Australian meteorology, it clearly indicates the apparent hopelessness of any such attempt in our present state of knowledge, and certainly no satisfactory results have been deduced from the other investigations referred to.

Almost every civilised country at the present time is provided with a principal meteorological observatory or observing station, generally assisted by various other stations of more or less importance, according to position or instrumental appliances, either wholly or partly supported by public money. Besides these there are always numerous careful and energetic private observers, who voluntarily furnish the central observatory with the results of their work. I know of no country or place of importance where settlement and civilisation have reached from whence meteorological records cannot be obtained; and if one can judge of the extent to which meteorological facts have been collected from the piles upon piles of manuscript records at the Melbourne Observatory, not only from these colonies, but from various regions of the broad ocean, from desolate islands and other places, leaving alone the weary number of volumes, sheets, and pamphlets which arrive from other countries, I think I am perfectly safe in saying that in no branch of inquiry has such an enormous amount of statistics been collected as in meteorology.

Now one of the chief, if not the chief, object in instituting meteorological observations in any country at the public cost, may be assumed to be climatology—for economic, sanitary, and, perhaps most of all, for agricultural purposes; to ascertain by a long extended series of observations the range of temperature, rainfall, movements of air, &c., to which the particular country may be subject. The broader aspect of the question is, as a rule, a secondary consideration—to be desired, but too extensive to be grappled with by observations extending only over a limited area; and so, while the accumulating records gradually serve the more immediate climatological requirements, they are laid by or are printed and disseminated. Except for the sake of criticism, these printed observations are only referred to occasionally by the student, writer, or traveller; and although there is now and then something said of the desirability of dealing with this enormous collection of facts, I think that about a thousand Keplers would be wanted for the task.

It will not be denied, however, that for local requirements some systematically conducted meteorological research is necessary and valuable in all civilised communities, more especially in countries like Australia, depending largely on agricultural and pastoral interests, as well as maritime commerce, and subject to the climatic vicissitudes which so often prevail. Assuming this, it will not be unprofitable to inquire how the observations can best be made in Australia to serve all the more immediate and local requirements, and at the same time assist in the general scheme of investigating the laws which govern the earth's atmosphere generally.

Before doing this, I would briefly indicate what is being attempted in other countries. The United States of America certainly stand in front as far as regards the magnitude and system of meteorological research, and the results obtained. The vast land-tracks in the U.S. over which meteorological observing stations have been extended have made possible in that country a system which few other nations could attempt. Provided with almost unlimited means, and the assistance of a whole army of military men as observers, the signal service of the United States has been enabled to meteorologically blockade a large portion of the continent. Aided by all the facility that can be conferred by a network of telegraph lines where priority and promptitude of despatch is insisted upon and given, the American meteorological system is undoubtedly the most complete in the world. The principal outcome of this great scheme is the issuing of daily weather charts and bulletins showing the meteorological conditions all over the States, and the publication of forecasts or "probabilities" (as they are called) of the weather a day or two ahead, indicating the track and intensity of marked disturbances, or the approach of fine weather. It is stated that over 80 per cent. of these predictions are realised, and if that be so, the result will not be so incommensurate with the magnitude and cost of the system as might at first be imagined. It is to be hoped, however, that in this magnificent undertaking some of the higher meteorological problems may be attempted and solved; and it is not unworthy of remark that General Myer, the director of this service, has enlisted the co-operation of nearly all the meteorological observatories in the world in obtaining simultaneous observations—that is, the meteorological conditions in force at each station at one definite time, that time being forty-three minutes after noon, Greenwich mean time.

From inquiries made during my late visit to Europe, I ascertained that 250,000 dols. was the annual vote for the American signal service, and that that amount included no salaries for observers, all of which come from the military votes. In Great Britain £10,000 is voted annually for meteorological purposes, and the commission of inquiry in its recent report on the department recommended an increase to £14,000 or £15,000.

The meteorological system of Great Britain includes both ocean and land meteorology. The former comprises means for furnishing the necessary instruments, &c., for observation to ships of both the Imperial and mercantile navy, and collecting and tabulating the results; while the latter includes, besides the ordinary systematic observations, a very complete system of weather telegraphy and storm warnings. Every morning, Sundays excepted, telegrams are received from about 50 places, more than half of which are in the British Isles, and the rest in other European countries. These telegrams are immediately discussed, and weather-charts founded on the results are at once published and disseminated. By this means the movements of the atmosphere over Northern Europe and the adjacent ocean become known. The approach of storms can be generally predicted with reasonable certainty, and warning at once given to the threatened coast line by telegrams, which are made widely and rapidly known by the storm-signals and other means. At the same time all the purposes of agricultural meteorology are subserved by the weather-charts, and the carefully prepared bulletins published in the daily papers. While, therefore, the more strictly local and practical requirements are thus admirably served, by reason of the oceanic observations and the widely spread area from which daily telegrams are received, the more theoretical demands from which to deduce information concerning the relations that prevail between the atmospheric movements and conditions in different parts of a considerable portion of the earth's surface are supplied.

France, Belgium, Denmark, Holland, Germany, Sweden, Russia, Austria, and Italy, all co-operate in similar work; but while America and England undoubtedly contribute most liberally, each of the nations mentioned grants State funds for meteorological purposes varying from £500 to £6000 annually. The latter sum, if we take into consideration the value of money and cost of computing power in most of the countries named, would represent an amount equivalent to, if not more than, the annual grant made by the British Parliament.

These brief references will convey a pretty correct notion of what is being done for meteorology in the Western world. I have only to mention that in South America, Cape Colony, India, China, Japan, Mauritius, and other places, systematic observations are made, to show that a pretty round sum must be expended every year for the purpose of recording what the weather has been, with the glimmer of a hope that the power of predicting what it will be may be eventually secured.

The outcome of all this expenditure of money and labour is at present easily summed up. In America it is said, and I do not doubt it, that immense and increasing benefit is conferred on the community by prompt publication of the "probabilities." In Great Britain and Northern Europe most of the dangerous storms are foreseen, and much loss of life and property no doubt prevented; for the rest of the world, with some few exceptions, the results are confined to furnishing climatic statistics generally of mere local interest, the piling up of volume upon volume of books filled with regular readings of instruments and descriptions of atmospheric appearances, which are exchanged between the observatories and scientific institutions of the world, forming so much building material for our future meteorological architects.

It will be evident from what I have already stated that meteorological observation holds a prominent place in the world's work, and that there is no niggard contribution from State or other public funds to aid in the undertaking; and while it will also be seen that in addition to the collection of statistics, which are in themselves valuable, a foretaste of what may be hoped for from systematic investigation has been actually realised in both Great Britain and America, it cannot but be admitted that meteorology has not yet become a science. To those who know the difficulty and complexity of the problems involved, this is no matter for surprise. Nevertheless, if, after all the time, money, and labour spent upon observation, and the enormous mass of statistics collected, we are compelled to this conclusion, the question forces itself upon us whether or not the inquiry of nature has been in the right direction, or whether there are not other modes of inquiry necessary to elucidate what the usual modes of observation have as yet failed to do. These questions I cannot pretend to answer. I feel confident, however, that our inquiries must be extended in new directions before further theoretical knowledge can be secured.

The present system of meteorological observation consists in measuring and recording at each particular locality the variations of temperature, pressure, movement, and humidity of the atmosphere, the amount of heat radiated from the sun by day and sent back from the earth into space by night, the amount of water evaporated from the earth's surface, and the amount returned to it in the shape of rain. To these may be added as matters of observation at some places the electric condition of the air, the temperature of the exterior crust of the earth, and the variations of terrestrial magnetism. Although nearly all observers agree that these constitute the orthodox items for observation, they are not at all agreed as to the best methods of obtaining them; there is a diversity of apparatus, different methods of exposure, and different times for observation. Some observations considered of paramount importance in one country are neglected in another, and so on. In order, however, to establish one universal and accordant system, a congress of European meteorologists was formed a few years ago, which has met from time to time at the various cities of Europe to discuss matters connected with this part of the subject. Recommendations have already been issued and co-operation invited by the congress, but the existing differences in matters of detail are so numerous and great that it is likely a considerable time will elapse before the congress can hope to succeed in establishing that uniformity of procedure so necessary in meteorology. Most of the observations are made near the surface of the ground, and even in this part of the subject difference of opinion exists: some prefer 4 feet, others 5 feet, 6 feet, 7 feet, or 10 feet, while many physicists attach great importance to the establishment of observatories at considerable altitudes, either on mountain-tops or by means of captive balloons; and there can be little doubt that observations made at altitudes varying from 2000 feet to 10,000 feet would add very materially to meteorological knowledge. Within the last few years, also, the state of the sun's surface has been regarded by many as being in some way connected with climatic variations, as we know it has upon the magnetic conditions of the earth.

I must now say a few words concerning what has been and is being done in Australia in this matter. For many years past meteorological observations of a more or less perfect character have been made in the various colonies, and annual means of temperature, rainfall, &c., deduced. Of later years the number of observing stations has been largely increased, with greatly improved instrumental aid; and many of the questions asked by the public, meteorologists have been able to answer; the chief characteristics of the climate have become known, and some of the laws which govern the movements of many of our atmospheric disturbances have been ascertained. But regarding the great local question of dry and wet seasons, and similar matters of the greatest importance in Australia, we are as ignorant as ever. I have now been intimately connected with Australian meteorology for nearly 25 years, and have gained some experience as to our requirements in that respect, of which I shall have a few words to say presently. At the present moment we have five properly furnished meteorological stations, where observations are made at least three times a day. Four of these are on the coast, three of which are lighthouses. Besides these we get observations once or twice a day made with standard instruments from seven stations, and records of rainfall and state of weather from 23 stations. Most of these are supplied with instruments at the cost of the State, while many observers furnish returns more or less complete with instruments belonging to themselves.

Some months ago, after my return from Europe, I determined to try and bring our meteorological system into a somewhat better shape. Each colony possessed a pretty complete machinery for first-class observation, and every month, or every year, the printed results were exchanged. My inquiry into the working of the weather telegram system in Europe convinced me that, now all the colonies are connected by telegraph, a similar system, on a smaller scale, could be put into operation here with considerable advantage to the public, especially the maritime portion, and at a very moderate cost. The question had often been discussed between Mr. Todd, of Adelaide, Mr. Russell, of Sydney, and myself, but matters had never appeared ripe until last year, when I formally asked the co-operation of these gentlemen, which was cordially given. Plans of operation were discussed and agreed upon, and in January last a system of Australian weather telegraphy was commenced. This system consisted of the exchange of observations in cypher by telegraph between Adelaide, Melbourne, and Sydney twice a day (Sundays excepted), the observations being those obtained at selected stations furnished with properly tested instruments. The stations were so selected that most of the coast-line along which passes our principal traffic should be represented, as well as districts which may be taken as typical of Central Australia; and with the view of having information of the dip of the monsoons and equatorial currents, stations along the trans-Australian telegraph line, as far north as Port Darwin, were also chosen. The information exchanged is of the usual kind—readings of barometers, thermometers, rain gauges, observations of wind, state of sea, appearance of sky, &c.

The first object in view in establishing this system was to prepare every afternoon a synopsis of the weather and state of the sea along the coast line, and also eventually to issue a weather chart, showing graphically the substance of the weather telegrams. It was intended to publish this information by posting the charts and bulletins at the various telegraph and shipping offices where they were likely to be of value.

The second object hoped for was the increase of knowledge of the meteorology of Australia generally, and additions to the very scant theoretical information we now possess.

Up to a certain point this system may be said to be established in Melbourne, but beyond it seems at present somewhat difficult to get, on account of the irregular and unpunctual manner in which the telegrams from the neighbouring colonies come to hand, rendering it impossible to satisfactorily attempt the publication of either weather bulletins or charts. Whether this is owing to defective telegraph arrangements, or a want of appreciation of the importance of the matter on the part of the various Telegraph Departments, I cannot say; but it must be obvious to all who know anything of the matter that unless there be prompt despatch and delivery of weather telegrams, it will be useless to try and make any immediate use of the information for the public benefit. In England, America, Belgium, &c., weather telegrams have precedence of all but pressing State business, as it is well known that without it they would be useless. These difficulties are, however, I hope only temporary, and are almost inevitable at the beginning of all new undertakings. I have good hopes therefore that the system will ripen into a most useful institution, which will, I am sure, be quickly and fully appreciated by the public. It is hoped that Western Australia, Tasmania, and Queensland will before long be included in the scheme; for the two former are, from their position, of great importance, and will increase in no small degree the prospect of further theoretical knowledge.

The meteorological observations comprised in this system leave a large amount of local inquiry unsatisfied, which can, however, I believe, be adequately provided for by a simpler method than is required for Australian weather telegraphy. While the six or seven selected stations in Victoria must be kept in the most efficient working order, with a full supply of instrumental means, local climatology and weather statistics can be furnished by a more numerous class of secondary stations, which should supply a brief daily report by telegraph of the state of weather, wind, temperature, and rainfall, and keep a record of the same, from which the usual monthly and annual means can afterwards be deduced at the Observatory for publication in the meteorological statistics. Such stations should be established in every township of importance, and it is a question whether this might not best be done by the municipal authorities, for it is not at all improbable that they might take sufficient interest in the matter, simply for the sake of the local information, to provide the necessary instruments and secure the requisite observations.

Our rainfall varies so largely with locality, that in order to obtain trustworthy statistics—so necessary in matters of water supply, drainage, and other public works—a rain gauge should be kept at every police station throughout the country. There are over 300 public barometers on the English coast for the use of fishermen and others, and in Victoria there are seven or eight. A few more of these instruments, if they could be taken care of (which some of those now in position appear not to be), would be advantageous. They are, however, not nearly so much required on our coast as in England.

The eager inquiries from all classes for weather news, especially during our critical seasons, render it desirable to adopt some simple means for furnishing the information sought. This is now done to a considerable extent by the Central Telegraph Office, but threatens to become a too cumbrous tax on that service if it is not systematised. If the localities from which reports are to be received were properly selected, and a simple code adopted, confining the reports to state of wind and weather, rainfall and temperature, omitting barometer readings entirely, a much more comprehensive and comprehensible bulletin of the weather prevailing throughout the colony would be furnished to the public than is now the case, without taxing the Telegraph Department so much as at present. By these means I think all the requirements of a temporary and local character would be fully met, while all the higher and more theoretical questions would be probably better dealt with by confining our attention to a few well-selected and well-equipped stations than by more numerous half-furnished observatories indiscriminately chosen. It is more economical, and more likely to be fruitful. The establishment of a station at a considerable altitude is the only addition to the present scheme that is required, and this I hope to accomplish before long on Mount Macedon, at an elevation of 3000 feet.

ART. IV.—*Notes on a Remarkable Meteor seen May 20th at Ballan.*

BY LOUIS LE GOULD, C.E.

[Communicated 14th June, 1877.]

ART. V.—*Notes on the Design of Telescope Tubes.*

BY W. C. KERNOT, M.A., C.E.

[Read 12th June, 1877.]

THE problem which I desire to bring before the Society to-night is that of the design of tubes for telescopes, and my remarks will have especial reference to telescopes of large size, such as for example the great Melbourne Reflector. These gigantic instruments are usually reflectors, and generally consist of a large and a small speculum, with the necessary subsidiary apparatus; and the function of the tube is to support these optical appliances in their correct relative positions. Should the tube be of a flexible and yielding nature, it will, by virtue of its own weight and the weight of the specula, bend down or deflect when it is in any position other than vertical; and this deflection will vary in amount and direction in the various positions the instrument is made to assume when directed to different points in the heavens. Hence if the optical arrangements are in correct adjustment in one given position of the instrument, they will cease to be so when it is moved to any other position.

As all known materials are more or less elastic, it is manifestly impossible to construct a telescope tube which shall be altogether free from this objectionable deformation. Nevertheless it is both possible and desirable to choose such a material, and to arrange it in such forms, as to reduce the inevitable deformation to a minimum; in other words, it is requisite to determine in what shape the material should be arranged in order to attain a maximum of stiffness, and to the question as thus limited I shall confine my further remarks.

In the Melbourne Telescope the large speculum is a very ponderous affair indeed, containing with its surroundings some tons of metal; while the small mirror situated at the opposite end of the tube is by comparison a mere featherweight. Hence the point of attachment of the tube to the declination axis (upon which alone it is supported) is placed very near to the end where the large speculum is fixed. The lower portion of the tube from the main speculum to a point a short distance on the other side of the declination axis is a hollow cylinder of riveted plates of metal very similar to the outside shell of a steam boiler. From that point to the extreme further end it consists of an open latticed arrangement of metal bars. In the Great Paris Reflector—a somewhat similar instrument in other respects—the whole tube consists of a continuous cylinder of boiler-plate. This latter arrangement, while admirable in point of stiffness, is objected to as giving rise to a certain circulation of currents of air of unequal refractive power, and thus impairing the optical performance of the instrument. The former system—that adopted in the Melbourne Telescope—is free from this somewhat serious objection.

We have thus arrived at these conclusions—1. That the greater part of the length of the tube of a large reflector must consist of an open framework of thin bars. 2. That this framework will be supported at one end only, where it is united to the cylinder tube, and will be loaded by its own weight and that of the small speculum. 3. That the framework must be so arranged as not to intercept any of the rays of light in their course through the instrument. 4. That the framework must be so designed as to secure a maximum of stiffness with a given amount of material; and 5. That it must be equally stiff in every direction.

In order to comply with condition 3, the bars must be placed altogether exterior to the solid cylinder of rays proceeding to the main speculum, and may be appropriately arranged in the surface of a cylinder or a prism of polygonal section. And in order to comply with condition 4 it will be necessary to revert to the fundamental principles of design of framed structures, and to adopt a method of investigation similar to that employed in designing girders, roofs, and bridges. In fact, the design of our telescope tube is but a particular case, or extension of the old familiar problem of designing an open framed bridge girder; the main difference being that, while the bridge girder is required to resist forces in one plane only, the telescope tube is, by condition 5, required to resist forces in various planes.

The effect of the force of gravity upon each particle of material in the telescope may be resolved into two portions —one along the length of the tube, the other at right-angles to its length. The first of these will attain its maximum value when the tube is vertical, and will vanish when it becomes horizontal; the second will attain its maximum when the tube is horizontal, and will vanish when it is vertical. The effect of the first set of forces will be to shorten or compress the tube longitudinally, thus bringing the specula nearer together. But this result is not a practical evil; for it is, in the first place, excessively minute, and, further, is completely neutralised by the action of focussing the instrument. The second set of resolved parts—those at right-angles to the length of the tube—tend to bend the tube, and thus throw the specula out of their proper relative positions opposite each other; this is a more serious evil, as it at once impairs the action of the optical part of the instrument.

In designing our tubes, we need therefore have regard only to forces at right-angles to its length.

A properly-designed framed girder for a bridge will be found almost invariably to consist of two massive parallel straight members or booms, connected together by a system of more slender straight bars, forming with the parallel booms a system of triangles. The essential conditions of strength and stiffness are in this case—1st, that the structure should consist of an assemblage of triangles; the triangle being the only polygon the form of which is absolutely fixed when the length of its sides is known, and therefore the only figure which will maintain its shape in spite of external forces without requiring its various parts to endure a cross-bending action; and 2nd, that all the sides of the triangles should be straight, for seeing that they are called upon to endure longitudinal compressions and tensions alone, a crooked or curved form is plainly inadmissible. No one would think of making a pillar intended to carry a heavy load, or a tie-rod to endure a heavy tension, other than straight.

Now, our framed telescope tube, like the bridge girder, must consist of a series of rectilinear triangles, and it must also have its massive longitudinal booms. Two booms will not now, however, suffice, for no longer are the forces we have to contend with, as in the bridge girder, all in one plane. The tube must be a girder in at least two different planes. Now, two ordinary girders, intersecting each other at right-angles, would be well adapted, as far as strength and stiffness are concerned, but are optically inadmissible; and therefore it is necessary to fall back on a prismatic section, each side of the prism being a complete girder. A prism of four sides—a square section—would be strong and stiff, but somewhat unsightly. It has been employed by no less an authority than Warren de la Rue in the reflector which he used for obtaining his celebrated photographs of the moon. I have here a model (Fig. 1), in which I have endeavoured to show what appears to me the most favourable disposition of material, all things considered. It is hexagonal in section, having booms at the angles, which together contain about half the material of which the tube consists. The booms are united by a series of small, straight, diagonal bars, making an angle of 45° with the booms, this being the mathematically demonstrable angle of economy in such structures. The latticed tube ends in a stiff, hexagonal angle-iron ring, as shown. The salient feature of the model is the size and number of the booms; and this is a very favourable arrangement in view of stiffness, for, as Bindon B. Stoney has shown in his excellent work on *Strains in Girders and Framed Structures*, the deformation of a girder due to compression or extension of its booms is a large quantity compared with that due to the compression or extension of the smaller bars uniting the booms.

In contrast to Fig. 1, let us consider Fig. 2, which is a representation of the actual tube of the great Melbourne Telescope. Here we shall, I think, find a systematic infraction of all the canons above laid down. In a properly designed framed structure all the bars are straight; in the Melbourne Telescope they are all curved. In a properly designed girder a large proportion of the material is placed in the form of longitudinal booms; in the Melbourne Telescope absolutely none is so employed. The proper angle of economy and efficiency is 45°; in the Melbourne Telescope this angle is nowhere found.

The action of the various bars of the Melbourne Telescope, when under strain, is rather intricate; I will, however, endeavour to trace it. When the tube is horizontal or inclined, the effect of gravity is to produce a longitudinal tension of the upper side and a longitudinal compression of the lower side. To resist these stresses we have a series of curved bars placed at an angle of about 30° with the lines of stress. These on the upper side tend to straighten when under stress, and those below become more curved. Hence arises a general bulging in of the upper or extended side, and a general bulging out of the compressed or lower side of the tube. This action is plainly visible in the model when loaded. Those parts of the tube which connect the top and bottom together are subject to equal inclined stresses—the bars that slope upward toward the open end to compression, the others to tension; the former tend to become more bowed, the latter to straighten; and as they are riveted at each intersection, these two actions probably antagonise and balance each other.

The angle-iron rings which are placed at intervals along the tube do not, as far as I can see, fulfil any important function. I think the tube would be improved much if they were removed, and longitudinal booms inserted instead.

In order to verify experimentally the preceding conclusions, the two cardboard models represented by Figs. 1 and 2 were constructed. They are of equal length, and will permit the unobstructed passage of cylinders of rays of light of equal diameter. They were constructed from the same sheet of cardboard, special care being employed to use an exactly equal area of cardboard in each model; and both in constructing and testing them every possible precaution was taken to place them under absolutely identical conditions. The test load was a weight of 12 ounces avoirdupois, applied at right-angles to the length of the tube at its upper or free end, the other end being firmly fixed to a massive frame. After each experiment the tube was rotated on its axis, so that the test load should act on a different plane. In this way Fig. 1 was tested six times with the test load acting in planes passing through two opposite angles, and six times in planes passing through the centre of two opposite sides; and Fig. 2 eight times in various directions equally distributed round the circle. The mean results of these experiments were as below:—

Fig. 1. Deflection over angles, mean of 6 results, ·0325 in.
„ „ „ sides „ „ ·0314 in.
Fig. 2. „ mean of 8 results, ·0876 in.

Fig. 2

Fig. 1.

During the trial the bulging in and bulging out of the extended and compressed sides of Fig. 2 were plainly visible; but no such distortion of Fig. 1 was to be detected, although its diameter was repeatedly tried with callipers.

ART. VI.—*Notes on the Coast Line Formation of the Western District, and Proofs of the Uniform Condition of Meteorological Phenomena over long periods of time.*

BY MR. T. E. RAWLINSON, C.E., &C.

[Read on the 14th June, 1877.]

TWO years ago a very interesting paper, by Mr. R. Etheridge, on the sand dunes of the coast of Victoria, was read before this Society; and I purpose following up the subject by a few notes of personal observations on the same subject, connecting it with volcanic phenomena of the locality.

My observations are chiefly confined to the portion of coast line from a few miles east of Warrnambool to a few miles west of Belfast.

My object in doing so this evening is to bring forward evidence which I consider conclusive in reference to establishing the fact of the permanence in this locality over great periods of time of climatic conditions, and the several changes in the coast line during the same period.

The present coast line from the River Hopkins, east of Warrnambool, to the Yambuk Lake entrance, about ten miles west of Belfast, is the third and last line of beach, and consists chiefly of pulverised shells; and, as Mr. Etheridge points out, echini spines and other marine remains, to which I may add enormous quantities of calcareous operculums, which, from their great strength, have borne with impunity the bruising which has mostly destroyed the parent univalves, although in places there are many of these univalves yet left on the dunes, together with the helios limpet and more ordinary bivalves of the present sea.

In all cases where I have tested the so-called sand with acids, 80 per cent. and upwards has dissolved, leaving a small residuum of reddish mud or clay, and the remainder particles of silica (or sand).

From Belfast, for a distance of from four to five miles easterly, I have often found pure flint nodules, with the outward white coating precisely the same in appearance as those obtained out of the chalk hills of Kent; and if it were not for the number picked up from time to time at various places on the line of hummocks, I should have been disposed to think their occurrence purely accidental, the more especially as I know of no other place where they occur near to Belfast, nor do I know nor can I conjecture the agency at work in their formation.

Between two and five miles east of Belfast I have been much surprised to find the frequent recurrence of human remains (nearly always in pairs), which had become bared and the bones mingled together, owing to the action of the wind on the drifting sand. I have counted as many as 50 undoubted remains, without taking into account scattered bones which may have belonged to other groups; but in only one case have I seen a perfect skeleton, and this was just above high-water mark, the sand around it being tinged a darker shade, the skull being a little distance away, and perfect. Owing to matters of business preventing my attending to the affair at that time, I lost the opportunity presented of securing the skeleton, owing to the wind and other causes having disturbed the remains. That all the remains were human cannot be doubted, because of the presence of the leg, thigh, and arm bones, the ribs and vertebræ, and frequently the skulls, with the front teeth of the upper jaw wanting.

From frequent enquiries made of the oldest residents in reference to the remains, I could obtain no information; and natives who used to muster in Belfast under the genial hospitality of their protector, Mr. Dawson, when first questioned on the subject evidently knew nothing of it; but after they had time to consider the object of the questioning, they, with the well-known courtesy of the race, had a reply which they evidently considered was the answer wanted.

Some years afterwards, in conversation with Mr. Goodall, the Superintendent of the Framlingham Aboriginal Station, he informed me that he had no doubt he could obtain what information there was to be had from an old Port Fairy blackfellow on the station; but on my expressing doubt as to the value of such evidence, he replied that from long acquaintance with them he felt sure he could question them and obtain truthful replies to his answers, unmodified by qualifications and inventions given with a view to please. Shortly afterwards Mr. Goodall informed me that the old blackfellow said there had been a great shooting; that "Blackfellow had been rounded up and shot by whitefellow." Mr. Goodall expressed himself as perfectly satisfied that the answer was given in good faith, and was true; and this will account for the singular occurrence of the remains in couples, which so frequently, and as far as my observations went, always occurred, the perfect skeleton on the beach excepted.

The above being true (and I think it very probable), it is but a confirmation of those accounts so frequent in connection with the early settlement of the country, of the wretched natives in their ignorance interfering with the white man's flocks and herds, and provoking these terrible reprisals. It constitutes murder of the same class with that of a Queen's ship, armed with the most perfect weapons and skilled men, shelling a native village in Polynesia, and destroying wholesale, in revenge for some isolated outrage by one or two of the natives, who in all probability but retaliated for some injury previously sustained at the hands of the white man.

To return from this digression, I beg to note, in passing, the great change which has occurred within the last twenty years in the appearance of the sand dunes. When settlement first took place in the West, and for years afterwards, the coast line was clothed with verdure; and west of Belfast the honeysuckle (*Banksia*) and she-oak (*Casuarina*) grew in abundance; whereas, now, the dunes are denuded of vegetation, and the trees gone, with the exception of a few very brief isolated instances; and in many cases the material of the dunes is drifting inland. In places where the action of the wind has been localised, and cut gullies in the dunes, the formations alluded to by Mr. Etheridge may be noted in abundance—namely, the filling in the cavities formerly occupied by roots of the sedge grasses, reeds, and other vegetation, with calcareous concretions, preserving the common appearance of pith and stem; but the whole is very brittle, and not in any way partaking of the character of the older formation fossils.

Between Belfast and Yambuk the dunes have in places been converted into an indurated limestone, of so firm and glassy a character that a friend one day brought me in triumph a piece of it which he pronounced to be flint, and nothing short of an adjournment to a neighbouring chemist's would convince him to the contrary.

Inland from the coast, between four and nineteen miles from Belfast to the west, this indurated limestone is very prevalent, with the exception of an overflow of lava between the eighth and tenth miles; but how far it extends under the lava I do not know. The limestone is water-worn, is an excellent road material, and is suitable for building, and makes a strong mortar. It has many of the ingredients of an hydraulic lime, but Mr. Foord does not esteem it highly in this latter respect.

In use I found it to make the best mortar of any I have used in the colony.

Nearly the whole of the coast line from Warrnambool to Yambuk is modified by the outflow of lava from Mount Rouse, which is situated about thirty-six miles from the coast northerly.

In remote ages, when Mount Rouse was active, the whole of this region must have been one of sterile desolation over a great portion of its area, the lava stream extending over a breadth of many miles from Mount Rouse across the Hawkesdale district, and round by the high limestone cliffs of Tower Hill Marsh (an ancient coast line) to the sea, spreading out in a fan-like shape from the Sisters in Armstrong's Bay to about four miles west of Belfast.

The lines of demarcation of the lava-flow are tolerably well defined, and leave little doubt as to its source, for on the north-west, about twenty-four miles from Belfast, we have at the deep Creek the Mount Rouse lava on one side and ancient basalt on the other, which extends a considerable way north, dividing the outflow from Mount Rouse from that of Mount Napier and Mount Eccles, to which I purpose alluding presently; whilst on the east we enter on to the out-throw from Tower Hill, which is of an entirely different character to that from any of the surrounding vents, namely, those of Mount Gavoc to the east (lava), Mount Rouse to the north (lava), and Mount Napier and Mount Eccles (largely of vesicular lava); whilst Tower Hill has been wholly of ash (vesicular bluestone in a comminuted state), red-hot stone (glassy in structure), in isolated showers, dust, and vapour, which now forms the tufa of the neighbourhood.

The basalts of Mount Rouse have formed Port Fairy; whereas the indurated tufas of Tower Hill, and the indurated sand dunes of the coast, have formed Lady Bay, the lavas of Mount Gavoc having been checked in flow westward at Yangary Creek—a small stream marking the dividing line between the products of the Tower Hill eruptions and those of Mount Gavoc, which latter outflow has been further checked on the south-west by the ancient sand dunes on which Warrnambool is built. It is possible that the lavas of Mount Rouse and Mount Gavoc may blend in the country between Russell's Creek and Woolsthorpe.

To the west of Belfast, about from twenty-five to thirty miles, we come on to the outflow of lava from Mount Napier and Mount Eccles—the former having had its chief outpour through what is known as the Lowth Swamps, until it joins the Mount Eccles outflow near to Lake Condah and thence to the sea.

I have been informed that the overflow of water from Lake Condah, at one season, disappears under a portion of the basalt, and after a passage of several miles emerges again in considerable streams into Darlot's Creek, which latter empties into the sea near Portland Bay.

I may mention in this place that near to Yambuk there is one place where in flood-time a very considerable body of water enters a cavity in the indurated limestone before spoken of, and disappears, but where its exit is I never could learn.

Over nearly all the coast limestone formation there is evidence of hollows existing in the limestone, because in driving along there is the peculiar rumble as if passing over a wooden bridge or vault.

The indurated limestone has been either formed under water or submerged subsequently; but I think the evidence of formation under the sea is reliable, for I have noted what I believe to be casts of the common limpet in the rock.

I am further inclined to believe that the outflow of the lava has been at a period when the sea washed the coast line of limestone bluffs, to which I have before alluded, as forming the northern boundary of the Tower Hill marsh, and which now forms the third line inland of old sea coast. The evidence of the coast lava having been submerged to a much greater extent than at present is, I think, proven by the rounded and water-worn forms of the rock masses—in many cases having a cup-and-ball form, which can scarcely be due to atmospheric influences alone—and the water-worn appearance of the indurated limestone between Belfast and Yambuk.

A few miles inland from Warrnambool, in the direction of Woodford, and across the River Hopkins at Allansford, in the parish of Tallangatta, there exist large formations of indurated limestone, similar in character to that described near Yambuk, at a considerable elevation above the sea, and containing abundance of marine fossil remains, indicative of formation below water.

Having thus far endeavoured to sketch in the general geological features of the district, I will now give a general view of the existing and ancient coast lines, with the evidence in favour of the views enunciated.

In the preceding notes I have pointed out the conditions which modify the line of coast as at present existent, but to those above named I must add the agency of ocean currents, which, although frequently influenced superficially by prevailing winds, all my observations have tended to confirm those made by me sixteen years ago off the coast of Gippsland as to the existence of an oceanic current from the westward, permanent in its character, and only influenced superficially by easterly and southerly weather; and it is due to the existence of such permanent current that all our harbours and rivers have an easterly or south-easterly exposure, excepting only in such exceptional circumstances as the entrance to Port Albert, in Gippsland; and this, even in its exception from the general rule, proves the law of current as stated from west to east.

From Warrnambool to Tower Hill the country consists chiefly of rounded mammaliferous hills of pulverised shell, limestone, ash, and tufa; but immediately west of Tower Hill we come upon evidences of an old inland coast line, which gradually rises into a long ridge consisting of pulverised shells, spicula, and other marine remains; amongst which, Mr. Castwood, of Belfast, has obtained sharks' teeth, from the inner or second line of ridge near to that town. Between this inland ridge and the coast exists a flat, which in part is occupied by a lagoon enclosed from the sea by the present line of sand dunes. The bed of the lagoon consists of deep alluvial deposits mixed largely with sand drift.

Inland of this second ridge, at a distance of about a mile, the land rises in steep hills, and, in some places, limestone bluffs, which extend from Tower Hill westward for from six to seven miles. The bluffs are chiefly of an indurated limestone, but the sloping hills have a thick bed of soft limestone, with abundance of shell spicula and other marine remains; and the whole has evidently been the sea-coast of what has in all probability been an indented bay, formed between the Tower Hill and the outflow of lava before described as coming from Mount Rouse.

The inclosed basin between the second line of ridge and the bluff is occupied by a bed of stiff black diluvium, through which flow the surplus waters of Tower Hill and the country to the north-east and the River Moyne, which latter rises in the marshes and stony rises south and west of Mount Rouse.

Until recently this flat was more or less a marsh during the greater portion of the year, but it has now been reclaimed by drainage.

On a portion of these flats west of the River Moyne, well shafts have been sunk to depths varying from 14 feet to 18 feet deep, and an original sea bed disclosed, with abundance of recent shells. From the River Moyne westward the land is chiefly undulating bluestone ridges, until the sea-coast or the limestone beds before described are reached.

The formation of the land and its three distinct coast lines as described indicate considerable changes of coast, and these changes must have occurred since the upheaval of the land to its present level; and so far from the line of coast being even now fixed, I have often thought when standing on the present sand dunes that I could detect in the paler colour of the sea a short distance from the present coast a new formation of coast line in progress, but the data on which I have arrived at this conclusion is not sufficiently positive to give reliable evidence of the fact; but, assuming such to be the case, the progress of formation must of necessity be slow owing to the long period requisite to accumulate fragments of shell sufficient to form these extensive mounds. The materials brought down by the river in floods can have little effect in hastening such formation, because although the outflowing current is strong enough to carry along the finer particles of mud sufficient to discolour the water, it has not velocity sufficient to convey the more solid matters held in suspension far from the mouth of the river.

Such a formation and the agencies which I conjecture to be in operation are very similar to those of earlier times, when the second line of ridge was formed enclosing the Tower Hill marsh and the outer line which encloses the lagoon and flats between the existing dunes and the second ridge; namely, a heavy sea on the coast breaking down and carrying back with its recoil particles of the coast held in mechanical suspension across a deep water channel, until the under draft meeting with a resistance of force sufficient to check its current precipitates the solids in a long ridge, which from continuous accumulations becomes at last a shoal enclosing a basin; and in time the shoal emerges as a bank, alternately dry and wet, on which the wind can act, and then begins the process of accumulation in ridges and the filling in of the basin with vegetable deposits and growth until dry land appears.

In one place at Warrnambool the wave action from some cause has become destructive, as evinced by the erosion of the shell limestone, undermining it, and breaking down the fallen materials. The outlyers of these rocks now form dangerous reefs over which the sea breaks for about half a mile seaward of the coast line of the dunes. From what has fallen under my own observation, however, I believe the wave action along the Victorian coast is chiefly conservative, as a proof of which the long ninety-mile beach of Gippsland is an excellent example; the dunes of Gippsland bear evidence of formation from similar causes to those suggested as having been active on the western coast.

Of the long continuance of the climatic conditions existent in Victoria the out-throw from Tower Hill affords very striking evidence in the great prevalence of its products to the east and south-east of the mount, a direction which would be taken now by ejected matter in any time of great atmospheric disturbance.

The crater of Tower Hill is from five to six miles in circumference, and rises in places to 320 feet above the level of the lake, which occupies a large portion of its area, whilst the island from which it appears to have received its name rises a little higher in mounds and peaks, with one well-defined crater and the broken remains of others. When in its early times of activity, the crater must have been a yawning gulf of the area described, and probably from 600 to 1000 feet deep; but as its activity lessened the cones of eruption formed in the interior, and these having broken out from time to time in new vents, moulded the peaks nearly as they now exist

Surrounded as Tower Hill is by extinct volcanoes, ranging at various distances from thirty to forty miles—all of which poured out molten lava in abundance—it is somewhat singular that amongst the deposits from Tower Hill there is evidence only of showers of red-hot stones, comminuted basalt, or ash-dust and vapour. The stones are glassy in fracture, and are obtained in the sides of the crater and adjacent pastures; but the ash and the dust and vapour which form the tufa extend around for several miles' distance, but more especially to the south and east in the direction of Warrnambool, precisely as if ejected under existing meteorological conditions. It is to the vast volumes of steam ejected, and the heavy rainfalls which would accompany these great atmospheric disturbances concomitant with violent eruptions, that I attribute the induration of the sand dunes on which Warrnambool is built into strata of rock bending equably over in the form of mammaliferous hills; and as each layer or bed of sand became blown over and covered the former layer, fresh precipitation of moisture would dissolve, and the solution would penetrate and cement the loose particles of shell together; and so the process would continue for such time as Tower Hill continued to eject matter.

Evidence of the formation of these dunes on dry land is occasionally given by the exposure of the imprint of footmarks of some three-toed animal or bird, which may have been either emu or kangaroo, the impressions being sufficiently distinct as a footprint, walking on sloping ground, but scarcely so clear an impression as to indicate precisely the nature of the animal.

On the flank of Tower Hill, near Yangery, a shaft was sunk through the layers of ashes and tufa to a depth of from 70 to 80 feet and a bed of ancient turf exposed; but this depth I believe to be a minimum.

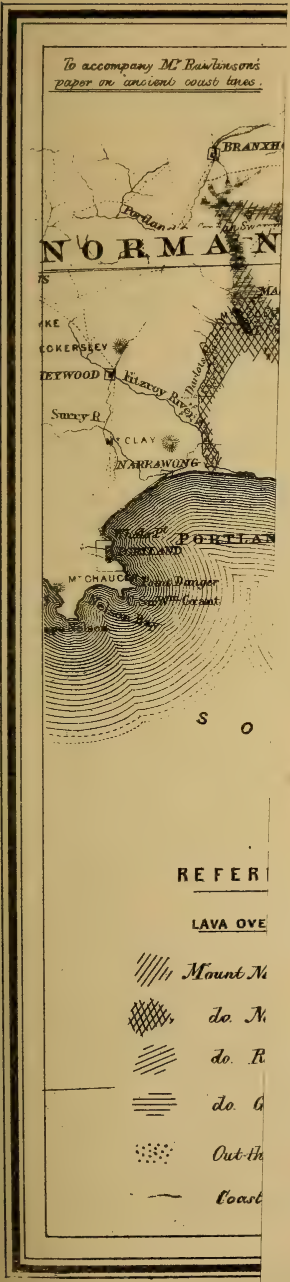
From a careful consideration of all the preceding facts, and from reasoning based on them, I have been able to arrive at only one conclusion, namely, that between Warrnambool and Yambuk the form of coast line has been determined by the outflow of molten lava; that three coast lines have been formed in succession between Tower Hill and Belfast, and that in all probability there is now a fourth in course of formation; whilst at Warrnambool the outliers of rock are but the original dunes partially dissolved and cemented together by the volumes of vapour and of rain either ejected from or induced by the action of Tower Hill in remote times; and lastly, from the vast preponderance of Tower Hill out-throws existing in greater quantity and to a much greater distance in an easterly and south-easterly than in any other direction, that meteorological conditions under circumstances of great atmospheric disturbance were in remote times the same as at present—and if in times of great disturbance of which we have evidence, then also in periods of comparative repose, and hence climatic conditions over very remote periods were the same as now.

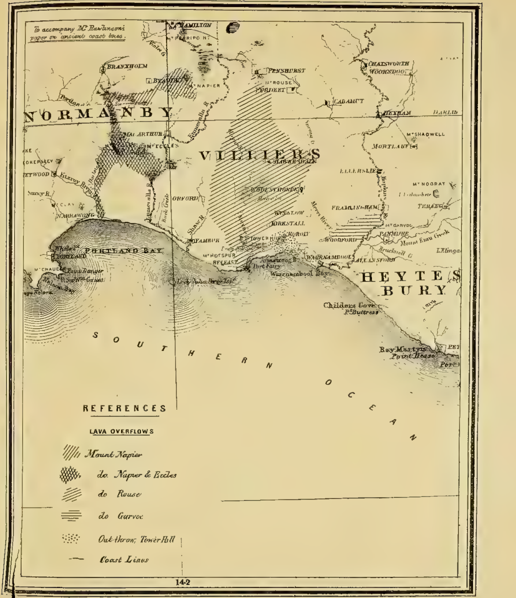
ART. VII.—*Notes on the Recent Earthquake.*

BY R. L. J. ELLERY, ESQ., F.R.S., F.R.A.S.

[Read 12th July, 1877.]

To accompany Mr Rawlinson's paper on ancient coast lines.




To accompany Mr Rawlinson's paper on ancient coast lines.
HAMILTON
PERRIPO NT
BRANXHOLM
BYADUK
Mt NAPIER
PENSHURST
Mt ROUSE
PURDEET
CHATSWORTH
WOORNDOO
CABAMUT
HEXHAM
DARLIN
NORMANBY
MAC ARTHUR
Mt ECCLES
VILLIERS
Mt SHADWELL
MORTLAKE
ELLERSLIE
Mt NOORAT
TERANG
FRAMLINGHAM
Mt GARVOC
PANMURE
Mount Emu Creek
Bucknell C
L.Elingamite
WOODFORD
WOOLSTHORPE
WYNSLOW
KIRKSTALL
KOROIT
TOWER HILL
Mt HOTSPUR
BELFAST
Port Fairy
Armstrong B
WARRNAMBOOL
ALLANSFORD
Warrnambool Bay
HEYTESBURY
Childers Cove
Pt Buttress
Bay Martyrs
Point Hesse
Port
PET
YAMBUK
ORFORD
Shaw R
Buck Creek
Eumeralla R
Fitzroy River
Surry R
Mt CLAY
NARRAWONG
HEYWOOD
LAKE CONDAH
Whaler Pt
PORTLAND
PORTLAND BAY
Mt CHAUCER
Point Danger
Sir Wm Grant
Nelson Bay
Cape Nelson
Lady Julia Percy Isd
Moyne R
Hopkins R
Merri River
Violet C
SOUTHERN OCEAN
REFERENCES
LAVA OVERFLOWS
Mount Napier
do. Napier & Eccles
do Rouse
do Garvoc
Out-throw, Tower Hill
Coast Lines

Art. VIII.—*Notes on Barometer Construction.*

By George Foord, F.C.S.

[Read 12th July, 1877.]

At the last ordinary meeting of the Society my name was on the list for reading an account of a proposed new form of barometer—a somewhat free translation of a paper appearing in a recent number of Poggendorff's *Annalen*—it being understood that papers possessing this degree of originality may from time to time be brought upon their own merits under the notice of the Society. For want of time the reading was postponed, since which postponement it has occurred to me that there were other proposed forms of barometer which it might be also interesting to consider; moreover, that a few hints concerning barometer tubes, and the precautions to be observed in selecting, preparing, and filling them—points which have fallen within the range of my own personal experience—might prove useful. Most of those who follow physical inquiries in the colony find the necessity of at times helping themselves, often to the extent of repairing, and occasionally of constructing, the instruments upon which their work depends; and therefore it is believed that an interchange of views and experience concerning minor details of construction—such as those now offered—may not be wholly devoid of interest.

I will then, with your concurrence, proceed in the first place to give a few hints calculated to assist those who may choose for the first time to try their skill in barometer building; and I will afterwards make reference to the forms of barometer proposed respectively by C. Bohn, by Guthrie, and an old proposition of Descartes incidentally mentioned in Mr. Guthrie's paper, and which is not dissimilar in principle to a form brought under the notice of our Society last session, and which originated with Mr. Venables.

First, then, as to the glass tube to be used. Its selection is a matter of primary importance. Callipers or gauges will enable us to ascertain how far the bore of a glass tube, otherwise applicable to our purpose, is of the same diameter at the two ends; for such gauges we may use very taper cones of copper or brass, or acute-angled plates of copper, brass, or zinc. Or we may choose to be more exact, and properly calibrate our tube throughout; although it must be here admitted that even for a syphon barometer it is only a few inches of each end of the tube which is required to be of uniform diameter. For calibration, if the interior diameter of the tube be small, say not exceeding two, or at the most three millimetres, we may pass a cylinder of mercury of known weight from end to end of the tube, accurately measuring the length of this thread of mercury progressively during its course; this will give data from which we may calculate the mean diameter of the bore of the tube in every portion of its length.*

For the calibration of wide tubes we may close one end, and, fixing the tube in a vertical position, weigh or accurately measure into it definite constant quantities of mercury. Or a method well calculated to avoid air bubbles may be practised by fitting the lower closed end of the tube with a glass reservoir, furnished with tubular terminations and glass or steel stopcocks. This reservoir with its tubes has the form of the letter U, the reservoir forming the thick arm of the letter (see Fig. 1). The parallel vertical tubular branch representing the thin arm contains a stopcock of supply, while a second stopcock for discharge of the mercury from the reservoir is placed at the lower portion or bend of the U. The whole requires to be fixed on a vertical board, and a funnel with capillary lower termination, of a length

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* For purposes for which it is convenient to gauge, with a metallic gauge, the interior diameter of the two ends of the glass tube, the calculation for the estimation of the relative diameter of the intermediate parts becomes very simple, as the following example chosen as affording a simple illustration will show :—Say diameter at each or either end is found by the gauge to be 4 millimetres, and that we introduce a cylinder of mercury measuring in this part of the tube 10 millimetres in length. Suppose that we pass this column along towards the centre of the tube to a position in which its length is exactly doubled, becoming 20 millimetres, the cubic measurement of the mercury is $4^2 : \cdot7854 : 10$; but for our purpose, as the proportion ·7854 to unity is common to all the sectional areas we may discard this factor ·7854, and thus we deal with $4^2 : 10 = 160$. This in the portion of the tube where the length of the mercurial cylinder is doubled, occupying 20 millimetres, divided by the latter ($\frac{160}{20} = 8$) will give a quotient of 8, the square root of which, say 2·84 millimetres, is the diameter of the centre of this portion of the tube; and so indeed for any other part, the square root of the quotient obtained by dividing 160 by the length of the mercurial column in that part will give the local diameter. Of course in tubes selected for their apparently near approach to a perfectly cylindrical form the length of the mercurial calibrating column will be nearly uniform throughout, but whatever differences there may be are calculable from results obtained by the method described. See illustration A.

FIG. 1

greater than that of the barometer tube to be calibrated, must be used. Immediately under the bowl of this funnel is a stopcock which, when the point of this long funnel tube is lowered to the bottom of the barometer tube, enables us to regulate the supply of mercury, so that the surface of the fluid mercury rises slowly and equably, filling the tube without locking in a single bubble of air against the inner glass surface of the barometer tube. The glass measure fitted to the lower end of the barometer tube, as already described, is a spheroid with tubular ends. There is a narrow vertical glass tube forming its upper opening, and on this narrow glass tube a measuring mark is made; a second mark is also placed on the tube below the lower orifice of the bulb. With this arrangemeut we can calibrate the barometer tube. We first fill the tube under trial with mercury; we then open the stopcock of supply and allow mercury to run off until it has reached the trait x below the bulb. We now mark on the barometer tube the position of the upper surface of its mercurial column. We next open the stopcock of supply, until we have filled the measuring bulb to its upper mark y, when we mark the level to which the upper surface of the mercury has descended in the barometer tube. The supply cock being shut off, we next open the discharge cock, allowing mercury to flow slowly out until the lower mark is reached. In this way the measuring bulb is slowly and accurately alternately filled and emptied between the two gauge marks, and after each filling the level of the mercury in the barometer tube is carefully registered on it. This is continued until the barometer tube is almost or quite emptied, by which time we have marked it with subdivisions throughout its length, each of which we know to be of capacity equal to the rest, and from their several distances apart the diameter of every portion of the tube can be computed. The temperature of the mercury and the weight of the bulb measure of mercury should be noted, and when extreme accuracy is the aim there are other influences to consider and allow for; but the *modus operandi* is essentially what I have described whenever a barometer tube, or indeed a straight glass tube of any kind, is to be calibrated. The data for correcting the bulk of the mercury for temperature, &c., &c., are fully set forth in physical treatises, and therefore I need not further allude to them in this place.

If we consider the mode of manufacture of these glass barometer tubes we shall easily understand their liability to the conical as distinguished from the cylindrical form. A hollow stout cylinder of soft semi-molten glass is formed on the end of the blowing tube, and a second heated blowing tube is attached to the outer end of the ductile mass. The two workmen, each holding one of these blowing irons, retreat from each other until the glass tube is drawn down to the requisite diameter, say until they are twenty or thirty feet or more apart. A ladder of suitable length has been laid on the floor, and on this the glass tube is now laid and detached from the blowing rods at each end. It is eventually cut into six-feet or three-feet lengths, in which state the tube is ready for removal to the annealing lear (if it be annealed at all). The "butts," that is to say, the two outer lengths which were in immediate contact with the blowing irons, are sensibly conical, and the other segments of the entire tube are liable in degree, according to their position, to this conicity, and therefore it is a point of primary importance to gauge the tubes during selection in the manner already described, so as to obtain pieces which are sensibly cylindrical.

There are certain other points in selecting the glass tubes which will require attention—clearness of the glass, freedom from knots, and similar defects, &c.; but these are too obvious to require further mention.

As barometer tubes are required in most cases to be of stout glass, it therefore becomes necessary that they should have been effectually annealed; and here enters into the consideration a curious point of interest. I think I need not hesitate to say that much of the glass tube met with in commerce is either imperfectly annealed, or, as in the case of tubes with thin walls, it has not been annealed at all. The question of the degree of annealing which each kind of tube requires is regarded I believe in a purely commercial spirit; providing what will sell, and especially regarding the consideration of cheapness of production. As there is more in this statement than might catch our attention, I ask your patience while I go into the question a little more fully. Unannealed glass is glass in a condition of strain or unequal tension, and that portion of it which is unduly stretched is liable, on slight prompting, to rupture; such glass will not bear sudden vicissitudes of temperature, or sudden mechanical shocks, or the slightest scratch upon its strained inner surface. But glass may be in a condition of high tension and may at the same time possess very marked properties of permanence. If we optically examine vessels of De la Bastie's toughened glass we find them showing in a beam of polarised light the black cross indicative of strain, and we know that these specimens of glass will resist mechanical shocks of great violence, and that they have some other marked properties conducive to permanence; but if sufficient external force for the fracture of one of these vessels be employed, it does not simply break as annealed glass would break, but goes off with a report and is shattered throughout into a complete ruin of small particles. The "Bologna vial" and the "Prince Rupert's drop"* are each permanent in this sense, and each under proper conditions liable to disruption; and, in fact, we have to distinguish between irregular and symmetrical strain in order to gain a clear insight into the question of fracture of glass tubes, especially fracture due to imperfect annealing. Just as the Bologna vial is safe as long as you hammer its external surface, but flies into fragments as soon as you scratch ever so slightly its strained interior surface, which has cooled and contracted after the exterior layers have become solid, so a large proportion of the glass tubes found in commerce are permanent enough as long as we do not suddenly heat them, and so long as we do not bring hard substances in contact with their inner surfaces. Experience has taught the glass manufacturer that, unlike pieces of complex form, thick glass tubes with little annealing, and thin glass tubes with none at all, or next to none, are sufficiently permanent to serve most of the purposes of commerce. Take a stout glass barometer tube and pass through it an iron wire so as to rub the inner walls of the tube with the latter, the chances are great that after this treatment the tube will very soon crack; indeed it is unsafe to touch the interior surfaces of stout glass tubes with iron at all, as no instrument made with tube thus treated will be afterwards reliable. Regard the inner surfaces of your glass tubes as possessing in degree the physical properties of the inner surface of the Bologna

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* The latter are called by the French "Larmes Bataviqne;" concerning the properties of which bodies the reader is referred to an interesting memoir by M. Victor De Luynes in the *Annales de Chemie et de Physique*, 3rd series, Vol. XXX. p. 289.

vial, treat these surfaces accordingly, and you will thereby effect much towards the permanence of whatever instruments you form from glass tubes.

But there are two kinds of glass (chemically speaking) of which barometer tubes are made; these may be distinguished in general terms as "crown glass" and "flint glass"—I might say Continental glass and English glass, as "crown" glass tubes prevail, as a manufacture, on the continent of Europe, while most of the English glass tube is of the "flint" variety. Besides the silicic acid and alkali the crown glass contains a basis of lime, which is replaced in the flint glass by lead oxide, so that "lime glass" and "lead glass" are equally distinctive terms. The lead glass is soft, the lime glass is hard; the lead glass is easily fusible, the lime glass is less easily fusible; the lead glass has less cohesive strength than the lime glass, as may be easily seen by trying the breaking weights of rods or tubes (of equal stoutness) of these two qualities of glass.[*] Lead glass is more pellucid than lime glass; tubes of the latter being mostly striated throughout by lines which in reality are air bubbles drawn into cylindrical cavities or threads of extreme tenuity. Although the strength of lime glass may recommend it for the construction of barometers to be used in the field, on the other hand lead glass offers advantages for instruments intended for indoor or laboratory use. The lead glass is easier worked, is sufficiently strong for use in careful hands, and in this material tubes free from defects and of beautiful uniform transparency can be easily obtained.

Whatever the pattern of the barometer, the tube from which it is to be made must be first examined as to equality

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[*] *Experiment on cohesive strength of lead and lime glass tubes:*—

Relative weights of the glass tubes—	
A, lead glass	1123 grains
B, lime ,, ...	836 ,,
Length of the tubes...	each 15 inches
Bearing (wood) edges	10 inches apart
Exterior diameter of each tube	very nearly ½ inch
Breaking weight of A	32½ lbs. avoir.
,, ,, B	46 ,, ,,
Specific gravity of A	3·27
,, ,, B	2·509

The tubes were gauged and selected so as to be as nearly as possible of the same exterior diameter and diameter of bore; the breaking weight was gradually increased by progressive addition of lead bullets to a tared suspended scale until fracture ensued.

of bore, and the exact diameter of the bore is also to be ascertained, because when the tube is closed and filled, and especially when bent into the syphon form, the ascertaining of these points is no longer readily accomplished. The tubes chosen for making into barometers will be often longer than is requisite for the instrument, and the end cut off may be almost or quite the same diameter as the upper end of the barometer; when this is the case it may be worth while to carefully label and set aside this end piece, which would at any time answer any question concerning the curve of the meniscus or any of a kindred nature which might arise. Concerning capillarity, a suggestion may be offered:—With any tube about to be employed, or with the end piece of tube cut off as just mentioned, a measurement of the effect of capillarity may be made by a method given in Bunsen's Gasometry:—Measure a column of mercury in the tube *per se*, and measure the same column after covering it with a few drops of corrosive sublimate solution: in the former case you have the meniscus proper to the given diameter of tube in its integrity; in the latter the mercury assumes a horizontal upper surface, and the difference of height of the two columns is that due to those physical causes which are collectively spoken of as the influence of capillarity.

Before proceeding to clean the inner surface of the tube it will be well to become acquainted with what has been ascertained concerning chemically clean glass, as especially set forth in the papers of Tomlinson.* In the *Chemical*

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* When you have prepared with all precautions your supply of mercury for the cistern and for filling the tube, I will suppose in a clean porcelain vessel, with a nicely-polished glass bell jar for a cover, in a relatively dust-free apartment, you may try a simple experiment which is suggestive of the necessity of extreme cleanliness in barometer construction. Let the experimenter elaborately wash his hands, and then press his finger against the pure mirror surface of the mercury; he will, if I am correct, produce a minute and faithful oleograph of the skin structure—a picture of the skin surface—drawn in sebaceous and epithelial particles, which the cuticle, however well cleansed, is always ready to throw off. Now if you take up Deschanel's *Manual of Physics*, or other elementary work of the kind, in which the barometer is figured and described, you will see a wood engraving of the Torricellian experiment:—the hand inverting the tube filled with mercury, and the finger about to be placed on the open end on the mercurial column, before its insertion in the cistern—all very good for lecture table demonstration, but certainly violating the rules according to which a good barometer should be filled and erected. You cannot blow through a tube or touch the end of it without making a fouled surface; and although I am not prepared with any suggestions for the best method of meeting this

Dictionary of Watts, article "Barometer," will be found an account of the formation of the large bore barometer of the Kew Observatory; it will there be seen that the tube was polished out with alcohol and whiting (precipitated chalk, probably). Fuming nitric acid is an efficient oxident of greasy substances, and immersion of tubes in this acid before the final polishing, or first in oil of vitriol and next in nitric acid, would conduce to a satisfactory result; but whatever be done in the way of polishing out the tube, extreme care in avoiding the slightest scratch or abrasion of the inner glass surface must be observed. If iron wire be used for carrying the polishing plug, the wire must be covered completely with lamp cotton; the latter should have been previously purified by digestion in ether or bisulphide of carbon. But even with these precautions there is a risk of filaments, and perhaps, on the whole, it is best to avoid covered wire altogether. Brass or copper wire are less dangerous, but whalebone, or cane, or soft non-resinous or de-resinated woods have some peculiar recommendations.

I here may point out in reference to the cleaning of glass tubes generally, and especially to the cleaning of curved tubes with complex bends, and when whalebone of sufficient length is obtainable, that it possesses a property which cannot be too pointedly indicated to those who have not hitherto recognised it, and who are engaged in experimental physics. By its means some problems in cleaning the interior of complex forms of glass vessels can be solved which, to the best of my knowledge, are soluble by no other known means. A rod of whalebone is taken and shaped to our requirement; we intend to pass it through certain tubular crooked ways to reach a certain point on some remote inner surface; the material is elastic enough to pass through the tortuous duct, but when this is accomplished we have little or no control over the inner end of the slight constrained whalebone rod on which we depend for doing the work. But the possibility of doing

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requirement, it is still important to point out the difference between modes quite effective for lecture table demonstration, and those to be observed in the construction of instruments intended to meet all the requirements of precise physical research. Indiarubber finger stalls, collodion films, gutta-percha moulded valves, and similar contrivances, suggest themselves; but without attaching weight or preference to any of these, it still remains as a fact worthy of our best attention, that we cannot bring the hand into contact with pure mercury or chemically pure glass without in some measure fouling their surfaces.

the work resides, as I shall show, in the material nevertheless. If we carefully warm it over a spirit lamp we can bend it into curves corresponding with those of the crooked tube through which it is to pass, and when each of these bends has cooled we find that the whalebone rod has acquired a permanent set. We thus model an instrument whose axis is coincident with that of the crooked tube, and the elasticity and pliability of the rod remains. It gives and recovers itself as we humour it through the channels, and when we have put it in position it is free to be moved to a limited but mostly sufficient extent, so as to exercise the desired friction at the proper place, detaching a minute insect or a speck of dirt or mould, as the case may be. Doubtless this bending property of whalebone may be utilised in the hands of the physicist and chemist in other ways. Of course wood may be bent by heating or steaming, as instanced in boat-building, and in the familiar instance of walking-stick handles; but in the case of whalebone we have at the same time the permanent set and the elasticity of the material—a very valuable combination.

Concerning the use of cane rods for cleaning the interior of glass tubes, a suggestion may also be made. The elasticity of the ligneous material and its even cylindrical form recommend the cane for this purpose, but its siliceous glaze is obviously a dangerous element; this glaze can be readily removed by scraping with a knife, and cane rods thus stripped will be found sufficiently elastic, strong, clean, and safe for purposes of the nature considered.

For converting the tube open at both ends into the closed, and when required into the bent and shaped barometer tube, the enameller's blow pipe is used. I shall not enter into details on this point of the construction, as it is a matter of personal education and skill, and general directions of more or less value are to be found in technical works; but it will suit the limits of this sketch if the essential requirements of this class of operations are concisely stated. In closing, joining, or bending glass tubes they must be gradually heated to the required temperatures; the thicker the substance of the glass, or the less perfectly it is annealed, the more care will be required in gradually and equably raising its temperature. In closing the ends of tubes a little blowing for producing a hemispherical termination is mostly necessary. Remember that if this be done with the lungs the expired gases are charged with organic contaminations; a purely mechanical air pressure, as that supplied by a compressed indiarubber ball or condensing syringe, is free from this objection.

If sealed junctions are necessary for the construction of the barometer, these are not satisfactorily effected by pressing merely softened glass surfaces together; the glass tube ends to be joined must be well melted in the flame, then joined, and the joint must be retained in the molten condition in the flame until the whole of the softened portion has become identified into one homogeneous mass. Attention to the necessity of annealing such work as far as practicable will influence its durability. The air-driven gas flame used should, when lead glass is the subject in hand, be sufficiently oxygenated to prevent reduction of lead oxide to the metallic state and consequent blackening of the tube. One final remark, especially addressed to beginners in the work, is the advice to mark out in pencil on a smooth pine board the dimensions of the piece to be made at the lamp; this outline is used as a gauge with which to try the dimensions and angles of the piece, by juxtaposition, as it proceeds.

So much concerning the glass tube, whether for cistern or syphon barometer. Let us in the next place pay a few minutes' attention to the mercury. The mercury must be pure and dry, and free from all superficially adherent particles. When we allow a beam of sunlight to fall through a shutter hole into an otherwise dark apartment, we see that the air is permeated throughout with minute floating solid particles—motes which gyrate and eddy with every motion of the air, and which gravitate so slowly that in very few positions indeed is the air free from them. Among these particles are the germs which insinuate themselves between the lenses of telescopes, start into vegetative life, and feed on the glass surfaces, deadening them, just as the familiar lichen establishes itself upon and assists the decay of the hard surfaces of igneous rocks. I refer to these bodies with the object of calling your attention to the great necessity of employing the utmost care in the construction of glass instruments of the nature of the barometer, and the great difficulty of effecting absolute cleanliness of the glass inner surfaces, and the mercury to be employed, even when very great precautions are taken. Fortunately it is not difficult to ascertain when mercury is sufficiently chemically pure and mechanically clean, and fortunately very much of the mercury of commerce is found in a state of almost or quite chemical purity. Moreover it is fortunate that if the mercury to be employed contains lead, tin, or other such chemical impurity, it is a matter of no great difficulty to completely separate these metals. In the chemical handbooks you will find directions for several methods of treatment in the wet way; and you will find not infrequently an objection raised against purification by distillation, but nevertheless I venture to state that with all ordinary samples of mercury the method of distillation will be found easy and simple. Should the mercury contain traces of gold and silver—no infrequent occurrence in Victoria—in that case the humid methods described in the books would fail to remove these metals, distillation being the only effective mode of doing so.

First, it is easy to ascertain the purity of a sample of mercury. You warm and dry it very thoroughly; then you fold a piece of clean dry writing paper into a cone, having an exceedingly fine opening at the apex. The warm mercury is poured into this cone, and allowed to run out at the fine aperture in a very thin thread or stream, and collected in a perfectly clean white porcelain basin; any fine particles of dirt will adhere to the paper, and are thus removed, and the mercury collected in the basin, if pure or nearly so, will present a perfect mirror surface. But this brilliancy is not of itself a sufficient index of absolute freedom from base metals. Take half an ounce or less of this mechanically cleaned and warm mercury, and cause it to gyrate in a porcelain dish, also clean and warm; the metal is mobile enough, and a slight shake of the hand will make it circulate freely, when one of two results will happen—the dish will remain unsoiled, the mercury preserving always the spheroidal form and its perfect brilliancy, a certain indication of its freedom from base metallic impurities; on the other hand, if there are present the slightest traces of lead, tin, &c., the mercury will form a "talus" or queue, with tarnished surface, and will leave a stain or streak where it has passed over the glazed porcelain surface.

I notice in certain books a statement about the oxidation of mercury at common temperatures, which appears to demand a remark in this place. With impure mercury there is doubtless, even at common temperatures, oxidation—

oxidation of the metal forming the impurity; and this oxidation will be attended with the fouling and breaking up of the mirror surface by the formation of minute globules of mercury—a grey mass which the adventitious oxide prevents aggregating once again into the mirror form. But I think it may be correctly stated of pure mercury that, although it may be converted into red oxide at a comparatively high temperature, at ordinary temperatures of the atmosphere it undergoes no perceptible oxidation of any kind. Henry Watts* reiterates Gmelin's statement that "mercury remains unaltered when agitated for any length of time with oxygen gas, common air, hydrogen, nitrogen, nitrous oxide, nitric oxide, carbonic acid gas, or alcohol;" and I believe that statement is strictly true as applied to pure mercury and the ordinary constituents of atmospheric air.

If the mercury is found to be impure by the tests already given, or if it leaves the slightest residue—say of gold or silver—after evaporation of a small sample, it may be distilled. A cast-iron retort, with wrought iron exit-tube, is used for the purpose. It is furnished with a lid or cover with turned joint, and fastened with screw-bolts or key-wedges; a lute of moist clay secures the joint. The lid of the retort may be furnished with a stopper, which permits renewal from time to time of the charge of mercury without breaking the luted joint. The temperature at which the metal "boils," or is said to boil, is rather high, say 662° Fahr. or 350° C.; but the capacity for heat of the vapour of mercury, as compared with that of aqueous vapour for example, is so low that a small quantity of fuel will do a large amount of distillatory work, and the distillation is therefore rapid. Among the papers of the Royal Society of London, in the *Proceedings* of that body, and probably also in its *Transactions*, is a valuable contribution by W. R. Grove on the "Phenomena of Ebullition," in which it is shown how great an influence the gases dissolved in water exert upon the phenomenon. Water deprived of air can be converted into vapour, but in a manner which it would be incorrect to call boiling. As we apply heat, its temperature gradually increases, and eventually mounts beyond the ordinary boiling temperature; finally the super-heated water is in part converted into vapour by a sudden explosive act,

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* *Dictionary of Chemistry*, article "Mercury."

very different to what we call boiling. Now, oil of vitriol, methylic alcohol, and mercury—most probably on account of the absence of dissolved gases—are each converted into vapour with more or less tendency to sudden bursts and "bumpings," as they are called, and in these cases the distillates are liable to contamination with portions of the fluid, scattered and thrown over rather than distilled; and some kind of artifice is requisite in all such cases for obviating this source of an imperfect result. Many years ago a French chemist (M. Violette) recommended the use of super-heated steam for the distillation of mercury—a promising suggestion enough; but a purification completely satisfactory may be effected by simpler means. Three or four circular discs of iron wire gauze are allowed to float on the mercury in the retort, covering its whole surface; or, what is better, a layer of three-quarters of an inch of small cut or wrought iron brads are allowed to float on the metal; either of these forms a mechanical barrier, holding back the mechanically dispersed fluid mercury, but allowing sufficiently free escape for the mercurial vapour. For the reason already given a very small stream only of cold water, running over a cloth laid over the exit-tube of the iron retort, is requisite for re-condensation of the mercury. The lower end of the exit-tube is also bound round with a few folds of calico, which, projecting beyond it, form a tubular conduit, sufficient for confining and conducting the condensed mercury into a pan of water, and at the same time sufficiently pervious to the atmospheric air to prevent the water in the collecting pan being drawn up into the retort as a result of condensation of mercurial vapour at the end of the operation. I believe that a more extensive acquaintance with the efficacy of this simple method of distillation would cause its employment in preference to the several methods of chemical treatment.

A few observations on boiling out and other modes of filling glass tubes with mercury may now be added. Boiling out means raising the mercury to the temperature at which it freely forms metallic vapour, and so expelling the atmospheric air from the tube; it also means raising the mercury to a temperature at which its oxidation takes place when in contact with atmospheric air. The warm mercury is added in small doses to the inverted tube, and the boiling is brought about by heating the tube at a point a little below the mercurial surface; the boiling out thus proceeds from the closed end to within an inch of the open end of the tube. The tube is now filled up with hot mercury, and eventually it is suitably closed and inverted in its cistern of boiled pure mercury. To what extent or how syphon barometers are boiled out I am unable to state. Barometer tubes may be boiled out or filled with warm mercury without boiling out. The great standard barometer of Kew Observatory, which has a bore of one and one-tenth inches, was filled by the aid of the air-pump, and without boiling out. The Torricellian void above the mercurial column is stated to have been, when the instrument was completed, quite air free. I venture to express an opinion that the boiling out of barometer tubes is a mistake. The formation of oxide of mercury may not be grossly palpable; but I fear it is hardly possible to avoid the formation of some oxide, and that the quantity, however small, may have its effect upon the sensitiveness of the instrument. Possibly the intervention of microscopic crystals of red oxide of mercury between the metal and the glass may ultimately favour the entrance of air into the void. The mode in which mercury distils, and the absence of specific knowledge concerning any power which mercury may possess of absorbing or occluding gases, would appear to suggest that as far as the mercury itself is concerned the boiling out is unnecessary; or, if necessary for depriving the mercury of air, or gas, or vapour, of any kind occluded in its substance, as on that account ineffectual, for if the metal has this property it must soon again take up what we have expelled at the exposed surface in the cistern, and when saturated eliminate them into the void, while all our experience of the comparative permanence of the Torricellian vacuum renders this supposed property of mercury improbable, the small and slow creeping in of air being quite in unison with the fact of there being no real adhesive contact between the metallic column and the glass tube. Moreover, glass tubes, especially those of complex form, are jeopardised by the boiling process. A carefully and fortunately selected tube, well prepared, and therefore valuable far beyond its money cost, may be broken during the boiling by the turbulent and sudden bursts of mercurial vapour; or, if not actually broken during the boiling, it may be reduced to such a state of molecular unrest as to break with apparent spontaneity, some time after it is finished and mounted, or on receiving some slight concussion. We should also remember, as pertinent to this question, that it is not only losing the materials and the outlay of valuable time expended on the construction of the instrument itself, but by the loss of such an instrument after it has been brought into use, a break in the continuity of our results is brought about, and we resume observations with a new instrument, whose index, error, or deviation is different. It would seem that with the Sprengel-pump and other modern appliances at command for obtaining voids as good as have been hitherto by any means obtained, boiling out has become unnecessary and undesirable.

Concerning the mounting of barometers and the mechanical means for dividing the brass or other scales, I may state that these are beyond the scope of the present notes; but to those who essay to construct for their own use this instrument, I may mention one form of mounting which offers the advantage of simplicity in the materials of construction, enlisting glass and mercury only for the tube and its scale, and therefore to that extent simplifying corrections of the reading. On the mercurial tube mounted on a board and dipping into a glass cistern there is fitted an outer glass tube; the latter is divided, forming a scale which reckons from a glass rod fixed on to the lower end of this outer tube. This outer tube can be raised or lowered by a light cord or wire passing over a small pulley, and attached to a winch of glass rod working in a cork socket near the mercurial cistern. Before an observation is made this tube is raised or lowered until its zero pointer coincides with the mercurial surface in the cistern. The temperature is then taken; the reading made and the correction of the column for temperature concerns merely the expansibility of mercury and glass. There is a drawing of this arrangement attached to a Sprengel pump in the illustration to Mr. Mica Smith's paper on "The Motion of Bodies under the Influence of Radiant Energy" in a recent volume of our *Transactions*.

This completes what I have to communicate respecting the selection and preparation of barometer tubes and the mode of filling them, and I will therefore now proceed to the description of three several proposed forms of the instrument, each of which possesses features of interest, and perhaps I may correctly also state that each appears to be not wholly free from structural defects. First, that proposed by C. Bohn is described in Poggendorff's *Annalen* 1877, first part, p. 111, the paper being entitled "On the Construction of an Air-free Barometer, quickly, easily, inexpensively, and without boiling out:"—

"The syphon barometer has well-recognised advantages over the cistern barometer, but it possesses also its own particular disadvantages.

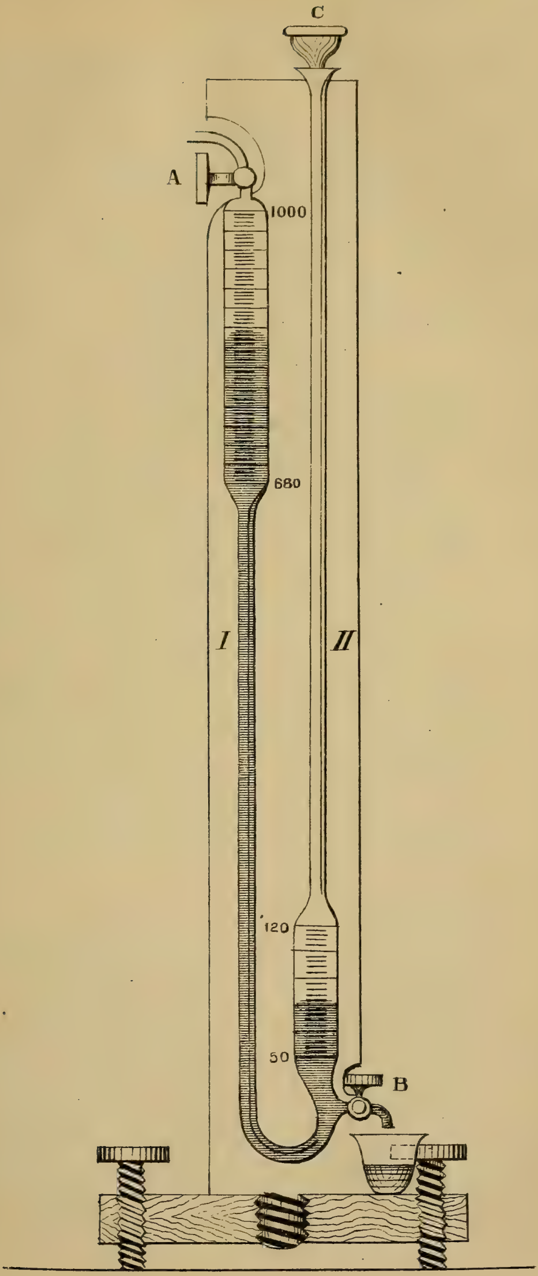
"In the first place, while the boiling out of barometer tubes is an operation not devoid of risk, this risk is still further augmented in the case of the syphon form, and in any case the operation is a tedious one. Further, the mercury in the open arm of the latter suffers the well-known oxidation, besides other kinds of fouling; its meniscus is then no longer identical with that in the closed limb, it changes by degrees into a concavity, the metal clings unequally to the inner wall of the glass tube, which it soon renders dirty. In fine, the compensation for capillarity aimed at in the syphon barometer holds good, even under the most favourable circumstances, for only a very short time.

"But these disadvantages attending the use of the syphon barometer can be avoided in the manner about to be described. An instrument of general application can be made quickly, without boiling the mercury, at small cost, and without the requirement of any special skill.

"A glass tube of about two metres long is bent into the syphon form; the two arms, as shown in the sketch (Fig. 2), are of unequal length; the shorter (I.) bears at the upper extremity an air-tight single-way glass stopcock. The longer arm (II.) is open at top. Near the bend, at bottom, a short branch tube carrying a mercury-tight single-way stopcock is attached (soldered on); the latter opens outwards or can be shut off.

"For economy of mercury the tubes, for a large proportion of their length, can be chosen of rather small diameter, only immediately below the stopcock A for a space of about 320 millimetres the tube must be wider; also for a space of from 70 to 90 millimetres close over the stopcock B, in the longer arm, the tube must be of a diameter identical with that under A. This glass tube is now perfectly cleansed (I find it best to finish with strong alcohol), then it is dried by aspiration of several hectolitres of hot dry air

FIG. 2.



through it, while the tube itself is supported over a warm stove or other suitable source of heat. The caoutchouc connector leading to the aspirator is attached to the tube over A; on the open end of the long arm a chloride of calcium tube is also attached by an indiarubber joint.

"The mode of filling is the following:—First the tube, very carefully dried, is fastened to a narrow wooden board in the manner shown in the engraving. This board ends below in a screw, which is screwed into a base also of wood, and which is supported by three wooden levelling screws. The board has at its upper end a ring for the purpose of hanging up the instrument.

"Thus mounted, with the stopcock A open and the stopcock B closed, well cleaned dry mercury heated to about 100° C. is poured into the tube through a small funnel with capillary termination, which holds back all dust. The mercury drives before it slowly and gradually the air in arm I., causing it to escape through stopcock A. Finally mercury also passes through the stopcock A and the tube above it. Now A is shut and B opened; the mercury now consequently falls out of the arm II. until its surface in this limb has descended to the point of junction of the branch tube, while in arm I. a column approximating the true barometric column remains suspended. The space thus existing above the mercurial column is not quite air free, although in a highly attenuated condition. The instrument may be made to act as a mercurial air-pump upon the air which adheres to the inner surface of the glass tube and on that drawn in by the warm mercury. For a few minutes, however, the instrument is allowed to remain at rest in the condition just described.

"In the next place the stopcock B is closed, the stopcock A also remaining closed; heated mercury is again poured into the open tube II., filling it completely; the small quantity of air contained in the vacuum chamber is compressed into a very small bubble close under the stopcock; A is then opened, allowing this bubble to escape, and afterwards mercury; after this mercury is again fed in again at C, when a stream of air-free mercury flows through A, sweeping with it mechanically all air attached to the glass inner surfaces; after several grammes have thus flowed out A is closed, B is opened, allowing once more the efflux of the mercury from the latter. The chamber above the mercury column is now almost perfectly air free. It is again worked for a few minutes as a pump; B is now closed, and for the third time the arm II. is filled up with mercury. With the naked eye I could never, at this stage, even discover a small bubble of air under the stopcock A, and with the aid of a lens I could very seldom discern one. A is once again opened, allowing a little mercury to flow through, and for greater security the prescribed routine may be repeated five or six times. On the last occasion of doing this the mercury is allowed to escape through B only until its upper surface stands in the tube at a level somewhat higher than B. Millimetre divisions are engraved or marked on this wide portion of the arm, the common zero point being at the bend. When the instrument stands exactly vertical (by virtue of the adjusting screws), then the difference of the readings of the mercury columns in the two arms is identical with the real barometric column.

" Mercury can at any time be readily run off at B, or filled up through C, so as to obtain a fresh upper surface of the mercury in the arm in which it is exposed to the air (the outer arm, II.), and regulated so as to fall within the limits of the divided portion of this arm; at the same time the perfectly air-free condition of the Torricellian chamber may be proved. When this condition of perfect freedom from air holds good, the uniform difference of altitude of the two columns holds good, whether the mercury stands at a greater or less height above B; but should air have penetrated into the vacuum chamber a slight difference of reading will be found to accompany this alteration of level of the mercury in II., for as the air space in the vacuum chamber is diminished, the counteracting pressure of the air which has entered it will be proportionately increased. The approximate compensation of capillarity is also by the same means ascertained. The facile repetition of the measurement by means of independent observations under the altered conditions as above described appears to the writer to be of great utility and void of all error.

" In the first instrument constructed on this principle the stop-cock A did not close quite air-tight. When the arm I. was for the last time entirely filled with mercury, and when the stop-cock A was closed, the author covered the latter with a solution of collodion; this provision, intended to effect an air-tight joint, was found to answer admirably;

notwithstanding variation of temperature the chamber remained air-free for months, during which the apparatus remained under the writer's observation.

"The board carrying the completed barometer can be unscrewed from the base and suspended on a wall.

"The above described instrument is well suited for use as a portable barometer. It is first emptied of mercury, with precautions ensuring that dry air only can enter in replacement of the quicksilver; for this object chloride of calcium tubes are attached at A and at C. The stopcock A is then closed, and C is stopped with a small cork. During travelling moisture cannot penetrate into the tube, thus dried carefully once for all. The board unscrewed from the tripod, with its attached glass instrument, is fitted into a padded case, which can then be carried suspended over the shoulder as a fowling-piece; with a sufficiently strong case even the brusque treatment incidental to railway carriage can be safely borne. The mercury is carried with the instrument in a securely corked stoneware bottle, of the kind commonly used in commerce for the transport of small quantities of this metal.* The third item of carriage is the wooden triangular base.

"Arrived at the observing station the tripod is screwed on, the previously dried mercury (the warming of which is now quite unnecessary, and which indeed was perhaps superfluous on the first occasion) is poured in, and within a quarter of an hour after the minutely described routine of filling, the barometer is ready for observation.

"This form of the barometer is recommended for isolate barometrical stations, and for similar positions; the drying out takes place in the laboratory, the glass pieces for which operation, attached to the board, are carefully packed and sent in the usual box. The filling takes place on the spot. If an assistant unqualified by previous scientific technical education be employed, it might prove advantageous to enclose the barometer with a glass case. Incidental to the inspection of the station would be the replacement of the upper surface of the mercury in the open tube, the verification of the instrument, &c.

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* Stoneware bottles containing mercury are rendered relatively safe from accident by a cover of several layers of brown paper securely pasted on to their outer surfaces.—G. F.

"The bend of this instrument it is advisable to form of tube of very small diameter; in which case, even with awkward carriage of the filled instrument in the laboratory, and even when it is violently shaken, air cannot pass from the open to the closed limb.

"During numerous comparisons of this instrument with an excellent standard barometer, of unusually large cross section, it yielded excellent results.

"The first instrument made by the author, rather faulty in the dividing and in the grinding in of the stop-cock A, he has sent to the Kensington Exhibition of Scientific Instruments. It had a not very suitable iron stand.

"Aschaffenburg, 25th July, 1876."

Guthrie's proposition aims at increased sensitiveness in the reading. In the first place he makes mention of a proposition long lost sight of and due to Descartes, in which is employed a column of dense fluid mercury; but in conjunction with a super-posed column of a much less dense fluid, in terms of which latter the atmospheric pressure is measured. Descartes' proposition included an aqueous solution of tartar emetic above the mercury; the object of employing this fluid solution being that of ensuring the expulsion of air Mr. Guthrie proposes to substitute glycerine or heavy hydrocarbon oil instead of the tartar emetic solution. Guthrie states, in reference to the diagram (Fig. 3) which he gives of this form of barometer, that "the sensibility of such a barometer would obviously be, if the upper liquid were without weight, directly proportional to the ratio between the sectional areas of the cylindrical chamber and the upper tube (if also the open limb were of infinite area). But the upper liquid having weight, the limit of sensibility is the comparative density of mercury and the liquid (say 16: 1);* accordingly this limit is secured when the cylindrical chamber has four times the diameter of the upper tube."

Professor Guthrie adds his own suggestion of a syphon barometer with a horizontal capillary tube of relatively great length connecting the column and cistern, the measurements being made on the capillary tube, in which a small bubble of air or fluid is intercalated dividing the mercurial cylinder (Fig. 4). Without doubt the indications of change of such an instrument are very sensitive; indeed,

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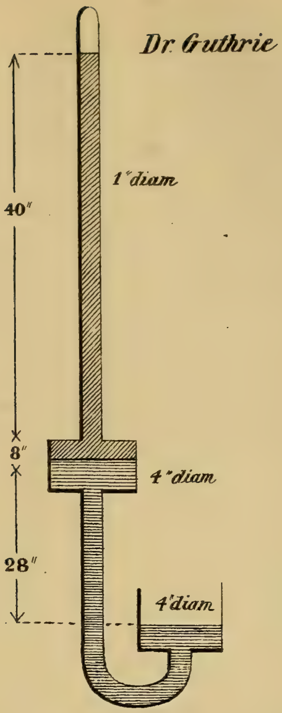
[*] Hypothetical gravities, for simplicity of illustration.

Fig. 3.



SYPHON FORM.

*Column of Glycerine, 48" = Mercury	... ...	3"
„ Mercury ...	... ...	28"
Value of Column in inches of Mercury	... ...	31

Now, suppose an extreme fall of three inches of mercury represented by the fall of the two fluids in this barometer; of this the diminution of the glycerine column will be $\frac{15}{16}$ of $\frac{1}{3}$ of the whole barometric fall; the alteration of the levels of the two surfaces of mercury will be each one half of the remainder.

DIMINUTION OF COLUMN.

$\frac{15}{16}$ of 16" of Glycerine = 15" = Mercury	...	·9375
$1\frac{1}{32}$ Mercury + $1\frac{1}{32}$ Mercury	... ...	2"·0625
Fall	... ...	3"·0

VALUE OF RESULTING COLUMN.

33" Glycerine = Mercury	2"·0625	Total Resulting Column. = 58"·9375
Mercury Column, 28" – $2\frac{1}{16}$" =	25"·9375	
	28"·0000	

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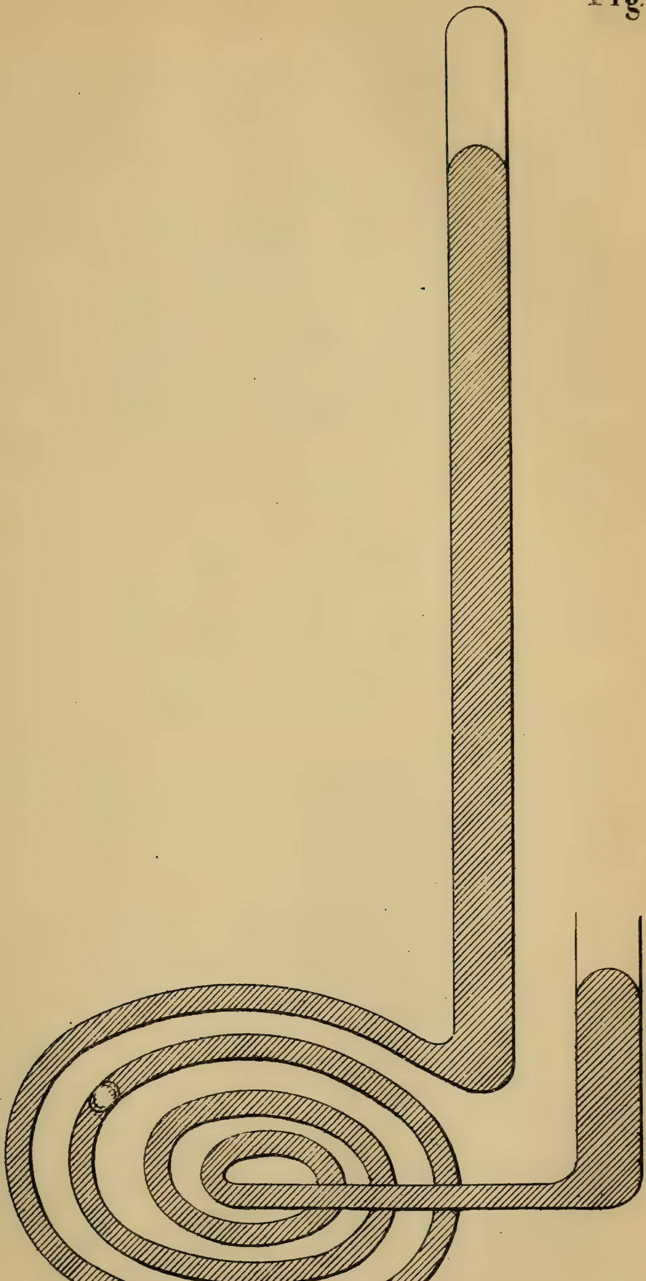
* Hypothetical specific gravities, following Dr. Guthrie's example.

Fig. 4.



the air bubble observed with a lens is seen to be in continual oscillation; but simplicity, portability, and some other desirable properties, seem to be sacrificed for the sake of sensitiveness in this instrument; although, on the other hand, it should be added that if for the first time the principles involved in the aneroid form of barometer were presented to the mind, the carrying them into practice for constructing a truly serviceable barometer would seem almost beyond hope; while experience has taught us that this form of barometer, even as small as a lady's Geneva watch, can be produced at relatively small cost with certainty and in endless quantity; and that the aneroid barometer is assisting in a large amount of valuable climatic and hypsometrical observation.

ART. IX.—*On some New Marine Mollusca.*

By REV. J. E. TENNISON-WOODS, F.G.S., F.L.S., Hon. Memb. Roy. Soc. N.S.W., Corr. Memb. Roy. Soc. Victoria, Tasmania, and Phil. Soc., Linn. Soc. N.S.W., &c.

[Read 9th August, 1877.]

THE following shells were placed at my disposal for description by Prof. M'Coy, of the National Museum of Victoria. I had been engaged for some time previously, preparing a census of the Tasmanian marine molluscan fauna, and on completing my lists and making the necessary comparisons at the National Museum I came across several in the extensive collections there which appeared to be new and undescribed. Permission to describe them was very cheerfully accorded by the learned Professor, whose obliging courtesy to me on all occasions where he could forward my small efforts in the interests of science I take this opportunity thankfully to acknowledge. It will be seen that the fauna here described is not in any way divergent from the recognised forms. A *Birostra* is, however, quite a novelty in Australian seas. Amongst all the species there is not one which even approximates to the extinct fauna of our tertiary beds, except in the case of the *Limopsis* just mentioned. N.B.—All measurements in French millimetres.

BIROSTRA M'COYI, *n.s. B. t., parva, lævi, nitente, anguste ovata, utrimque attenuata, superne subacuta, pallide rufescente, labio albida pallide lutea, conspicue incrassato, postice dilatato, canali brevi, tenuiter curvato.* Long. 23, Lat. 7 mil. Hab. Waterhouse, N.E. Tasmania.

Shell small, smooth, shining, narrowly ovate, alternate at each end, subacute above, pale reddish; lips whitish and pale yellow, conspicuously thickened, dilate posteriorly, canal short and slightly curved.

The only species of this rare genus found hitherto in South Australian waters. The type specimen in the National Museum is unique.

OLIVELLA AUSTRALIS, *n.s., O. t., turrita, fusiformi, spira elata apertur. æquanti, lævi, nitente, alba, pallide fulva reticulata et fasciis tribus albis zonata; sutura vix impressa; apertura angusta, antice dilatata, labro tenui acuto, columella simplici.* Long. 16, Lat. 4½ mil. Clark's Island.

Shell turreted, fusiform, spire produced and equalling the aperture; smooth, shining, white, reticulated with fulvous brown, and zoned with three white bands; suture scarcely impressed; aperture narrow, anteriorly dilated; outer lip thin acute, columella simple.

Differs from *O. nympha* in being coloured, and from *O. pardalis* and *O. leucozona* in its pale reticulated chesnut markings and three white zones. Its shape is also peculiar. I do not think enough is known about the genus to say whether it is liable to variation or not, and whether the species named are all only varieties. They are all rare, and therefore, one would imagine, less liable to vary.

MANGELIA HARRISONI, *n.s. M. t., anguste fusiformi, utrimque attenuata, lævi, gracili, tenui, translucida, spira elata, acuta, apert. æquanti, lactea, basi castanea, apice vero fulvo tincto, pallidissime (ult. anfr. tant.) luteo 4 zonata; anfr. 8, declivis, oblique costatis, costis lævibus, rotundatis, parum elevatis, superne obtuse angulatis, antice obsoletis; sutura bene impressa; apertura angusta, oblonga, labro tenui, labio reflexo.* Long. 14, Lat. 4. Clark's Island.

Shell narrowly fusiform, attenuate at both ends; smooth, graceful, thin, translucent; spire prominent, acute, equalling the aperture, milky white; base chesnut, but the apex stained, fulvous, and on the last whorl zoned with four bands of very pale yellow; whorls eight, sloping, obliquely ribbed;

ribs smooth, rounded, slightly raised, obtusely angular above, obsolete anteriorly; suture well impressed, aperture narrow, oblong; outer lip thin, inner lip reflected. Very rare.

Differs from *M. compta* of N.S.W. in the ribs being closer, and the absence of spiral striæ. The general form is also different.

MANGELIA TRACHYS, *n.s. M. t. parva, fusiforme turrita, opaca, solida, alba, maculis fulvis conspicue nebulosa; anfr.* 7 (2 *apical. lævibus, albis, obtusis) crebre crassicostatis et conspicue liratis ; liris supra costas transeuntibus, et ibi nodosis, costis in ult. anfr.* 9; *sutura bene impressa, unilirata; apertura oblonga, subquadrata, labro conspicue incrassato, postice profunde sinuato, sinu obliquo, columella simplici, canali brevi.* Long. 6, Lat. 2. Brighton.

M. shell small, fusiformly turreted, opaque, solid, white, conspicuously clouded with fulvous spots; whorls seven (the two apical smooth, white, obtuse), abundantly costate with thick ribs and very conspicuously lirate ; the liræ passing over the ribs and there nodose ; ribs in the last whorl nine; suture well impressed, with one fine raised line: aperture oblong, subquadrate, outer lip conspicuously thickened, deeply sinuous posteriorly; sinus oblique, columella simple, canal short.

The sinus, instead of going back into the shell, is confined to the thickened lip, and is oblique to the aperture.

RISSOINA KERSHAWI, *n.s. R. t. minuta, pupæformi, subcylindracea, fulvo saturata; anfr.* 6, *tumide convexis oblique crebre costatis, apice obtuso, apertura subcentrali, orbiculata, labio reflexo.* Long. 3, Lat. *vix.* 1½. Long Bay, Tasmania. W. F. Petterd.

Shell minute pupæform, subcylindrical, saturated fulvous brown, whorls 6, tumidly convex, obliquely closely ribbed; apex obtuse, aperture subcentral, orbicular, lip reflexed,

The aperture, which is almost central under the axis, and the uniform brown colour, distinguish this species.

RISSOINA SUPRASCULPTA, *n.s. R. t. minuta, pyramidata, alba, opaca, apice mammilato et verticaliter sito ; anfr. (vertice excluso)* 6, *ultimo et penult. rotundato* 3-*striato, reliquis granulatis, basim versus marginatis, supra suturas canaliculatis (canalic. transverse striata), apertura pyriformi, labio tenui, reflexo.* Long. 4, Lat. 1½. Long Bay, Tasmania.

Shell minute, pyramidal, white, opaque, apex mammilated and placed vertically; whorls 6, exclusive of the virtex, last and last but one rounded and tri-striate, the rest granulose, margined towards the base and canaliculate above (this channel transversely striate); aperture pyriform, lip thin reflexed.

BITTIUM SEMILÆVIS, *n.s. B. t. minuta, turrita, castanea; conspicue eleganterque carinis et costulis clathrata; anf.* 12, *quinque apicalibus lævibus, nitentibus, basi læva. labro tenui.* Long. 5, Lat. 1. N.W. Tasmania.

Shell minute, turreted, pale chesnut, conspicuously and elegantly latticed with keels and ribs; whorls 12, the apical 5 smooth, shining base, smooth lip, thin.

The smooth apical whorls are peculiar, and perhaps this portion is decollated with age. The only specimen known to me is in the Melbourne National Museum. Possibly it would come under some of Mr. Adams' genera near to *Cingulina.*

LIOTIA MINIMA, *n.s. L. t. minuta, orbiculari, spira parum exserta, alba, pellucida, spiraliter striata, apertura valde incrassata, umbilico granis nitentibus marginatis.*

This very minute *Liotia* seems devoid of ornament, except the regular spiral groove. It has, however, a remarkably thickened varix round the aperture, and a granularly margined umbilicus; in all which respects it differs from any species known to me.

THALOTIA MARIÆ, *n.s. T. conica simillimi sed paulo parviore, carinulis haud granulosis, striis inter carinulis latis, rotundatis, luteis; lineis albis longitudinalibus, angulariter undulosis et maculis roseis, et flammulis roseo purpureis, vel atro-purpureis variegata; apertura subquadrata, intus argentea, lirato, columella haud dentata.* Long. 17, Lat. 12.

Differing from *T. conica,* Gray (with which shell it has been hitherto confounded) in not being granular, though the peculiar spotted colouring makes it appear so. It is almost regularly tesselate on the upper part of the whorls. It is more tumid, solid, and darker in colour than *T. picta,* Wood, and *T. pulchella.* Not uncommon in Hobson's Bay, though much more numerous outside Port Phillip Heads. I have never known it to occur in Western Victoria or Tasmania.

THALOTIA TESSELATA, *n.s. T. t. parva, subumbilicata conica, pallide olivacea, alba maculata vel tessellata; anfr.* 7, *sub-*

convexis, ubique subtillissime spiraliter et oblique transversim striatis; 5 carinis munitis; carinis latis, planatis, supra et infra latioribus et prominentioribus, basi convexa, carinata; apertura subquadrata; labro acuto tenui, intus marginato, labio albo, conspicuo, fauce argentea, margaritacea, lirata. Alt. 6., Lat. 4½. *Interstitiis inter carinas interdum liratis.*

Shell small subumbilicate, conical, pale olive, spotted or tesselated with white; whorls 7, subconvex, everywhere finely obliquely, spirally tranversely striate; furnished with five keels, which are broad, flattened, and the upper and lower ones broader and more prominent; base convex, keeled, aperture subquadrate, outer lip acute thin, margined within; inner lip white, conspicuous; throat silvery nacreous, lirate. The interstices between the keels sometimes striate.

THALOTIA DUBIA, *n.s. T. t. turbinato-conoidea, solida intense roseo purpurea et roseo-flammulata; anfr.* 7, *convexis* (4 *apicalibus planatis), carinis* 4, *parvis, distantibus conspicue granulatis, instructis; granulis parvis, concinnis, roseo-purpureis; interstitiis granulose liratis, periostraca lutea sericea indutis; sutura profunda, late subcanaliculata, basi planata, spiraliter lirata et radiatim striata; apertura subquadrata, incrassata, conspicue multidentata; columella tuberculata marginata et crebre dentata.* Long. 18, Lat. 15. Clark's Islands.

Shell turbinately conical, solid whorls intensely rose-purple and rose-flamed, whorls 7, convex (the four spiral flattened), keels 4, small, distant, conspicuously granular; granules small, neat, and rose purple in colour; interstices clothed with a yellow silky periostraca; suture deep, broadly subcanaliculate; base flattened, spirally lirate and radiately striate, aperture subquadrate, thickened conspicuously multidentate; columella tuberculate, margined and closely toothed.

In general form resembling *T. conica*, but smaller and more closely ornamented. The mouth is also an approach to a *clanculus.* Rare.

MINOLIA VECTILIGINEA (Menke), var? *M. t. orbiculata, depressa, tenui, diaphana, profunde, perspective umbilicata; anfr.* 5½ *rapide decrescentibus rotundatis, ad peripheriam obtuse angulatis, undique spiraliter crebre tenuissime striatis et subtillissime transversim oblique striatis, umbilico albo, concavo, ad marginem angulato, apertura rotundata. Eleganter atro et olivo marmorata, ad peripher. olivo et albo* *tesselata, vel in lineis longit. dispositis strigata. Maj. diam.* 11, *min.* 9, *Alt* 8. Hobson's Bay.

Shell orbiculate, depressed, thin, diaphanous, deeply and perspectively umbilicate; whorls 5½, rapidly decreasing, rounded, obtusely angular at the periphery, thinly and very finely striate all over with transverse and oblique spiral striæ. Umbilicus white, concave, angular at the margin, aperture rounded. Elegantly marbled black and olive, tesselated at the periphery with white, or sometimes striped in lines. Common.

This shell is much varied in the markings, and in its young state is often rose, or brown, or orange in colour. It is of course no more than a variety of the variable *Minolia vectiliginea,* but I give my own diagnosis as Menke's list is difficult to meet with, and, as I think, hardly sufficient.

TAPES VICTORIÆ, *T. t. inæquilaterali, oblongo-ovata, subtumida, antice abbreviata, rotundata, postice sub-lata, elevata rotundata, et concentrice crebre costata; costis rotundatis sub-elevatis, inæqualibus, in medio sæpe desinentibus; umbonibus parvis, antice sub-arcuatis; ligamento lanceolato conspicuo; dentibus cardinalibus valv. dext.* 2, *valv. sinis.* 2 *anteriorib. bifidis; pallide carnea, lineis fulvis divergentibus, litterata ad margines punctis intensioribus maculata; pagina interna lutea antice et postico fulve purpureo tincta.* Lat. —, Long. —, Alt. —. Hobson's Bay.

Shell inequilateral, oblong, oval, subtumid, shortened anteriorly, rounded posteriorly, somewhat wider, raised, rounded, and concentrically thickly ribbed; ribs rounded, sub-elevate unequal, often disappearing in the middle; umbones small, slightly curved anteriorly, ligament lanceolate and conspicuous; hinge teeth, two in right valve and two in the left, which are bifid, colour pale flesh, with brown divergent letterlike lines, which are more intense towards the margins; inner surface yellow, stained at each end a purple brown.

CIRCE PYTHINOIDES, *n.s. C. t. parva, crassa, suborbiculata, vix gibbosa, parum, quadrata, albida, postice atro-purpurea maculata, radiatim costata, costis irregularibus, rude nodose granulatis ad marginem sæpe divisis, antice et postice divaricatim, bifurcatim plicatis, umbonibus acutis, vix curvatis, lunula late ovata, purpurea, marginibus incrassatis, valde flexuosis, pagina interna nivea, dentibus crassis, conspicuis.* Long. 25, Lat. 22, Alt. 10. Victoria.

Shell small, thick, sub-orbiculate, scarcely gibbous, slightly quadrate, whitish, spotted black purple posteriorly; radiately ribbed, ribs irregular, coarsely nodosely granular; ribs often divided towards the margin, anteriorly and posteriorly divaricately and bifurcately plicate; umbones acute, slightly curved; lunule widely ovate, purple; margins thickened, very flexuous, interior snowy white; teeth thick, conspicuous.

There is a *Circe* something like this figured in Reeve *(Icon. V., fig.* 21*)* and identified with *C. gibba* occurring in the Red Sea and Philippines. It may be the species here described, but it is quite distinct from *C. gibba*. The differences from both figures and descriptions are as follow:—It is smaller, almost orbicular, has a series of divaricating ribs sloping away on *both* sides at an acute angle from the first and last central ribs, giving rise to a sculpture like the genus *Pythina.*

ARCA M'COYI, *n.s. A. t. alba, periostraca fusca plus minusve induta, oblonga, quadrata, medio sinuata et hiante, postice latiore et carinata, confertissime concentrice granulosé costata; granulis subspinosis, rotundatis, obtusis, supr. carin. longioribus et radiatim dispositis; umbonibus parvis, acutis, planatis, curvatis, area angusta, postice attenuato ; dentibus parvis linea curvata dispositis ; marginibus denticulatis, pagina, interna, nitente, nivea.* Long. 7, Lat. 14, Alt. 6. Var ex, N. S. Wales, *tumidioribus.*

Shell white, more or less covered with a dusky periostraca, oblong, quadrate, sinuate and gaping in the centre, broader and keeled posteriorly, very closely concentrically granulously ribbed; granules sub-spinous, rounded, obtuse, longer upon the keel and radiately disposed, umbones small, acute, flattened at the sides and curved; area narrow, attenuate posteriorly; teeth small and disposed in a curved line, margins denticulate; internal surface white and shining.

This shell is so near *Arca gradata* (Brod. of West Columbia) that I doubt if it be distinct. The species have a wide distribution. The E. Indian *A. imbricata,* Brug., and the West Indian *A. trapezia,* are common in Australia.

PECTUNCULUS FLABELLATUS, *n.s. P. t. late orbiculari, paulo vero transversa, crassa, tumidiuscula, radiatim valide costata ; costis* 25—35, *latis, planatis, ætate antice et postice confertis; marginibus late denticulatis ; dentibus card.* 16—20, *crassis ; alba, intense fulva intus tincta et* *extus plus minusve nebulosa et maculata.* Long. 44, Lat. 47, Alt. 44.

Shell broadly orbicular, *but slightly transverse,* thick, somewhat tumid, validly radiately ribbed; *ribs* 25 *to* 35, *broad, flattened,* becoming very close at the sides as the shell grows; margins broadly toothed; cardinal teeth 16 to 20, white; colour white stained, but intense fulvous brown within, and more or less clouded and spotted with the same colour on the outside. Victoria and Tasmania. Not common. Resembling *P. radians,* Lam., but differing in the particulars italicised above. It seems also to be almost without periostraca. Very near *P. laticostatus,* Lam., which Prof. Tate informs me is found at Spencer's Gulf and N. Tasmania. It may turn out not to be specifically distinct from that shell which is so abundant in our Miocene Tertiaries.

TRUNCATELLA MICRA, *n.s. T. t. minuta, alba, translucida, cylindracea; anf.* 4 *(decollatis) irregulariter costato-striatis, inflato-convexis; sutura impressa, apertura parva, semilunari, labro reflexo.* Long. 4½, Lat. 1½. Brighton, Victoria.

Shell minute, white, translucent, cylindrical; whorls 4, (decollate) irregularly costately striate, inflatedly convex; suture impressed, aperture small, semilunar, outer lip reflexed.

There are so many *Truncatellæ* described, which run so closely to each other, that I hesitate to add this species. It seems, however, to differ widely enough from all known to me to warrant my giving it a name. It was found by Mr. Kershaw.

The following freshwater shells were placed in my hands for the most part by Mr. W. Kershaw, the intelligent taxidermist and collector for the National Museum. It will be seen that I describe as new species several ciliated *Physæ,* which I regard as being very close to those already described by me as from Tasmania. Freshwater shells, it must be remembered, have always a very wide range, being carried about by aquatic birds in their migrations. Thus I have found many freshwater and fluviatile species common to North-east Australia and New Caledonia. Yet strange to say there is sometimes a great difference found in the species inhabiting freshwater lakes or streams within a short distance. The species common to Tasmania and Victoria are pretty numerous, and more may yet be found. *Bythinia Huonensis,* nobis (which Professor Tate considers should be made the type of a new genus) is common about Melbourne. *Physa Dulvertonensis,* Reeve, I have also seen, but no traces so far of the peculiar and large *Ancylus.* If the ciliated large *Physæ* here described are all varieties of *P. ciliata nobis,* the shell must be very variable; and all those of Victoria have a marked uniform character; it is very possible that some of them may have been described before, though after a diligent search I have not been able to discover where. Meanwhile it is very desirable that the species should have names and descriptions easily accessible to Australian naturalists, which I have accordingly given them in the descriptions which follow :—

PHYSA PILOSA, *n.s. P. t. subumbilicata, tenui, nitente, inflato, oblique, late ovata, lactea vel fulva, spira, fulva, subpellucida; anfr.* 3, *ultimo inflato et obliquo,* 2 *apicalibus parvis, acutis; regulariter longitudinaliter striatis, periostraca lutea, indutis, lineis regularibus pilosis vel punctatis instructa, sutura coronata, apertura oblique ovata, antice producta; labro tenui, labio reflexo.* Lat. 6, Long. 11, 1.11 mil.

This may possibly be only a variety of *P. crebreciliata.* It differs from it in being thinner, lighter in colour, with a very thin periostraca—the extremely small spire, with the oblique and interiorly produced aperture.

PHYSA CREBRECILIATA, *n.s. P. t. umbilicata, tenui, inflata; late ovata, cornea, fusca vel albida et diaphana; periostraca totaliter induta; anfr.* 3½, *duobus apicalibus parvis, penultimo perobliquo, longitudinaliter crebre striatis, et spiraliter lineis ciliatis crebre instructis, suturis periostraca coronatis, apertura late ovata, tenuiter incrassata vel bilabiata, labio conspicue reflexo.* Long. 7, Lat. 15 mil. Caulfield, Melbourne.

Shell umbilicate, thin, inflated, broadly ovate, horny, dusky or whitish and diaphanous, completely covered with a ciliated periostraca; whorls 3½, the two apical ones small, the penultimate peroblique, thickly striate lengthwise, and furnished with close spiral ciliated lines; sutures crowned by the periostraca, aperture broadly ovate, slightly thickened or bilabiate, lips conspicuously reflexed.

The cilia in this shell are in regular equi-distant spiral lines, and at the sutures the periostraca seems to mass itself in small rough folds, so as to make a spinous ridge.

PHYSA ARACHNOIDEA, *n.s. P. t. elongata ovata vel subcylin-*

dracea, crassiuscula, opaca, nitente, vel periostraca induta. obscure fulva, vel lutea et alba maculata, apice acuto; anf. 6, *rapide decrescentibus, leviter convexis et declivis longit. et transvers. striatis; striis granulato-punctato (sub lente tantum visis) punctis lineis spiralibus dispositis; apertura, obliqua, pyriforme, antice producta, intus cretacea; plica crassa, per umbilicum tantum visa.* Long. 12, Lat. 5½. Long. apert. 7, Lat. 3½. Mordialloc, Victoria. W. Kershaw.

Shell elongately ovate or sub-cylindrical, rather solid opaque, shining or clothed with a periostraca; shell brown or yellow, with white spots, apex acute; whorls 6, rapidly decreasing, slightly convex and sloping; striate lengthwise and transversely, striæ granularly dotted, which is only visible under the lens, dots disposed in spiral lines; aperture oblique, pyriform, produced anteriorly, chalky white inside; plait thick, but visible only by looking, as it were, upwards through the umbilicus.

I believe that the points or dotted spiral lines are derived from cilia, which, however, had disappeared from all the specimens examined by me. They would surely be found in younger specimens. Perhaps, after all, this is only a variety of the *Physa Dulvertonensis* of Tasmania.

Physa Yarraensis, *n.s. P.t. sub-umbilicata tenui diaphana pallide cornea, nitente, spira acuta; anfr.* 4, *convexis, declivis,* 2 *apicalibus parvis, tenuiter longitudinaliter striatis, apertura elongata, pyriformi, labro tenuissimo, antice producto, labio inconspicuo, plica crassiuscula.* Upper Yarra, Victoria. W. Kershaw.

Shell subumbilicate, thin, diaphanous, pale, horny, shining, spire acute, whorls four, convex, sloping, two spiral, one small; finely striate lengthwise, aperture elongate, pyriform, labrum very thin produced anteriorly, lip inconspicuous, plait a little thickened.

A shell with no very determinate characters, of small size and thin.

Physa Kershawi, *n.s. P. t. parva, anguste ovata tenui, periostraca sordida, rugosa, induta, parum diaphana, sordide fusca; anf.* 3½ *ad* 4, *superne conspicue angulatis et planatis, ad angulum regulariter (et sup. ult. anfr. distanter) carinatis; carinis rotundatis, elevatis; ad suturas anguste canaliculatis, apertura ovali, antice producta; labro tenui, ad carinas sinuato, labio reflexo, subumbilicato.* Long. 8, Lat. 4½. Upper Yarra. W. Kershaw.

Shell small, narrowly ovate, clothed with a sordid rugose periostraca, slightly diaphanous, dusky in colour; whorls three and a half to four, conspicuously angulate and flattened above, at the angle (and on the last whorl distinctly) keeled, keels rounded, raised; at the suture narrowly canaliculate, aperture oval, produced anteriorly; labrum thin, sinuous at the keels, inner lip reflexed, subumbilicate.

There is a faint resemblance between this shell and the New Zealand *P. tabulata* of Gould.

BYTHINIA VICTORIÆ, *n.s. B. t., minuta, turbinato-conoidea, viride lutea, sericea, periostraca atra plus minusve induta; anfr.* 4½-5, *rotundato-convexis, lævibus, longitud. tenuiter rugoso striatis; apice obtuso, apertura ovata, intus castanea, vel alba, labro tenui, labio vix reflexo.*

A minute shell, whose size, silky appearance, fine longitudinal striæ, and turbinately conical form, distinguish it from all its Australian congeners. Lake Connewarre, Geelong. Found in great numbers in *Confervæ* by W. Kershaw.

ART. X.—*On Various Forms of Galvanic Battery.*

BY R. L. J. ELLERY, F.R.S., F.R.A.S.

[Read August 9th, 1877.]

ART. XI.—*Extracts from Diary in Japan.*

BY F. C. CHRISTY, C.E.

[Read 13th September, 1877.]

JAPAN consists of four Islands, governed by an Emperor, Ministry, and Parliament.

The Ministry consists of Premier, Ministers of Finance, Foreign Affairs, Public Works, Education, Agriculture, &c., &c., with Vice-Ministers to each department.

Its members of Parliament are not elected by the people, but are the Chief Magistrates of the various kens, or districts, and are supposed to know the requirements of their people.

Yesso, the northern island, is about the 44th degree of latitude and under the 144th parallel of longitude. Here the winter is extremely severe; with almost constant snow during the winter months; the bear, wolf, deer, wild boar, otter, fox, hare, &c., are abundant; ptarmigan (grouse), woodcock, snipe, &c.; codfish, herring, salmon, in profusion. The cod, salmon, and roe of fish are salted and sent to the southern towns in hundreds of tons per annum, and form with rice the chief food, meat being little eaten.

Niphon, the main island, has the largest population; Yedo, the capital, contains 3,000,000 inhabitants.

The southern islands produce the best rice, and the largest amount of good coal and minerals, excepting gold, which is found principally in the north. Silk is produced in Niphon and the southern islands; a large amount of good rice is also grown around Yedo and Yokohama and southward.

The temperature at Yedo during the hottest days in the sun was 122°, in the shade 93°; and the coldest 25°. It is believed that the thermometer often shows 14° of frost, 18° Fahr.

The autumn and winter months, from October to April, are very dry and bracing, with clear bright atmosphere, and from April to October very wet; the chief amount of rain falling during the latter months. The rainfall at Yedo, as obtained from Observatory, is 72 inches. The atmosphere during the summer is excessively humid, and very dry in winter. Furniture contracts and breaks its joints in winter; whilst in a summer's day one's boots become mouldy, and kid gloves spotted, which it is absolutely impossible to prevent.

Strange as it may appear there is very little sickness in summer, and fevers are almost unknown.

Small-pox is very prevalent in winter, and appears when the cold sets in, disappearing with the spring rains.

Skating is fashionable amongst the European population of Yokohama; good ice usually lasts a month, or six weeks; it is necessary to shade it with mats, or the sun's rays thaw it.

The 10th of January, 1876, eight inches of snow fell at Yedo, and remained with frost six days, and began to thaw the seventh day. On 27th January, 1876, fifteen inches of snow fell at Yedo, and delayed trains; in some places it was four feet deep.

July, August, and September are very hot months; although the temperature is much less than in this colony, the heat is more oppressive. Sun hats (helmets) and white linen clothes are worn.

There is very little thunder and lightning, but severe earthquakes, which appear to travel east to west; eastward is Brise Island, which has upon it an active volcano, and Fujiyama, the holy mountain, nearly 14,000 feet high, is distant about sixty miles west from Brise Island. Yedo and Yokohama, which are eighteen miles apart, lie between these two mountains; and it is thought the waves or shocks travel from Brise Island to old Fuji. (See notes on earthquakes at end.) Fujiyama is clothed with snow about nine months of the year, and is ascended by hosts of pilgrims during July and August, who are stamped on the back with a large circular seal, or stamp of red paint, in proof of the ascent being made; the pilgrims are usually clothed in white loose tunics and trousers, straw sandals, and huge broad brim hats, made of flat rush or bamboo. Fujiyama is well wooded at the lower part, but barren towards the top, which consists of loose lava and ashes, with a deep inactive volcano basin at the summit.

The ordinary lilac rhododendron grows on the mountain. According to tradition Fujiyama rose from the plains in a day, or night; the day being a dark day of horror and destruction by earthquakes, &c. The superstitious believe that the earth is moved by a huge tortoise.

Japan generally is mountainous, a chain of mountains running from north to south, through Niphon, of 3000 to

10,000 feet altitude. It is watered by numerous rivers from these mountains, emptying into the sea. The rivers are some of them wide near the sea, but narrow and more rapid inland; they abound with trout and salmon—the salmon being local, that is northward, although the salmon trout, a delicate fish with pink flesh, is largely taken in Lake Biwa, near Kobe (southward). The lakes are numerous and extensive. The country near the coast is beautifully wooded with small groves of evergreen and deciduous trees. The features of this portion of the country are striking, the hills running out towards the coast in forms resembling barrows, very steep, with irrigated valleys between—each valley having its stream, or rivulet; the tops and sides of the hills being clothed with trees and bamboo groves, and dotted with farms. The woods are lovely, tinted with every shade of colour in vegetation; the deep green of the cryptomeria and pine, evergreen oaks and other trees, intermixed with golden feathery bamboos, the scarlet, blood-red, and pink maples, the light green of the deciduous oaks, ash, beech, birch, elm, horse and edible chesnuts, &c.—the latter being a common forest tree.

The cottages are frequently sheltered by a bamboo grove (the bamboo attaining a height of 60 feet), and have a garden, with plum trees, and lime trees 20 feet high, with their golden fruit and deep green foliage; persimon of light green foliage and chrome-coloured fruit, resembling golden eggs.

The parks are lovely, especially Uyeno and the Castle gardens, with its ornamental water and rocky cascades; particularly when the double-blossom cherry and peach are in flower. The cherries grow 50 feet high, and the pines, cryptomeria japonica, cephalotaxus, &c., to 80 feet, casting a deep shade. In many districts avenues of cherries are planted, and thousands of Japanese go to see them in blossom; it is one of the great holiday sights. Among the early blossoming trees are the wistarias, purple, lilac, and white; there is also a double blossom purple. The wistaria, or fuji, is one of the greatest favourites, some of them being over 100 years old. The stem is carried up straight and the branches trained overhead on horizontal bamboo trellis, with seats underneath; one tree will often cover a square of 50 x 50 feet. They are generally planted at the tea houses, for shady lounges; the tresses of blossom hang through the trellis overhead. In the woods the wistaria is everywhere to be seen, with its beautiful lilac tresses of blossom hanging in festoons from the branches of the forest trees; here the ivy clothes others, the old English mistletoe hangs from the boughs above, and the honeysuckle wreaths the underwood. Neat hedges divide the cottage gardens, and frequently enclose the gardens of the rich. A wild bitter orange is the best hedge plant, as it is impenetrable; but the euonymus japonicus, althea (hibiscus), with white and lilac blossom, and the cryptomeria are used; these all make neat hedges when well kept. The camellia, although wild, is usually planted along the roadside; it frequently attains a height of thirty feet, profusely studded with lovely red blossoms. The fan palm is also a favourite, and produces a beautiful effect; the hairy covering around the stem is used in lime plaster of dwellings. The pink and white daphne attain a height of five feet, as also the azalia, which grows wild, and is cultivated in every variety of colour in the temple grounds and gardens, as also the lovely olea fragrans, or Japanese mignonette, so called from its powerful and sweet scent; together with the charming lagerstrœmia rosea, a tree 20 feet high, covered with magenta blossoms. The umbrella pine (sciadopetys verticillata) adorns the temple grounds, as also a tree resembling araucaria bidwillii, excepting that it grows very straight, tall, and luxuriant, with light green foliage, said to be a cunninghamia. The grandest of all trees, and perhaps the most esteemed, is the ginko biloba, or salisburia adiantifolia, which attains a height of 80 to 100 feet, with a noble contour, the foliage pale green in summer and chrome yellow in autumn. The commonest of all trees, and one of the most stately, is the pine of the country, used for firewood and a variety of purposes (pinus massoniana); this tree is the common tree of the forest, the roadside, and the avenue, and is most frequently pictured in lacquer work and introduced in bronzes, &c. The berry-bearing shrubs are much admired and cultivated; the most prominent is the bamboo of heaven (nandina domestica), with its light feathery foliage and lovely scarlet or yellow wax-like berries; it is to be seen in almost every temple ground and cottager's garden, and decorates the houses at Christmas time.

The timber generally used is the cryptomeria japonica, scented and soft like cedar (sugi), for lining houses, doors, windows, and boxes. An ulmus or elm (planera japonica), for temples, outdoor work, and furniture, is the most used and most valued of all. Cupressus obtusa (hinoki) is much esteemed for its durability, closeness of grain, silky appearance, and freedom in working; it is used for all the best temple fittings, &c.

The timber most commonly used in the rough framing and roofs of houses is the matz (pinus massoniana). There are seven species of oak, three evergreen and four deciduous. The deciduous oaks are seldom allowed to attain large growth, but are cut young for charcoal, oars of boats, &c. The evergreen oaks are large trees and truly magnificent; one, the kashi (quercus glauca), has immense glossy leaves, and is used for planes and other carpenters' tools, being very hard and of close grain. The ash (fraxinus excelsior) is fine timber, but seldom utilised, being chiefly burned for charcoal; the wood is like the European ash, as also the foliage, but is more robust. The walnut is largely grown, although the timber is not utilised.

The houses generally are built of timber, with heavy timber roof, tiled, frequently of two stories; the peculiarity is that all the windows and doors slide in grooves, economising space; the windows are framed in small squares and covered with paper, with a sliding shutter outside, which is closed in wet and stormy weather. The houses are without fireplaces, but are warmed by hibachis, an earthenware or bronze vessel containing lighted charcoal; the houses are scrupulously clean, the floors generally matted with rush matting. The higher class houses are heavily framed, diagonally lathed outside, and faced with flat tiles, which are nailed on vertically and the joints seamed with lime mortar; these houses are dry, cool, and comfortable.

The cities and towns are all much alike, with narrow streets, unpaved, but frequently macadamised. Lately, brick houses and wide streets have been adopted in Yedo and Yokohama by advice of Europeans, and they are much approved. The streets of Yokohama are wide and altogether of European appearance—this town having been chiefly occupied by Europeans and Americans for a considerable period. Yedo has now also given way to the same innovation; and Ginza—the main street leading from the railway station to Nihom Bashi (one of the chief bridges)—has omnibusses continually running, and hundreds of horse-drawn vehicles, also thousands of Jinrikishas—a small, hooded vehicle, on two wheels three feet in diameter, with springs, cushioned for one person; it has shafts, between which a man runs; when two men are employed, the foremost draws by a rope; two men will run from twenty to thirty miles, the greater part of the distance from eight to ten miles per hour. Yedo and Yokohama are lighted by gas, superintended by a French engineer. Yedo is a fine city, with a magnificent river, and veined with canals—nearly all navigable for large craft. The Harbour Trust of Melbourne might benefit by a trip to Yedo, which would make them less sceptical of the certainty of making a canal from the Gasworks to Hobson's Bay—a paltry 1½ miles, whilst in Yedo and other towns of Japan there are hundreds of miles of navigable canals, nearly all opening into the sea, and walled from end to end with masonry.

The masonry is wonderful as it is beautiful; it is generally of parabolic outline, with a quick curve at the base, and becoming nearly vertical at the top, with an average batter of about 1 in 12. The masonry is all of dry, squared rubble, coursed; the walls of the moats round the castles attaining a height of from 50 to 100 ft. Some of the stones in Osaka Castle weigh by measurement 160 tons each. The castle is on a hill, probably between 100 and 200 feet above the surrounding country, encircled by swampy rice-fields, four miles across before any quarry is reached; therefore the presence of such enormous stones on an eminence so far away from any quarry is a marvel which no Japanese could explain. The only answer was that the castle had been built about 500 years, and no records kept.

The temples of Japan are truly superb. The decoration of the interior is lovely and chaste; the intermixture of colours, opposed to each other according to European taste, are so beautifully blended and subdued that the most sublime harmony exists, and there is only one feeling of all visitors—the marvellously lovely and glorious effect.

The exterior of the temples is majestic and grand, built generally upon round wooden columns of large diameter, stepped into blocks of stone, with immense overhanging roof, heavily tiled, beautifully neat in pattern; the roof hipped but externally concave in the line of rafter; the overhang, supported by rafter upon rafter protruding in succession, beautifully carved, adding to the massive grandeur. There is usually an entrance gateway, roofed with the same massiveness and beauty, with noble gates, hung by enormous wrought-iron strap hinges, and bound in every direction by copper, bronze, and iron. A long, paved causeway, lighted on each side by grotesque columnar stone lanterns, beautifully carved, leads to the temple. Spacious grounds of many acres surround the temple, planted with beautiful forest and flowering trees and shrubs. As a rule, the grounds, which are enclosed by walls, are most lovely. A flight of stone steps leads to the temple entrance, which is closed by massive doors. The temples are usually guarded at the entrance gates or at the temple by huge human figures, carved in wood, painted red or black, complete and lifelike; the expression of the features most effective.

The interior of the temple is superb; black polished lacquer floor, with gilt surroundings; the altar a miniature temple of emblazoned gilt; the deity of gilt with the halo around the head, reminding one of the Roman Cathedral. The whole of the ceilings of the temple are panelled and painted in gold, green, purple, scarlet, and black, in the most chaste and elegant patterns, so minute that the decoration must have occupied a lifetime to execute. The priests officiate, and the suppliants kneel with their hands raised and clasped in the form of Christian prayer, chanting the service and counting their beads; a font of holy or sweet water stands at the temple entrance.

The priests, with their heads shaven mostly, are jolly fellows, glad to show and explain everything. Outside, slung on a large wooden beam, is an enormous bell of bronze, many tons weight, beautifully embossed with various devices, and tolled by a huge battering-ram of timber drawn backwards and forwards by ropes.

There are two contending religions—Buddhism and Shintoism. Shintoism is the approved religion of the Government; both are ceremonially similar to the Christian religion, the creed being much the same: they each believe that God has been on earth to reform and save them.

The colossal figures in bronze of their god Daibutz are very wonderful, being from forty to fifty feet stature, beautifully finished and polished outside, and the features most expressive and lifelike. The whole figure is composed of bronze, cast in small segmental plates, about one inch thick, and brazed together.

The soil is generally volcanic, rich and dark chocolate, overlying in many districts a clay slate much similar to that of Melbourne. The Kobe district, 300 miles southward from Yokohama, is granitic, and there the soil is poor, composed of coarse grit sand.

Kobe is one of the chief open ports, and communicates by railway with Osaka, distant twenty miles, and Osaka with Kioto, distant another twenty miles, or forty miles of railway from Kobe.

Kioto is the ancient city of the Mikado, and the people of Kioto wish to regain the seat of Government from Yedo, where it now is.

It was intended to extend the railway from Kioto to Yedo—*i.e.*, connect the two, viz., the railway between Yokohama and Yedo, eighteen and a quarter miles, with the Kobe line—which would require three hundred miles additional line; but for the present this is abandoned.

Again referring to the nature of the country, there is a total absence of chalk, limestone only of various kinds having been found.

The minerals generally are copper (widely distributed), iron, lead, silver, zinc, and gold; gold deposits do not appear to be rich. Coal is also widely distributed, of excellent quality, and varying from very bituminous to hard, approaching the character of Welsh or anthracite. The price delivered in Yedo or Yokohama is 8 dols. (32s.) per ton. It is not more than 10s. per ton at the mines in the Southern Island.

The mining is controlled by a department with a large European staff; but it does not appear to pay, and the Japanese prefer mining in the old manner.

There are several colleges in Yedo; the principal one—the Imperial College—is a most splendid institution, with a number of excellent English professors. It is established as an engineering college, and has extensive engineering workshops, capable of manufacturing the largest marine engines, being equipped with the finest machinery. There are professors of engineering, natural philosophy, geology, chemistry, electricity, English, mathematics, surveying, and all branches of education. Attached is an extensive museum of models, &c.

Yedo is the principal city of Japan, and the seat of government, and where the Emperor resides. There are two parks—Uyeno and the Castle—and several lovely palace gardens, the resort occasionally of the Emperor. Uyeno Park probably is not excelled in beauty, grandeur, and variety of trees by any park in the world.

Near Yedo is the Katakushi, or experimental farm, and Horticultural Gardens, which hitherto have been presided over by Americans. The whole affair has been very costly, with very poor result.

The military organisation is principally at Yedo; the cavalry, infantry, and artillery and arsenal, are under the supervision of Colonel Munier and staff, who are sent out by the French Government at the request of the Japanese Government. The Naval Department is organised by English officers, selected by the English Government.

Japan has about one hundred thousand troops, well armed with the best breech-loading firearms, and artillery, and all well clothed in smart European costumes. The greatest credit is due to the French officers. Many of the Japanese officers appear to be as smart as their European instructors; and when in their gold lace or red uniform, &c., it is difficult to distinguish one from the other.

Throughout Japan there is an immense and most efficient police force, entirely controlled by Japanese officers.

The European banks are the Oriental, the London Chartered, the Shanghae, Comptoir d' Escompt, and German bank. These are all at Yokohama; Mitsuës, the Government bank, is alone at Yedo. The currency is the silver Mexican dollar and the Japanese gold yen, of about equal value, of 4s.

All the Legations are at Yedo, the British and Russian being the most imposing; these two having erected fine buildings on large commanding sites. The Italian and German are in proximity, but the French still remains between Yedo and Shinagawa, where the English Legation originally was, outside Yedo.

The Legations are all presided over by ministers, who have been especially well chosen by their respective nations; under the ministers are consuls and vice-consuls. Yedo is the great centre of commerce. The exports—which are silk, tea, china (porcelain), tobacco, rice, copper, and various articles, chiefly fancy goods—nearly all pass through Yedo to Yokohama by water or rail, except those which are shipped from other open ports; all open ports have a customs department.

The revenue of Japan, as published by the Japanese Treasurer, is £17,000,000 sterling, chiefly raised by a land or produce tax, and an import and export duty of 5 per cent.; also a multiplicity of small taxes levied upon their own people.

The people are a most distinct race, all having black hair, and black eyes slightly almond-shape, which is most observable in the ladies of high birth; in this there is a remarkable distinction, the ladies of high families possessing characteristic features in the thin aquiline nose, small mouth and lips, and full black eyes, slightly almond shape, remarkably fair, clear wax-like complexions, lovely teeth, and the most beautifully-formed hands and arms. The hair is studied to the last degree, most beautifully arranged and kept, no covering to the head being worn. The dress is elegant and chaste, the all-prevailing purple and scarlet being the favourite colours of the ladies, although many other lovely colours are introduced—always harmoniously.

The outer dress is silk, folded across the chest, leaving the neck bare, closed by a broad obe or sash around the waist, fastened in a large loose knot behind; and generally a scarlet under garment, showing in front below the outer dress. The outer dress is usually embossed or embroidered beautifully with floss silk, in various devices; the feet covered by a white sock, and the sandal or clog worn.

The gentlemen wear a long loose dress of silk in winter, and silk gauze in summer, folded across the chest, leaving the upper portion of the neck exposed; fastened round the waist with a narrow obe, the legs bare, but covered by the outer garment, which reaches the ankle; socks and sandals, or clogs, being worn on the feet; no covering to the head, the hair drawn tightly back from the forehead, gathered and tied at the crown in a short queue brought forward flat upon the head. Two swords were worn until quite lately, being now prohibited by Government. The swords—one long and one short—have curved blades and wooden scabbards, the swords being of the finest steel with the sharpest edge, and much prized according to quality. It is said that a Japanese considers it a disgrace to draw his sword and sheath it without drawing blood, if drawn in anger.

The gentlemen ride on horseback. The horses are cobs, about fourteen hands, and very enduring; the trappings elaborate, large Eastern saddle and cloth, heavy stirrups enclosing the foot, and heavy head mountings, with silk reins, &c., all extensively worked.

The norimon of basket-work, sometimes entirely enclosing the traveller and sometimes open with a handle or rail running along the top (overhead), carried on the shoulders of a man in front and one behind, is the mode of travelling through the interior where the roads are bad.

There are several main roads, each one called a tokaido; moderately well kept, upon which horse vehicles can travel some considerable distance; but the roads generally are mere bridle tracks, unformed and unmade, upon which pack-horses alone can travel. All the produce which cannot be sent by water is brought upon pack-horse, even to timber, and it is astonishing what a quantity of heavy material is so conveyed.

The people are exceedingly polite and obliging in the interior as well as in the coast cities. No foreigner is permitted to travel beyond treaty limits without a permit (passport); the treaty limits are thirty miles around Yokohama, and about the same at other ports.

Japan is divided into provinces and kens, with a Governor to each province and police magistrates in each ken. All travellers on demand have to produce their passports or permits; on refusal, are arrested by the police and escorted back to their place of residence, there to be brought before their consul.

A large variety of poultry is kept, and game is abundant. Fowls average about 9d. each; ducks, 1s.; geese, 3s.; turkeys. 8s.; pheasants, 1s.; woodcock, 1s.; snipe, 3d., &c.

Sheep do not thrive, the country being apparently too wet; all the mutton is imported from China. Cattle of a small size are plentiful, as also pigs. Good beef is 8½d. per lb.; mutton, 1s. 5d.; and pork, 10d. Vegetables are plentiful and cheap. Fish is abundant in considerable variety, very good, and reasonable in price.

The principal fruits are plums, several excellent varieties; the persimon (kaki) eaten fresh and dried like figs in large quantities, and of several varieties, a delicious fruit. Loquats, oranges, cumquats, and a coarse variety of lime. Inferior pears, peaches, and apricots—good small green flesh, and water melons. Inferior grapes; a good variety, but the climate is not sufficiently warm to thoroughly ripen them.

Agriculture is one of the largest industries, and suited to the peculiar features of the country as there prosecuted. The land is all surveyed each year, and the breadth of produce recorded, and a tax levied on each producer. The high land, where irrigation cannot be applied, is cropped with barley, wheat, millet, buckwheat, pulse, root and green crops, &c. There is a large variety of leguminosæ, especially beans, which form a favourite food. Buckwheat and barley are also largely grown, and used as flour in cakes; the horses are also fed upon steeped barley. Wheat is not largely cultivated.

Rice is the staple food, and the rice fields with the waving rice in ear when green, and also when changing colour, produce a fine effect, the whole valleys appearing as one level sheet of green or golden-yellow when ripe.

The rice is sown in small seed-beds, well worked, manured, and irrigated, on the 1st of May and few following days; the seed is sown broadcast very thickly upon the surface, and about one inch of water remains over the seed. From the end of May until the 5th June the paddy or rice fields are being prepared for the transplanting of the rice from the seed-beds.

The rice fields or plots are from a half to two or three acres in extent, thoroughly level, and surrounded by a bank of earth about 12 or 18 inches high and 12 inches wide on the top. All these plots are levelled by a water-level, a bamboo split in half and placed horizontally upon a vertical stake and filled with water; the bamboo must thus be quite horizontal or the water would run over the ends, where the bamboo staves are sighted. Throughout the fall or decline of the valley these plots are one lower than another, the water being admitted to the highest and passed from one plot to another by openings in the banks surrounding each plot.

These plots are usually dug or rather turned over by a heavy drag fork, which is struck into the soft ground by the husbandman and then pulled towards him, thus effectually turning over the surface of the rice plot to a depth of 12 inches; water is then admitted into the plot, and a horse draws a rake or harrow, which is pressed down from behind by the husbandman or lifted when clogged; a little rice husk or green weeds appear to be the only manure given at this stage. After thoroughly stirring and mixing the soil into mud, the rice plants are taken out in bunches from the seed-bed and transplanted singly by hand in rows or drills about 9 inches apart in the rice plots, and 2 inches of water is run into and kept over the surface of the plot. The transplanting begins about the 5th of June and ends about the 25th; the rice comes into ear in September, and is reaped in November and December, and laid upon the banks of the plots; afterwards carried to the side of the valley, and the straw drawn through an iron comb fixed upon a trestle. The grain being thus stripped from the straw, is conveyed to the farmer's store. The rice-straw is tied around the stems of the alder and other trees which surround the rice fields, and is used for fodder for horses, &c.

Liquid manure is sometimes applied to the rice, but as a rule the manure used for the previous crops is sufficient. Before the rice is reaped the plots are drained by allowing the water to flow away through the apertures which feed from plot to plot. As soon as the rice is cleared the ground is broken up, and a root crop, or barley, or buckwheat, or some other crop grown which can be removed in time for the next rice-planting, Barley is harvested before the middle of June. These crops are manured by liquid manure poured along the drills from a hand-ladle; this is the most important, as no other manure is used, and yet the same cultivation has gone on for centuries with a constant growth of rice year after year upon the same land. Japan is thus entirely self-supporting. All excreta or fæcal matter is carefully retained in tanks or earthenware jars, which are emptied once or twice a week by the agriculturists, who fetch it in deep wooden buckets and carry it across their shoulders for miles to their farms; it is also taken long distances in these buckets slung across a pack horse; also by barges along the canals. There are in many places municipal large tanks for receiving it, ready for water carriage.

The application to the plant is very important. It is carried to the farm, there stored in an open tank preserved from the rain by a thatched roof, but exposed to the atmosphere; fermentation at once takes place, the gases pass away, and it is then poured along the drills by the side of the growing crop and frequently upon it, which it does not injure, probably because fermentation in the atmosphere has taken place.

It is estimated that the excreta from eight adults keep an acre in the highest cultivation, producing at the rate per diem of one pound of grain or pulse and one and a half pounds of green vegetable. This with a little fish and eggs forms the food of the Japanese. In other words, it is estimated that eight adults live from the produce of one acre, and keep it in heart as above stated. To go minutely into this subject would make the paper too long, but it has been carefully calculated. In England the excreta from 800 to 1200 persons is used per acre without profitable result, as stated this session at the Institute of Civil Engineers of London.

The rice grain is husked or shelled in wooden mortars by a concave wooden pestle, a number of which are worked by a wooden shaft, fitted with wooden pegs forming cams, the shaft being driven by a waterwheel constructed entirely of wood. Stone-husked rice is not liked, the wooden pestles producing a high polish upon the kernel.

Many species of roots are eaten; the sweet potato (dioscorea batatas) most largely, and is very delicious when properly cooked. There are also two species of roots, one grown on dry ground and one in the rice fields; each of these have leaves like the arrow head or arum (calla); all these three, as well as the ordinary potato, are called imo.

The beautiful lotus (with its lovely, large, lily-like white or pink blossoms, and large deep green leaves, floating upon the water or waving in the wind) is considered a great delicacy. The root is boiled or steamed, and has a slightly sweet but most agreeable flavour.

Of the root crops grown on dry ground the giant radish (daicon) has the largest consumption, perhaps; it is eaten in every way—boiled fresh, dried and boiled, &c. It is coarse in flavour, in size it is about 24 inches long by 2 inches diameter. Carrots and leeks are largely grown; onions and turnips sparsely. The whole country is irrigated where possible; the irrigation is simple, perfect, and inexpensive.

The white mulberry is cultivated to a large extent, but chiefly in small patches by farmers whose families raise silkworms; a large amount of silk is produced from bombyx mori by cottagers. The bombyx of the oak (the yamamai) also produces a considerable quantity of coarse silk; in a wild state a silk is likewise obtained from the bombyx (which feeds upon the ailanthus as well as the oak), the cocoon of which is open like network. The silk is chiefly reeled by hand, but one establishment in Yedo reels by water-power.

The woven silks have not been equal to those of foreign production, and the Government have imported filateurs from France to improve the silk manufacture.

Paper-making is one of the arts developed to the greatest extent. The paper is said to be manufactured by cottagers and farmers from the bark of the mulberry (the inner bark being separated from the outer), macerated by boiling, and pounded into a pulp with rice-water and spread out in thin layers; the outer bark being made into a coarse paper. Several European paper-mills have been erected where the paper is made from rags, &c.; these mills produce good white paper. The Japanese paper is of yellow cast, but is extremely tough, and is used for waterproof coats, windows, umbrellas (parapluis), tobacco pouches, and a variety of other purposes, and last, not least, for pocket-handkerchiefs.

Very many of the birds are identical with those of Europe. The sparrow is seen everywhere in large quantities; and although pyrgita montana, the tree sparrow of Europe, it breeds almost entirely in houses, and has exactly the habit of the London sparrow; but the plumage of the female is similar to that of the male.

The hawfinch (cocco thraustes vulgaris), bullfinch (loxia pyrrhula), crossbill (loxia curvirostra), bramblefinch (fringilla montifringilla), redpole (Lynota linaria), siskin (carduelis spinus), greenfinch (cocco thraustes chloris), house swallow (which migrates, appearing again on 5th April), skylark, pippet-lark, long-tail titmouse, large tomtit, small tomtit, wren, golden-crested wren, jay, waxwing, nuthatch, &c., are the same as those of Europe, with English song and call—that is, the song and call are exactly similar to those of the same species in England. There are numerous others, such as the linnet, which differ from the European species, and very many which are not found in Europe. The birds of prey are, many of them, identical with those of Europe.

The reptiles appear to differ from those of Europe. There are several species of snakes which are very abundant, many of them frequenting the trees; all are harmless excepting the marmouchi, which closely resembles the adder of England.

The most wonderful reptile is the Sieboldia maxima, a large animal about four feet in length, very robust, and nearly black, with four legs and flattened tail, resembling in character the water eft or newt; it is found in the rivers, and is harmless. Baron Siebold had a fine live specimen, which required two persons to lift it from its bath; it appeared to be sluggish in its movements.

The insects are perhaps the most interesting to the naturalist, especially the Lepidoptera, as so many are identical with those of Europe. Referring to a few of the papilionidæ, or butterflies, the following are identical with those of England:—Papilio machaon, pieris rapæ, pieris napi, leptoria, candida, gonepteryx rhamni, colias hyale, argynnis paphia, argynnis aglaia, argynnis adippe, vanessa io, vanessa antiopa, vanessa polychloros, vanessa cardui, limenitis sybilla, lycœna, phlœas, polyommatus argiolus. These are English species, but the butterflies generally in Japan are very numerous and lovely.

The following are some of the moths identical with those of England:—Smerinthus ocellatus, acherontia atropos (considered a different species in England, and named acherontia styx, but the larvæ and imago appear to be identical), sphinx convolvuli, chœrocampa elpœnor, macroglossa stellatarum, clisiocampa neustria; dendrolimus pini is abundant, but whether identical is doubtful; gastropacha quercifolia, stanropus fagi, clostera curtulæ, cerura furcula, cerura binula, porthetria dispar, psilura monacha, porthesia chrysorrhœa, porthesia auriflua, spilosoma menthastri, spilosoma lubricepeda, spilosoma urticæ, spilosoma salicis, arctia caja, enthemonia rusula, miltochrysta miniata, lithosia complana, lithosia quadra.

NOCTUIDÆ.

Several of lytœa, or rustics, as also most of the agrotis; segetum, and others; many of the graphiphora, orthosia, mythimna, segetia, caradrina, grammesia, glæa, amphipyra, lemuris, calocampa, xylophasia, hadena, euplexia, mamestria, Thyatira, scoliopteryx, acronycta, ceratopacha, cosmia; most of the xanthia, orbona, and gortyna, phlogophora, cuculia, plusia, heliothis, ophiusa, mormo, and catocala.

To go through the thin body moths would occupy too much time; but the larger number of English species are found in Japan.

In enumerating the above it must be understood that the numerous species omitted because not identical with those of England are far more beautiful than those mentioned. The papilio, or swallow-tail butterflies; the apatura, or Emperor; the thecla, or hair-streak; the parnassus, or Apollo, &c., are very grand. Also the large family of sphingidæ, particularly the clear wings or sesia, which are magnificent; and the species of catocala are lovely beyond description.

The humble bees are numerous; several species identical with those of England. Also the hornet, which is abundant; of this there are two or three species, one identical. The wasps differ; all have their nests on trees or some other dry place, the ground being too wet. It is curious to see the nests in rose bushes, &c., slung from a bough; and although they are very numerous in species and in quantity they are not troublesome. The coleoptera are very fine, with many new species.

In referring particularly to the very many species identical with those of England it is remarkable, because Japan consists of a series of islands so very distant and isolated from England, and goes far to disprove Darwin's theory that the farther species are from species—that is, the more they are diffused by distance—the more they must differ, having to struggle for existence over so great a space.

This paper must be received as a series of notes, not as a carefully written paper, as it has been written hurriedly; but it is hoped that there will be some matter which may prove interesting, as the whole may be relied upon as facts gathered by actual observation, although even then slight errors creep in.

F. C. CHRISTY.

5th September, 1877.

EARTHQUAKES OBSERVED BETWEEN THE 1ST JANUARY AND 17TH OCTOBER, 1876.

January 20th, 8.40 p.m.—Very severe vertical shocks; threw the wine out of champagne glasses, which were only half full; commenced by slight shock, immediately followed by severe shock, which lasted about three seconds, unaccompanied by noise; fine calm night, rained next day.

January 29th, 4 a.m.—Severe oscillating shocks; snowing all day, 15 inches deep on ground.

January 11th, 5.40 p.m.—Two very severe shocks, one immediately after the other. Whilst walking on the grass plot in front of dwelling the earth undulated from 1 to 3 inches; the trees rose and rocked as the wave rolled along; the wave appeared to travel from west to east. Second shock very severe, oscillating and trembling motion, causing the house to shake as though the tiles and windows would be thrown out of their places; no noise, excepting from the shaking of the house, which was so alarming that it was thought advisable to keep at a distance from it. The house is large, two story, heavily framed in timber, faced and roofed with tiles; the evening lovely and calm, with clear sky. First shock lasted 2 to 3 seconds, 2 to 3 seconds interval, then second shock lasting 3 to 4 seconds. During the day, which was unusually warm, a depressing sensation was observed.

February 13.—Three shocks during night; snowed all day.

February 26th, about 9 p.m.—Slight shock; day fine and warm.

March 9th, 12.10 (noon).—Sharp shock.

March 13th, at night, 12.20 a.m.—Moderate shock; gale sprung up, which lasted from 2 a.m. till 11 a.m., with rain; night very warm.

March 31st, 7.40 p.m.—Long, but not severe oscillating shock, apparently from west to east; lasted several seconds; weather calm.

April 11th, 2.25 a.m.—Slight shock; two seconds after, a severe shock. 4 a.m.—Slight shock.

April 12th, 7.10 a.m.—Severe shock.

April 17th, 6.30 p.m.—Sharp shock; day very fine.

April 21st, 5.30 a.m.—Slight shock.

April 25th, 5 a.m.—Slight shock. 1.58 p.m.—Severe and long shock. Day fine.

April 27th, 5 a.m.—Slight shock; strong wind.

May 3rd, 9.50 a.m.—Sharp shock; squall came up with rain.

May 7th, 9.30.—Sharp shock, lasted several seconds; rained all day.

May 21st, 10.20. a.m.—Slight shock; day fine.

May 24th, 9.30. a.m.—Slight shock; day fine, overcast in afternoon, rain at night.

June 25th, 6.15 p.m.—Very severe and long shocks; day cloudy and cold, with wind.

July 16th, 10 a.m.—Slight shock.

July 30th, 10.5 a.m.—Very severe undulating shock; day fine, very warm.

August 5th.—Slight shock; day fine, very warm.

August 20th, 4.30 p.m.—Slight shock; heavy thunderstorm, with vivid lightning.

August 24th, at night.—Slight shock; sultry, with rain.

August 27th, 2 a.m.—Slight shock, rained heavily. 9.10 p.m.—Slight shock, oscillating, lasted several seconds; sultry and overcast.

September 14th, 5 p.m.—Sharp shock.

October 16th, 6.30 a.m.—Slight shock; day fine and calm.

October 17th, 3 a.m.—Two severe shocks, and one slight one.

ART. XII.—*On the Probability that a Connexion of Causation will be shown to exist between the Attraction of Gravitation and the Molecular Energy of Matter.*

BY ALEXANDER SUTHERLAND, M.A.

[Read on the 13th Sept., 1877.]

IN his recent paper on "Force" Mr. Pirani asks what is meant when we say that one portion of matter attracts another. Is it to be supposed that just as a conscious being exerts a force upon an external object, so does one inanimate body exert a force upon another? To this notion he takes exception, and, as I conceive, with justice. For the idea that that which is itself devoid of energy should have the power of imparting energy to another body is opposed to all our intuitive beliefs.

Yet the fact remains, that when two bodies are placed in space at a distance from each other, and left to themselves, each begins to set the other in motion—that is, each imparts to the other a certain amount of kinetic energy.

Here we have a difficulty: on the one hand it is inconceivable that inanimate bodies should have the power of doing work, on the other there is every reason to believe that two portions of matter can do work upon one another. But in this connexion is not the word inanimate altogether misapplied? Now that we know all matter to be replete with energy, would it not be more correct to regard it as in certain respects animate? Seeing that it is possessed of energy, it must be possessed of the power of doing work, and if we could establish a connection between this internal molecular energy of matter and its power of doing work upon other matter, we should at once remove this inconsistency.

Our proposition would then be that two portions of matter animated by vast internal energies which are similar in all respects to the energies of animals, except that they are not accompanied by consciousness, have by virtue of this internal energy the power of doing work.

I desire in this paper to inquire how far we should be justified in thus seeking in the known molecular energy of matter the attractive power which this matter certainly possesses.

If there be two bodies at a certain distance from one another, each is found after a certain time to be possessed of kinetic energy, which was not previously in existence; and we have to inquire from what source this energy has been derived.

In accordance with the principle of the conservation of energy, the reply must be that it has sprung from some antecedent energy; for if the sum total of energy in the universe be constant then energy cannot be created, and cannot be produced from something which is not energy.

Now let us ask—What is the pre-existent energy from which the energy of these attracting bodies has been derived?

We must carefully avoid being misled by the use of such a term as "Potential Energy;" for in referring the energy whose origin we seek to what is called potential energy, we should at once beg the whole question. When Professor Rankine invented this term, he never intended that it should be used to represent any real form of energy. It is an analytical artifice of great use, but merely representing the potentiality as distinguished from the actual existence of energy. It is a condensed statement of the fact that if a body be left to itself it will after a certain time have acquired a certain amount of energy. But the question we propose is still untouched—From what source has this derived energy been obtained?

We have to decide in what direction we may, with most hope of success, seek this unknown source. Is it external to the attracting bodies, or is it internal? In other words, when two portions of matter in space begin to move towards one another, is this motion due to external energies *driving* them together, or to the internal energies of matter itself tending to *draw* the two portions together?

There can be little doubt that the latter is by far the more promising direction of inquiry. For we know that all matter is possessed of eternal energy, whose amount is far more than sufficient to explain all the known effects of gravitation. Each atom is for ever in motion, and therefore fraught with its own store of kinetic energy due to this motion, the gross amount of these molecular energies being far beyond any force to which living beings can pretend.

On the other hand, to refer the energy arising from gravitation to energies external to the bodies themselves, is in every way unsatisfactory. Newton at first declined to speculate on this subject, declaring that there was no known energy external to the bodies to which their resulting energy could be attributed. Pressed by the importunities of his friends, he formed a theory of the causation of gravitation, referring it to supposed external agencies; but he attaches no value to his speculations, as they are based on the utterly unscientific method of explaining the existence of a known effect, by assuming the existence of an imaginary cause invented for the sole purpose of explaining that effect.

The same objection is open to the theories of Lesage and Mossotti. *If* we allow to Lesage that the universe is filled with extra-mundane particles, moving at high velocities and impinging on all bodies, and *if* we allow that these bodies have a cage-like structure, then gravitation may be partly explained; but an hypothesis which calmly assumes two important propositions, for the purpose of partially explaining a third, introduces more difficulties than it removes.

In the same way Mossotti requires us to allow, first, that all particles of matter repel one another, which is a gratuitous assumption; secondly, that all particles of intervening ether repel one another, which is a second gratuitous assumption; thirdly, that particles of ether and particles of matter attract one another, a third assumption, with this special objection, that it assumes the whole question, when it speaks of attraction between particles. Here, then, we have three assumptions for the purpose of explaining a single fact. The mathematical part of the work is handled in a masterly way; but just as an equation is not solved, if we introduce unknown quantities and allow them to remain in our final result, so if we introduce assumed facts in explaining a known fact, there is in effect no explanation given.

But the internal energy of matter due to the motion of its molecules, is at present a well-established fact, and is free from the objection of being an hypothetical existence assumed for the purpose of explaining a known fact.

The case then may be stated thus: When two bodies are placed near one another and left to themselves, each acquires a certain energy. This must have been derived from some antecedent energy; but the only antecedent energy known to exist is that due to molecular motion. Hence we shall be justified in turning our investigations, whether experimental or mathematical, in that direction.

This is an explanation which has not been possible until within late years. Newton never dreamt that what we call inanimate matter is in reality animated by vast energies; had he known this fact he would perhaps not have regarded it as an absurdity that two such bodies should exert forces upon one another.

That gravitation is due to molecular energy is also the result of the following consideration drawn from the analogy between gravitation and the forces of magnetism and electricity. These three forces are the only known forms of attraction at sensible distances. They differ among themselves in many respects, but they are, in their main features, so similar as to form a class very distinctly marked off from all other existences. Now it is certain that magnetism and electricity are caused in some unknown manner by the energy of material molecules. But when the forms of energy are absent to which these two kinds of attraction are peculiarly due, the portions of matter in question are still endued with the other forms of molecular motion, and are still found to possess a power of attraction similar to, though much less intense than, the other attractions. Is there not a large measure of probability in the belief that also in the case of this universal form of attraction, the force is due to the universal form of molecular energy?

A more definite idea of what is meant will perhaps be obtained in this way:—When an electro-magnet attracts a piece of iron in front of it the following action goes on:—Molecular vibrations are originated in the battery and pass into the core of the magnet. From this core they are propagated out into space in the form of waves, and, in some undetermined way, the molecular energy of these waves is converted into the kinetic energy of the piece of iron. So in the case of a permanent steel magnet, it has been shown by Clerk-Maxwell, Verdet, De La Rive, and Wertheim, that the attractive force is due to the molecular state both of the attracting and the attracted body.

Now, take the case of a steel magnet which has been heated and allowed to cool. It has lost its special molecular energy, and its special attractive force; but it now possesses the ordinary form of energy common to all matter, and likewise possesses the ordinary form of attraction common to all matter. Since, then, in its former state, its attractive power is known to be due to the energy of its atoms, there is a strong presumption, in the absence of any other explanation, that the attraction and the molecular state in the second condition, are causally connected.

The following, therefore, is the theory to which the facts point:—When two bodies are placed near one another, the internal energy with which each is actuated is radiated into space, but such of it as is intercepted by the other is converted into kinetic energy in a manner analogous to that in which the molecular vibrations of an electro-magnet radiate and produce kinetic energy in the attracted iron.

If this theory could be shown to be true, it would explain certain facts which seem otherwise to be inexplicable.

For instance, suppose a mass of iron at the surface of the earth to weigh one ton; then, if it were to be carried fifteen hundred miles upwards from the surface, it would weigh only half a ton. Now, what would become of the lost weight? Faraday spent some months in trying to discover if weight lost in this manner is turned into electricity; but his experiments gave no hopeful result. No other explanation has been given of this apparent disappearance of something from existence. But it is possible, that though this particular mass of iron has lost something, yet that something has, nevertheless, not been lost from existence. And this is the result our proposition would give. For if we imagine a body in a certain position to receive a certain amount of the molecular waves proceeding from another body, then, when removed to twice the distance, it would receive only one-fourth the amount it previously received. The remaining three-fourths would be lost to this particular body, but would not be lost from existence—it would travel out into space;

and though it became attenuated the further it spread, yet it would as truly conform to the law of the conservation of energy as light does when not intercepted, but allowed to radiate into space. Thus, though our ton of iron loses half its weight, the loss could be easily accounted for without supposing the annihilation of anything.

Again, it is known that all space is filled by a medium which is capable of conveying molecular vibrations; that it conveys the motions of heat and light is certain; that it likewise conveys the motions which constitute magnetism and electricity was the belief of Faraday, and is now held by Thomson, Tait, Maxwell, and others who have written on the subject.

Now Newton demands a medium for the conveyance of the effects of gravitation. In his letter to Bentley, he says—"That one body may act upon another at a distance, through a vacuum, without the medium of anything else by and through which their action and force may be conveyed from one to another, is to me so great an absurdity, that I believe no man who has in philosophical matters a competent faculty of thinking can fall into it."

This assertion has been severely criticised. Still the reasoning on which Newton bases it is sound, and it is now generally held to be justifiable.

Now since the ether which is known to fill space has the power of conveying molecular vibrations, this fact tallies very well with the supposition that gravitation is itself due to waves of molecular vibration.

Our supposed origin of gravitation satisfies sufficiently well the necessary condition of supplying an explanation of the known laws to which gravitation is subject.

First, the attraction which a body exerts is proportional to the amount of matter it contains. This is consistent with our supposition. For it has of late years been conclusively shown that matter is simply a name for a collection of such energies as are capable of making an impression on the senses. Thus the qualities of a body are dependent on the amount it contains of the various forms of molecular energy; and its mass must depend upon the amount it possesses of some constant form of energy. Hence if we suppose that gravitation is proportional to this form of energy, it necessarily follows that gravitation is proportional to the amount of matter in the body.

The second law has a greater significance than this. The attraction of one body on another varies inversely as the square of the distance between them. If r be the distance between the two bodies, then one of the factors of the expression for their attraction is r^2. Now r is a surface quantity, and if gravitation were a simple force acting in a simple straight line from a particle of one body to a particle of the other, then it would be difficult to conceive of any explanation for the entrance of such a factor.

But in the case of magnetic attraction, or of any other form of radiation, we can see easily enough the origin of this term. For in all cases of waves propagated from a centre, the square of the distance naturally enters. As the wave moves forward, it expands equally in two directions, and the expansion in each direction being proportional to the distance traversed, the intensity of the wave is lessened in proportion to the square of the distance traversed. Hence the inverse square is the law for light, heat, magnetism, and electricity. If we find the same law in the case likewise of gravitation, it strengthens to a certain extent the supposition that the internal energy of matter is radiated through space in spherical waves which obey the ordinary law of such waves, and decrease in intensity in proportion to the squares of distances they have travelled.

In conclusion, it may be observed that of all the possible explanations that could be given of gravitation, the simplest and most likely is that the power of attracting lies in the mass of matter itself; and if we ask what it is in matter that gives it this power, we can scarcely have any other answer than that it is some form of energy due to the motion of the constituent molecules. It certainly would be a step in the establishment of that conformity of nature, to which all science tends, if it could be shown for gravitation, as it has recently been shown for electricity and magnetism—that it is the effect of molecular vibrations propagated through the same omnipresent medium which conveys the vibrations of light, heat, and actinism. Of course, no real advance will be made in such a theory until, by fresh experiments, or by mathematical investigations, founded on previous experiments, something like a reasonable explanation shall be given for the nature of the connection that binds the two together; till we shall be able to say how it is that a molecular disturbance propagated from one body is converted into an attractive force upon the other. And yet the present theories of electricity and magnetism are in the same state. It is simply known that they are the result of molecular waves, but the nature of the transformation is as yet a mystery. Clerk-Maxwell has given in six papers in the *Philosophical Magazine* for 1861-1862 a provisional theory for magnetism; but there has been no great advance made in this direction. That the full connection will ere long be discovered, is almost certain; and in the meantime it will not be without its purpose to point out that in the course of time it will, in all probability, be necessary to extend the same investigation to gravitation.

ART. XIII.—*Experiments on the Comparative Power of some Disinfectants.*

By James Jamieson, M.D.

[Read on the 11th October, 1877.]

The object of the present communication is to record the results of a series of experiments on the comparative power of certain disinfectants when applied in the form of vapour. While this department of the subject has undoubtedly great practical importance in many ways, it has been comparatively little cultivated, due no doubt, in some measure at least, to the difficulty which attends any attempt to carry out such investigations in an exact way. It so happens, therefore, that our knowledge on the subject of aërial disinfection is made up mainly of vague impressions, which may perhaps be tolerably correct, but which are greatly in need of a basis of well-established facts and scientific investigations.

It would be out of place for me to enter at any length on the general question of the nature of those remarkable processes included under the terms putrefaction and fermentation; but it is necessary to state the opinion I hold on the subject, which is that now generally accepted by men of science. It may be said, then, that putrefaction, fermentation, and other allied processes are in their essential nature chemical changes brought about in organic matters by the functional activity of minute vegetable organisms; these changes being of a destructive character, consisting in the reduction of complex substances into simpler ones. Certain phenomena which specially obtrude themselves on our notice, such as the formation of disagreeably smelling matters in putrefaction, and the copious evolution of gas in ordinary alcoholic fermentation, are mere accidents. Among the allied processes referred to must be ranked, I think, the changes going on in the animal economy in the course of certain acute diseases, which, from their apparent analogy with the phenomena of fermentation, have been long named zymotic. The investigations of some of the best pathologists of our own day have supplied evidence of a positive kind in favour of that theory; and with reference to a few of the acute contagious diseases there is, I think, satisfactory proof that they owe their origin to microscopic organisms belonging to the lowest order of plants. The doctrine of the parasitic nature of the ordinary epidemic diseases, founded partly on the analogy already mentioned, and more recently on the results of exact observation and experiment, has received a further confirmation from the beneficial results following the use of well-known disinfectants having a parisiticidal action in the cure, and still more in the prevention, of some diseases of the kind now under consideration. To prove the action of disinfectants in preventing or checking putrefaction in substances liable to it is easy; but when we have to deal with the living animal the matter becomes much more complicated, and hence perhaps the want of demonstrative force in the evidence adduced in favour of the action of disinfectants as preventive and curative agents in disease. An important step has been recently made by subjecting the virus or contagious matter of some diseases, such as glanders and vaccinia, to the action of disinfecting agents, and then testing its power of communicating the disease by inoculation. Such investigations have been carried on by Dr. Dougall, of Glasgow, and in a more thorough way by Dr. Baxter, whose experiments are fully described in the Reports of the Medical Officer of the Privy Council for the year 1875. It is there clearly shown that the ordinary disinfectants—carbolic acid, sulphurous acid, and chlorine—destroy the contagious property of the vaccine and glanders viruses when applied to them in the same manner and in the same strength as is found sufficient to destroy the organisms causing putrefaction, and thus to put a check to that process. The chain of evidence, therefore, seems very complete in favour of these two points—(1) that certain acute contagious diseases are caused by the introduction into, and multiplication in, the animal body of minute vegetable organisms; and (2) that it is possible to destroy the contagious power of the virus by means of disinfecting agents, and so prevent the spread of these diseases. There may be room for difference of opinion as to what diseases can be included in this class; but there has been almost absolute demonstration supplied of the correctness of one or both of these points with regard to certain, and among these are to be reckoned especially anthrax, glanders, remittent fever, diphtheria, and vaccinia. When the virus has taken root in the body, it is very questionable if we can do anything to stay its progress. This is owing to the fact that we cannot introduce these parasiticidal agents into the animal system, in amount sufficient to destroy the morbific organisms without at the same time doing irreparable injury to the delicate structures of which it is built up. But whilst we have thus to confess our impotence in the present state of our knowledge, and with the agents now at our disposal, I for one cherish the hope that the chemist, by means of the synthetical method of forming new compounds, will yet offer us some agent capable of doing all that is required. That salicylic acid has not done more to supply the want must have been to many, as it was to me, a grievous disappointment.

We are thrown back therefore on prevention as the great field of our activity in this department of practical medicine; and there we may with confidence look forward to triumphs greater far than have been already attained, considerable as these are.

As epidemic diseases generally spread by means of some virus, which has been formed in the body of animals suffering from them, and is conveyed in some way from these diseased animals to healthy ones, it is clear that if we could destroy with certainty all contagious matters as they leave the body the work of prevention would be done. That it is possible to destroy the viruses of all contagious diseases by mixing them with a sufficient amount of some disinfectant may be regarded as almost certain, since it has been actually done in the case of several of them. Unfortunately, we cannot always obtain the virus in substance, so as to operate upon it in that way; and we are compelled, therefore, to consider the possibility of attacking it when suspended in the atmosphere, or attached to walls or other surfaces in a dried state. That some diseases are conveyed by means of a dried contagium floating in the air seems to be certain, and therefore in the prevention of many diseases—such as scarlet fever, measles, small-pox—we have to face the problem of aërial disinfection, with all its difficulties. The only experiments made to test the effect of disinfectants, in the form of vapours, on a dried animal contagium, which I have seen detailed, are those on vaccine virus by Drs. Dougall and Baxter. The general result of these was to show that concentrated vapours destroyed the contagious quality of the virus when they operated for a sufficient length of time, just as the same agents in substance robbed fresh liquid vaccine of its power of communicating vaccinia. One other point is necessary again to adduce, and that is that the septic microzymes so abundantly found in ordinary processes of putrefaction are destroyed by the same agents used in nearly the same strength. These preliminary statements have now brought me to the ground and reason of my own experiments. Some of the animal contagia, as those of scarlet fever, measles, and some others, are almost unknown to us as objects of direct observation; but we have every reason to assume that they are subject to similar vital conditions with those which have been made the subjects of experiment, and therefore will have their virulence annulled by agents which act in that way, either on septic microzymes or on vaccine virus. My experiments have been made with these septic microzymes, which are always attainable, and whose death or continued existence can be proved with greater certainty than is possible in the case of the animal contagia by the method of inoculation, which is always liable to some fallacies. It is known that bacteria of different sorts, and especially these septic organisms, can live and multiply in a perfectly clear solution of certain saline matters, and the mixture known as Cohn's solution is admirably adapted for their cultivation. I used a slight modification of that originally recommended by Professor Cohn, composed of the following ingredients :—

Tartrate of ammonia	2
Sulphate of magnesia	1
Acid phosphate of potash	1
Chloride of calcium	$\frac{1}{10}$
Distilled water	200

When this solution is boiled and preserved from any contamination it remains clear for an indefinite time; but if the smallest portion of any substance containing the septic organisms, called by botanists the *bacterium termo*, is added, it gradually becomes milky, the rapidity with which this occurs varying with the temperature at which the fluid is kept. The mode of procedure which I adopted was as follows:—I obtained a supply of the bacteria by adding a few crushed peas to warm water and leaving the mixture till it emitted a putrid smell, when it was found on microscopic examination to be swarming with these and other organisms. Then, to obtain them free from admixtures, I inoculated a portion of Cohn's solution with a minute drop of this putrid fluid, with the result that in less than two days the previously limpid solution had become quite opalescent. The bottle in which it was contained was shaken up, so as to obtain a uniform mixture, and a piece of filter-paper soaked with this, and then carefully dried in the sun for several hours. This bacterialised paper was preserved between the leaves of a book, and small portions of it used as required. To guard against fallacies I used the following precautions:—A number of small phials were taken, containing each about a fluid dram of Cohn's solution, and after being carefully plugged with baked cotton wadding, they were kept immersed in boiling water for a few minutes, so as to ensure the destruction of any bacteria which might by chance have obtained admission. After cooling, a portion of the bacterialised paper, which had been subjected to some disinfecting process, was put into one of them, the plug being removed for as short a time as possible. For the purpose of saving time a number of phials were thus charged and put aside in some protected place at the ordinary house temperature. As a check I placed beside them one phial to which nothing was added, and another into which a piece of the bacterialised paper, pure and simple, of the same size as the others, was put. If the phial containing only boiled Cohn's solution remained clear, this was a proof that there had been no accidental contamination, while if the one to which paper not disinfected had been put became opalescent, it was evident that the bacteria in it were alive (in the sense that a dried seed is alive) at the time the experiment was carried on. No experiment was held to be satisfactory unless both of these tests were fulfilled.

The endeavour was made to apply the disinfecting process in such a way as to allow of the results attained being made a guide in the practical use of these agents in every-day life; and in the use of vapours the time required for destroying the bacteria was the point investigated, the concentration being that which could be attained by the usual simple methods.

I.—EXPERIMENTS WITH CARBOLIC ACID.

A wide-mouthed 8-oz. bottle was used, about a dram of crystallised carbolic acid being put into the bottom of it. The pieces of paper were suspended from a hook on the under side of the cork, which was fixed tightly in, and the whole left at the ordinary temperature of the atmosphere for carefully noted periods. A good deal of time was lost in feeling my way, in the absence of any knowledge at the time of similar observations.

(1.) Two pieces of the paper were exposed to the carbolic vapour for 9 hours and then introduced into the solution. In both cases opalescence began to appear in 42 hours, showing that the bacteria had not been destroyed; though as the phial into which undisinfected paper had been put began to be coloured in 36 hours, it appeared as if some of them had been killed, or at least in some way paralysed.

(2.) Two pieces exposed to vapour for 19 hours. Both remained clear.

(3.) One piece each 11 and 14 hours. Both remained clear.

Suspecting now that the air contained in the bottle had not had time to become saturated with the carbolic vapour in No. 1, which was begun as soon as the crystallised acid had been put into it, and in view of the positive effect in Nos. 2 and 3, I next tried some shorter periods.

(4.) One piece each exposed to the vapour for periods of 2¾, 3½, 5, and 7 hours. The first two became opalescent, whilst the others remained quite clear. This experiment I considered quite conclusive, as the opalescence began to appear in the following order:—With the undisinfected paper in 60 hours, with that exposed for 2¾ hours in three days, and with that for 3½ hours in 4 days. The longer time required in all than in Exp. No. 1 was due to the different temperature of the atmosphere, the first having been carried on in hot weather, and this in cold.

It follows then that with the fullest possible concentration of the carbolic vapour at ordinary temperatures an exposure of more than 3½ hours is necessary to ensure the destruction of the bacteria. As the conditions, in ordinary measures for disinfecting the air of a room by means of carbolic acid, can scarcely be made so favourable as in a closely-corked bottle, it must be evident that, as generally used, carbolic acid is useless for the purpose. To bring out this satisfactorily, however, I performed the following supplementary experiments.

(5.) A tin of carbolic powder was taken, and all the perforations in the lid opened. The powder was then shaken up and two pieces of the paper left suspended close above it, one for 10 and the other for 24 hours. The solution into which they were put became opalescent with both in 3 days.

(6.) Two pieces were sprinkled freely over with the carbolic powder, and left uncovered for 10 and 24 hours respectively. With both the solution remained perfectly clear after 14 days. The powder was therefore good and showed itself useful when applied in substance, but the result of the whole series is to show that leaving vessels containing carbolic acid or this carbolic powder in a room is useless as a measure for destroying contagion, and may indirectly be harmful by giving a false sense of security, and thus preventing the use of more efficient measures.

II.—Experiments with Sulphur Fumes.

(1.) One piece each exposed for 5 and 15 minutes to the fumes of sulphurous acid obtained by throwing sulphur on hot charcoal. The paper was suspended from a wooden box inverted over the vessel containing the charcoal pan, which was placed at the opposite corner. The box was not very close, and the fumes escaped freely. The solution containing the piece exposed for 5 minutes became cloudy in 60 hours; that with the 15 minutes piece remained transparent.

(2.) Two pieces again in a closer box, but without very copious evolution of fumes, one for 5 the other for 10 minutes.

Both caused the solution to become milky, though earlier by 12 hours with that exposed for only 5 minutes.

(3.) Two pieces for 3 and 10 minutes in a close-fitting box, the vapour being more copiously evolved. The 3 minutes piece became opaque in 60 hours, whilst the 10 minutes one remained quite transparent.

It follows from the whole series that whilst it is possible to destroy the dried microzymes by an exposure to sulphur fumes for 10 minutes, it can only be done under very favourable conditions. An exposure for 15 minutes, if at all thorough, will usually be sufficient.

III.—EXPERIMENTS WITH OZONIC ETHER.

These were carried on in a bottle in the same manner as with carbolic acid, about half a dram of the ether being put into the bottom of a wide-mouthed bottle of about 5-oz. capacity, the pieces of paper being suspended from a hook on the under side of the cork.

(1.) One piece each exposed to the vapour of ozonic ether for 10, 30, and 60 minutes. The 10 minutes piece caused opalescence in 4½ days, the same time as the bacterialised paper. The other pieces left the solution unaffected.

(2.) One piece each for 10, 15, and 20 minutes. The 10 minutes piece caused only a slight opalescence after 5½ days, the other pieces remaining transparent.

It is clear from these experiments that in ozonic ether we have a powerful disinfecting agent, from 10 to 15 minutes of full exposure being sufficient to destroy the dried microzymes, and presumably the specific contagia of zymotic diseases. It is true that the high price of the ozonic ether would preclude its free use on ordinary occasions. These experiments are the only ones with which I am acquainted, as carried out in an exact scientific manner, and they have considerable interest in their bearing on the external application of ozonic ether in the form of ointment, as recommended by Dr. Day, of Geelong, for the purpose of destroying the contagium, and thus checking the spread of scarlet fever. It is very possible that direct contact with any contagious particles will render them powerless; but in view of the time required with the most concentrated vapour attainable, it is scarcely possible that the amount escaping into the air in the course of the process of inunction can have any effect on dried particles of contagium, which may chance to be floating about, or resting on walls or other surfaces. On a small scale, and where the conditions approximate those of the experiments just detailed, the ozonic ether may therefore be used with advantage.

IV.—EXPERIMENT WITH CHLORINE.

The general impression in recent times is that chlorine does not deserve the great reputation it formerly enjoyed as a disinfectant, and, indeed, experiments have tended to show that when the gas is dry it has little or no power as a bleaching agent or as a parasiticide. I made one experiment in which the bacterialised paper was exposed, in a wooden box with a loosely-fitting lid, to the gas, evolved in the usual way by adding a few drops of muriatic acid to chloride of lime. The chloride of lime was rather damp, and a good deal of moisture was carried up with the gas. Three pieces of the paper were left suspended in the box for 1, 3, and 4½ hours respectively. The solution containing the 1 hour piece became milky in 4½ days, the other two remaining quite clear.

It appears then that, used in the manner described, chlorine, though not equal to sulphurous acid, is more powerful than carbolic acid. As ordinarily used, however, it can serve no good purpose, and sprinkling small quantities of chloride of lime on floors and other surfaces, in the hope of affecting any contagious matters floating in the air, must really be regarded as mere trifling.

V.—EXPERIMENTS WITH DRY HEAT OF 212° FAHR.

These may not have very much value; but as I have not met with similar ones, they may be given for what they are worth. In the absence of any more elaborate scientific armamentarium, I adopted the following procedure:—Two short, wide-mouthed bottles were carefully washed and then heated strongly in an oven, so as to ensure the removal of all moisture and the destruction of any organisms which they might by chance have contained. When still warm a piece of the bacterialised paper was put into each, and a good plug of baked cotton inserted into the mouth, which was further covered with a cap of the same material. They were then immersed in water, which was kept boiling for noted periods. The paper lying flat on the bottom of the bottles must have been exposed to a temperature nearly, if not quite, up to 212° Fahr.

(1.) One piece each for 10 and 30 minutes. The solution in both remained transparent, but I was somewhat doubtful of the trustworthiness of the result, as that which contained the unheated paper showed only a slight cloudiness after 4 days. This circumstance will be referred to again.

(2.) One piece each for 15 and 45 minutes. The solution with the 15 minutes piece became cloudy only in 4 days, the test bottle being opalescent at the end of 2½ days. The 45 minutes piece had no effect.

(3.) One piece each 15 and 25 minutes. Solution in both cases remained transparent after 12 days.

The conclusion come to, therefore, is that an exposure of these microzymes to a temperature of about 212° Fahr. must be continued for at least 15 or 20 minutes to ensure their destruction.

Two circumstances of considerable interest came out in the course of the investigation, which I have reserved for separate notice. The first was that when the bacterialised paper had been kept for between two and three months, the organisms seemed to have lost their power of reproduction. What the cause may have been I am not prepared to say, but that this happened was certain, and it caused a good deal of confusion and perplexity in my mind, till I suspected the state of matters and prepared a fresh stock, with which satisfactory results were at once obtained. The paper was kept between the leaves of a book, and was dry and exposed to very little rubbing. Could it have been that in course of time the desiccation of the bacteria became so complete as to be incompatible with continued vitality? Whatever the reason, it seems to follow that this particular species of bacterium cannot be kept in the dried state for very long periods without losing its vitality.

The other point is also, I think, of some interest, as showing the varying capacity of resistance offered to disinfecting processes by the germs of different low vegetable organisms. On a good many of the pieces of the paper which did not cause opalescence of the solution there appeared a copious growth of white mould, apparently the ordinary *penicillium*. The spores must have fallen on the paper at the time when it was exposed to the air, and they must have been subjected to the same destructive influences as the bacteria; and as they developed an abundant mycelial growth in several instances where the bacteria had undoubtedly been killed, it is evident that they possessed greater powers of resistance. In the detailed notes of my experiments I find that the mould appeared on paper which had been exposed to the vapour of carbolic acid for as long as 8 hours, a period of 3½ to 5 being sufficient for the destruction of the bacteria. On none of the pieces exposed to the fumes of burning sulphur was there any growth of mycelium. The ozonic vapour, again, though capable of destroying bacteria exposed to it for 10 or 15 minutes, apparently had not injured the spores of the fungus after 60 minutes. Again, whilst the chlorine killed the bacteria when applied for something over an hour, two pieces of paper, exposed to it for 3 and 4½ hours respectively, showed a copious growth of mould. Even to heat the *penicillium* spores showed greater power of resistance. Thus the mycelium appeared on each of the two pieces of paper which had been treated for 15 and 30 minutes respectively, the bacteria being killed in both instances. None appeared on the paper which had been treated for 45 minutes.

The conclusion to which I am brought, therefore, by the concurrent results of all these experiments is, that the spores of fungi are less easily destroyed than dried septic organisms, and presumably than dried contagium of zymotic diseases—as Dr. Baxter's experiments with dried vaccine showed its power of causing cow-pox to be destroyed by carbolic vapour in about 30 minutes, by sulphurous acid in 10 minutes, by chlorine in 30 minutes, and by a dry heat of 185° to 194° Fahr. for 26 minutes. He ventures to express the opinion —founded not on his own experiments, but on a few made by others on yeast and *penicillium*—that the influence of disinfectants on such fungoid spores affords no measure of their action on contagia, since the former are very much more susceptible to adverse influences than the latter. This opinion is directly contradicted by the results of the exact experiments here detailed, which show that any disinfectant which destroys *penicillium* spores in the dry state may be depended on to destroy bacteria, and so, presumably, contagia, which are even more easily destroyed, as a comparison of my observations with Dr. Baxter's on vaccine clearly shows.

ART. XIV.—*On Heat and Molecular Energy.*

BY H. S. PATCHING, ESQ.

[Read 8th November, 1877.]

ART. XV.—*On the History of Palæozoic Actinology in Australia.*

BY R. ETHERIDGE, JUN., F.G.S.

[Read 8th November, 1877.]

THE following condensed account of the study of the corals of the Australian palæozoic rocks may be found of service to those who may hereafter take up the systematic study of this group:—

In the course of the surveying voyage of H.M.S. "Beagle," under the command of Capt. Fitzroy, R.N., during the years 1832-36, Mr. Charles Darwin, F.R.S., naturalist to the expedition, collected two palæozoic corals in Tasmania. These were afterwards described by the celebrated actinologist, Mr. Lonsdale, in Darwin's *Geological Observations on Volcanic Islands,*[1] published in 1844, under the names of *Stenopora Tasmaniensis,* and *S. ovata.* The genus *Stenopora* was established expressly for these species in the work referred to, but was more fully defined in Count P. de Strzelecki's work, published during the next year (1845), *Physical Description of New South Wales and Van Diemen's Land.*[2] The full definition of the genus was accompanied by the description of two further species—*Stenopora crinita* and *S. informis*[3] —the former from New South Wales, the latter from Tasmania. In addition to the foregoing Mr. Lonsdale also described in Strzelecki's work another coral as *Amplexus arundinaceus,*[4] and mentioned the occurrence in the limestones of Yass Plains, New South

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[1] London, 1844 ; 8vo ; Appendix, pp. 161-163.
[2] London, 1845 ; 8vo ; p. 262.
[3] *Ibid*, pp. 264-65 ; pl. 8, fig. 5.
[4] *Ibid*, p. 267.

Wales, of a species allied to, if not identical with, *Favosites Gothlandica* (Fougt.).

In the *Annals of Natural History* for 1847 Professor (then Mr.) M'Coy published his celebrated paper "On the Fauna and Flora of the Rocks associated with the Coal of New South Wales," in which he gave numerous localities for Mr. Lonsdale's species, and in addition described as new, two more—*Cladochonus tenuicollis*, and *Strombodes Australis.*[1]

In the account of one of Ludwig Leichhardt's explorations —*Journal of an Overland Expedition*, &c.—the Rev. W. B. Clarke described a coral found by Leichhardt in the Burdekin River limestone (Queensland), about lat. 19° 58′ 11″ S. under the name of *Cyathophyllum Leichhardti.*[2]

During the years 1838-42 the United States Government organised the well-known exploring expedition under Captain Wilkes, U.S.N. The scientific results of this voyage were published in a series of magnificent volumes, the description of the recent corals, fossils, and geological notes being, amongst other things, undertaken by Professor Dana. In the Appendix to the volume devoted to geology[3] a large series of fossils from the palæozoic rocks of New South Wales are described, including references to some of the previously-mentioned corals. Lonsdale's species of *Stenopora* are referred to the genus *Chætetes* (Fischer), and a new species was described as *C. gracilis.*[4]

A paper by the Rev. W. B. Clarke, published in 1848, "On the Genera and Distribution of Plants in the Carboniferous System of New South Wales,"[5] contains the record of a "*corallite*" from the Newcastle coalfield, named by Leichhardt *Corallites Wiltoni.* I here quite failed to find any further reference to this species—in fact, I do not think anything further is known about it.

The importance of Professor M'Coy's paper on the New South Wales fossils forwarded by the Rev. W. B. Clarke to the late Professor Sedgwick, and which originally appeared in the "Annals" as previously noticed, was evinced

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[1] *Annals Nat. Hist*, 1847, XX., p. 227, pl. 11, figs. 8 and 9.

[2] London, 1847; 8vo; p. 212.

[3] *U.S. Exploring Exped.; Geology*, by J. D. Dana, New York; 1 vol. 4to, Atlas, 1 vol. folio.

[4] Pp. 711-712; Atlas, t. 11.

[5] *Quest. Jour. Geol. Soc.*, IV., p. 62.

by the republication of it in the *Proceedings of the Royal Society of Tasmania* for 1851,[1] with *fac-similes* of the plates. A useful and analytical review of what had been done for the palæozoic corals of Australia up to the time of publication of their work (1851) was accomplished by Professor Milne-Edwards and M. Jules Haime, in the "*Monographie des Polypiers Fossiles des Terrains Palæzoiques.*"[2] These authors consider the coral doubtfully regarded by Lonsdale as *Favosites Gothlandica* (Fougt), to be *F. Goldfussi* (D'Orbigny). They follow Professor Dana in placing *Stenopora crinita* (Lonsdale) in the genus *Chœtetes* (Fischer), and make the same reference but more doubtfully in the case of the other Australian species, *S. ovata* (Lonsdale), *S. informis* (Lonsdale), and *S. Tasmaniensis* (Lonsdale). Another coral described by Lonsdale in Strzelecki's work on New South Wales as *Amplexus arundinaceus*, is considered by Edwards and Haime to be indeterminable. They lastly remark on *Cladochonus tenuicollis* (M'Coy), that it is probably a young *Syringopora*.

The Rev. W. B. Clarke published a list of the Palæozoic fossils of New South Wales, in 1860, as an appendix to his "*Southern Goldfields of New South Wales.*"[3] Several genera and a few species of corals are cited.

In the third volume of the "*Histoire Naturelle des Coralliaires,*"[4] Milne-Edwards expresses much the same opinions on the foregoing Australian fossils to those enunciated mutually by himself and Jules Haime in their "*Polypier Fossiles.*"

Of the peculiar and problematical genus *Pleurodictyum*, Professor M'Coy has recorded the occurrence of a new species in the upper Silurian rocks of the Upper Yarra district, Victoria,[5] and has named it *P. megastomum*. No description of these species, so far as I know, has as yet appeared, the fact being merely mentioned in his paper.

In his "*Mémoires des Paléontologie,*"[6] my friend Professor L. G. De Koninck has given a valuable general list of palæozoic corals, arranged in tabular form, showing their distribu-

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[1] Vol. 1, pp. 313-314.

[2] *Extrait du tome V. des Archives du Museum d'Histoire Naturelle;* Paris; 4to; pp. 235, 273-74, and 347.

[3] Sydney, 1860; 8vo; 3rd edition, pp. 285-86.

[4] Paris, 1860; 8vo.

[5] *Annals Nat. Hist.*, 1867, XX., p. 261; Note.

[6] Bruxelles; 8vo.; 1857-71, pp. 78-82.

tion. A column is set apart for Indian and Australian species; unfortunately, however, no distinction is made between them. I can only recognize three as distinctly Australian, viz:—

Amplexus arundinaceus, *Lonsdale.*
Chætetes crinitus, *Lonsdale.*
Cladochonus tenuicollis, *M'Coy.*

Certain small areas of the Gippsland district were considered by Mr. A. R. C. Selwyn, F.R.S., to be, probably, of Devonian age.[1] Through the researches of Mr. Alfred Howitt, these localities have been well searched for organic remains, with the result, Mr. R. Brough Smyth informs us, of the discovery of forms which Professor M'Coy considers conclusive on this point. In addition to the *Spirifera lœvicosta (Val.),* and the remains of Placodermatous fish mentioned by Mr. Selwyn, Mr. Smyth now adds *corals,* an assemblage of forms which would indicate a close relation with the Devonian limestone of the Eifel.[2]

The "Nouvelle Recherches sur les Animaux Fossiles," &c.,[3] of Professor De Koninck, contain an allusion to some Australian species which may be noticed in passing. The *Stenopora Tasmaniensis* (Lonsdale) and *S. ovata* (Lonsdale) are placed as synonyms of the European form *Chœtetes tumidus* (Phillips), but under the name of *Monticulipora tumida* (Phillips). Professor De Koninck, following M'Coy, uses the name *Cladochonus* in a generic sense, and does not appear to participate in the opinion of Milne-Edwards and Haime, that most of the species described under that name are young *Syringoporœ.*[4]

The list of Victorian Fossils drawn up by Professor M'Coy forming a portion of Mr. R. B. Smythe's "First Progress Report"[5] contain the following summary from the upper silurian rocks of Victoria:—

Favosites, two new species.
Pleurodictyum megastomum, *M'Coy (m.s.).*
Stenopora, two new species.
Palæopora, two new species.
Petraia, two new species.

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[1] *Physical Geography, Geol. and Miner. of Victoria; Exhib. Essays,* 1866, p. 10.

[2] *Mining and Mineral Statistics; Exhib. Essays,* 1872, p. 40.

[3] Bruxelles; 4to; 1872.

[4] Pp. 143 and 152.

[5] *Progress Report* for 1874; *Geol. Survey of Victoria,* p. 34.

One of the most important contributions which has been made to Australian palæontology of late years is Professor Koninck's "*Recherches sur les Fossiles Paléozoïques de la Nouvelle Galles du Sud,*"[1] in which we have a most interesting and instructive account of the fossils collected by the Rev. W. B. Clarke, M.A., F.R.S., during his many wanderings amongst the fossiliferous rocks of New South Wales. The fossils in question, as determined by Professor De Koninck, are of two ages—Silurian and Devonian. The silurian species appear to represent two horizons—the upper Llandovery (May Hill sandstone) and the Ludlow.[2] The corals appertaining to these divisions are:—

(a.) UPPER LLANDOVERY.

Rhyzophyllum (?) interpunctatum, *De Koninck.*
Strombodes diffluens, *Edwards* and *Haime.*
Striatopora Australica, *De Koninck.*
Aulopora fasciculata „
Syringopora serpens, *Linn* (?)
Monticulipora (?) Bowerbanki, *Edwards* and *Haime.*

(b.) LUDLOW.

Ptychophyllum patellatum, *D'Orbigny.*
Cystiphyllum silurieuse, *Lonsdale.*
Omphyma Murchisoni, *Edwards* and *Haime.*
Cyathophyllum articulatum, *Wohlenberg.*
Halysites escharoides, *Lamarck.*
Monticulipora pulchella, *Edwards* and *Haime.*
Alvcolites repens, *Fougt.*
„ rapa, *De Koninck.*
Favosites cristata, *Blumenbach.*
„ Forbesi, *Edwards* and *Haime.*
„ aspera, *D'Orbigny.*
„ multipora, *Lonsdale* (?).
„ fibrosa, *Goldfuss.*
„ Gothlandica, *Fougt.*
Propora tubulata, *Lonsdale.*
Plasmopora petaliformis, *Lonsdale.*
Heliolites megastoma, *M'Coy.*
„ Murchisoni, *Edwards* and *Haime.*

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[1] Bruxelles; 8vo.; 1876, p. 140; Atlas of Plates.
[2] P. 64.

In a similar manner the Devonian species also appear referable to two horizons—an upper, probably equivalent to the upper Devonian, and without corals, so far as the specimens in Professor De Koninck's hands showed; and a second somewhat below the higher, but above that which in Europe is so well characterised by the presence of *Calceola sandalina* (Lamarck). The latter of the two divisions contains the black limestone of the Yass district.[1] The corals recorded and described by Professor De Koninck are:—

Phillipsastrea Verneuilii, *Edwards* and *Haime.*
Campophyllum flexuosum, *Goldfuss.*
Cyathophyllum vermiculare „
„ obtortum, *Edwards* and *Haime.*
„ Damnoniense, *Lonsdale.*
„ helianthoides, *Goldfuss.*
Amplexus Selwyni, *De Koninck.*
Cœnites expansus, *De Koninck.*
Billingsia alveolaris, *De Koninck.*
Syringopora auloporoides, *De Koninck.*
Alveolites obscurus, *De Koninck.*
„ subæqualis, *Edwards* and *Haime.*
Favosites Goldfussi, *D'Orbigny.*
„ basaltica, *Goldfuss.*
„ alveolaris „
„ polymorpha „
„ reticulata, *Blainville.*
„ fibrosa, *Goldfuss.*
Heliolites porosa „

Since the publication of the foregoing account of the lower and middle palæozoic fossils of New South Wales, Professor De Koninck has been engaged in the examination of the fossils of carboniferous age, contained in the Rev. Mr. Clarke's cabinet. The descriptions of these are now in course of printing, and Professor De Koninck has in the kindest manner forwarded me the advanced sheets. As his memoir will appear before these remarks reach the Royal Society, I feel that I am committing no breach of professional etiquette in stating that on the whole the carboniferous fossils of New South Wales correspond in a very considerable degree with the facies of the carboniferous limestone

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[1] *Loc. Cit.*, pp. 133, 134.

series of England and Scotland, combined with the presence of a few peculiarly Australian types. The similarity of the New South Wales palæozoic fossils examined by him with those of the L. carboniferous series in Ireland was many years ago pointed out by Professor M'Coy.[1] The confirmation of this opinion through Professor De Koninck's studies is particularly gratifying, especially when we recollect that in both instances we owe the material on which these opinions were founded to the researches of the father of Australian geology—the Rev. W. B. Clarke, M.A., F.R.S. The corals of carboniferous age determined by Professor De Koninck are—

Axophyllum (?) Thomsoni, *De Koninck.*
Lithostration irregulare, *Phillips.*
„ Basaltiforme, *Con* and *Phillips.*
Lophophyllum minutum, *De Koninck.*
„ corniculum „
Amplexus arundinaceus, *Lonsdale.*
Zaphrentis Phillipsi, *Edwards* and *Haime* (?)
„ Gregoryana, *De Koninck.*
„ Cainadon „
„ robusta „
Cyathaxonia minuta „
Cladochanus tenuicollis, *M'Coy.*
Syringopora reticulata, *Goldfuss.*
„ ramulosa „ (?)
Favosites ovata, *Lonsdale.*

One of the most interesting points to be noticed in this list is the reference to the peculiarly Australian species, *Stenopora ovata* (Lonsdale), to the genus *Favosites.* Professor De Koninck states that the pores perforating the walls of the calices are irregularly placed—some in the angles of the tubes, others upon the general surface of the walls.

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[1] *Annals Nat. Hist.*, 1847, XX., p. 311.

Art. XVI.—*On the Ratio of the Length and Height of Sea Waves.*

By S. R. Deverell, Esq.

[Read November 8th, 1877.]

Of the phenomena appertaining to water-waves none seem to have appeared more capricious to observers than the variable proportion of the height to the length of waves. Indeed such strange diversities are exhibited in this respect that writers have used themselves to speak of different kinds of waves as if they were of different species:—The short chopping sea; the steep high sea; the long high sea; the long roll, of medium height and length—that measured tread of old ocean, as an Arctic voyager has expressed it, which so gladdens the eyes and the heart of the Polar sojourner when he first strikes it; finally, the tremendous "comber" of navigators which from overhead threatens to bury the ship: these are often referred to as originating rather from different causes than as being so many transitions or attitudes of the same thing or entity. There is, again, the mysterious ground swell, which old seamen firmly believe to arise in some occult manner from the bottom, proceeding in slow, languid oscillations, but breaking with an everlasting roar and violence on the shore to which it is bound. Mere magnitude does not appear to be an essential characteristic of any of these forms, for they may all be met with in various degrees of size. Scoresby mentions waves in the Southern Ocean a quarter of a mile from peak to peak; but this can be by no means unusual, for in that vast sea, which may in truth be said to be the native home of the great waves, five waves to a mile is a very ordinary occurrence in a westerly gale, and the writer has counted five to a mile when the waves have not been more than six or seven feet high. The length of a wave in fact is by no means a criterion of its height: its actual magnitude is rather measurable by the area of a vertical length-section than by the height. Again, as regards the speed, the velocity, says Mr. Reed, seems to depend almost entirely on the length of a wave and not at all upon the height. It should be remembered that existing knowledge on these subjects, to which general attention has only been attracted during a few years past, is at present in an immature or rather embryonic state, as indeed is continually pointed out by its most eminent followers. The views and suggestions of any observer, however humble, are of value; and the store of information which the British and French Admiralties—ever rivals in scientific progress—are now engaged in collecting, through their naval officers, in all parts of the world, must soon tend to formulate a completed theory of the subject. The extraordinary length of some waves in comparison with their height has often attracted the notice and the vague surprise of observers long even before the attention of mathematicians was drawn into the inquiry. In a recent Admiralty circular Mr. Froude cautions officers observing waves that they must not neglect those of almost imperceptible height but from 600 to 1000 feet in length, which greatly influence the rolling movement of a ship. On the southern coast of Australia there is a well-known and remarkable difference in this respect in the character of the swell from the eastward and the westward. The south-east and the south-west directions there extend over equally great stretches of ocean, but while the swell from the south-east is a short chopping sea, high and steep (usually 8 or 9 feet high and 150 feet long, or as 1 to 17), that from the westward is a long heavy roll, usually about the same height (8 or 9 feet) but 150 *yards* instead of feet in length, or as 1 : 151. What I would here attempt to show, or rather to suggest, is that the varying ratio of height to length signifies or rather represents none other than the process of increase or subsidence of waves, and that if we could follow a sea-wave from its genesis to actual extinction we should be able to observe it through all the various phases as to height and length which have been enumerated.

That a certain force of wind acting for a given time will produce a wave of definite form is, I suppose, undoubted; and I presume it will not be questioned that the same conditions will always produce the precisely similar wave whose height is in a given ratio to its length. A certain force of wind, again, sufficient to obviate the loss by friction, will sustain in it this form; but if the sustaining force be withdrawn, then, however far its momentum will carry it—and it is known to carry it thousands of miles—the wave must thence gradually decline; and it is in this decline, viz., from its maximum height to final disappearance or extinction, that the ratio of height and length must, in this view, vary through all the degrees observed in waves.

But what is meant by the decline or the subsidence of a wave since the actual bulk or magnitude is neither measurable by its height nor its length, but by the area of a cross section ? A volume of water has been raised to a certain height above the ordinary level; and in declining its height must decrease until the curve of its profile gets flattened out to a straight line. In what manner is the length thereby affected ? Inquiry will, I think, show that the length is not only *relatively* increased (which it would be by remaining constant while the height alone decreased), but itself increases—that is, absolutely, in the act of the wave's subsidence.

Now, although we cannot accompany a wave in its onward progress across a sea to note the changes it undergoes in its transit, yet be it remembered that the same laws which influence deep-sea waves, however vast, likewise direct the movement of the smallest ripples, scanning which the eye may under favourable circumstances take in at a glance the phenomena here indicated.

For instance, if a fresh breeze be blowing on a small piece of water so as to produce a series of riplets, and these travel into a part which is sheltered from the wind, it will be observed that at genesis the wave is steepest—*i.e.*, the ratio of the length to the height small, and that as long as the wind has a direct active influence in sustaining them the height preserves a large proportion to the length. As soon, however, as the direct support of the wind ceases the wave begins to decline by, be it observed, spreading out in length and decreasing in height. The annexed diagram is made from observation in a spot favourably situated. The genesis of the riplet is at A (Fig. 1); from A to B, the point of maturity, it increases in size, the ratio of height to length being greatest during increase. In the mature stage (from B to C) the same ratio is maintained. At C, however, the wind has ceased its support, and thence to D the wave gradually subsides to extinction—*i.e.*, until the height becomes indefinitely small, and the length indefinitely great—in other words, the surface becomes flat.

Such a diagram may be said to represent the life of a sea wave in miniature, for although it is the *fac-simile* of the progress of a ripple only, from birth to extinction, the same reasoning obviously extends to that of the heaviest sea. For, be it observed, the largest sea must have had its origin in a primary wavelet, as at the point A; and we have only to extend the period of increase from A to B further towards D, as in the annexed figure (Fig. 2), to obtain the larger waves. The magnitude of the wave, in fact, is proportional to the period of increase, while being increasingly urged by the wind during the progress of the wave from A to B, and this time must obviously be dependent upon the extent of the *fetch* of free water over which the wind may extend; so that the strength and range of the wind being the same, the magnitude is proportioned to the fetch. A storm-wave therefore of forty feet in height may have the same profile as a ripple, from which indeed it must have sprung, and in the same way the declining ground-swell of an ocean has its miniature *fac-simile* in a pond.

The annexed diagram (Fig. 3) may practically illustrate the foregoing remarks. A represents accurately the average profile of the permanent south-west swell in the Southern Ocean in latitude from 40° to 48° S., arising from the prevailing winds around the Pole. The curve is taken from entries of a number of profiles drawn from observation in a recent voyage of the ship "Newcastle" from Melbourne *viâ* Cape Horn to London, and the same curve and dimensions are identifiable throughout in the same latitudes. B in like manner represents the profile and dimensions drawn to the same scale of presumably the same permanent south-west ground swell as it reaches the southern coast line of Australia, averaged from many sketches of such profiles taken on the spot. The outline A therefore represents the swell in its active or mature state at or about its maximum ratio of height to length in a stage when the height and the bulk of water moved oppress the mind with a sense of sublimity; and B represents it in its decline, when, after having traversed forty degrees of a great circle, or more than two thousand miles, it approaches dissolution. The height here is comparatively nil, and the length has increased almost to flatness. Yet this enormous swell had its origin in the Polar sea, as an initial wavelet, the relative magnitude of which could only be represented in the diagram by a dot.

Instead, however, of tracking a wave through this vast distance we may picture it as fixed and subsiding in a single spot without interfering with the logical sequence of the argument, inasmuch as it thus represents the same wave, filled by the same instead of by changed particles of the liquid to which its embodiment has been transferred.

Let, therefore (Fig. 4), *a, b, c, d, e* represent the profile of a wave from trough to trough, the dotted line *f, g* being the mean or smooth water level. So far as the subsidence is concerned we may wholly disregard the actual movements of the particles, and conceive an indefinitely thin layer of the liquid to be instantaneously fixed or congealed in the shape of the wave *a, b, c, d, e*. Here *i, c* is the height, and *a, i, e* the length of the wave.

It will be seen that the area *b, c, d, h* is that portion of the liquid which has been raised above the mean level of the ocean; while the areas *d, g, e* and *b, a, f* are that of the water which has been *thereby* depressed below the mean level; whence the area *b, c, d, h* above the mean level is equal to the sum of the areas *d, g, e,* and *b, f, a* below the mean level, since the filling of the lower areas by the upper would render the surface flat.

Conceive now that the rigidity is slackened, so that the ideal lamina becomes semi-viscous. The onward velocity of a wave keeps it from sinking suddenly, as does that of a hoop or a top; its decline, therefore, is not due to its onward velocity, and the slow sinking of a semi-viscous fluid may justly represent the process of its actual subsidence.

Taking this view then to be correct, we may, under such an assumption, consider the wave as wholly divested, not only of any onward motion, but also of any rotatory movement of the particles. This is nothing more than conceiving the form of the wave to be embodied of the same particles instead of successive ones.

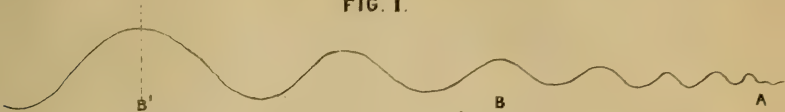
If the sinking of the upper area merely filled up the lower areas, the length of the wave would still remain the same, viz., *a, i, e;* but observation shows that the length absolutely increases. Let the height of the wave have subsided to c', then instead of the profile being the curve f'', b'', c' d'', e'', which it would be if the length remained unchanged, it is represented by the curve a', b', c', d' e', whose length (the dotted line e', i, a') exceeds *e, i, a ;* i', c' now represents the height of the declining form.

Now, in order to simplify matters, we may—the two halves of the wave being symmetrical—treat only of the half shapes, viz., *i*, *c*, *d*, *e*, and *i′*, *c′*, *d′* *e′*. The area *c′*, *d′*, *h* is now equal to area *d′*, *e′*, *g′*, and *i′*, *c′* is the reduced height of the wave, the reduction taking place from both sides of the mean level. The actual quantity of subsidence is measured by the difference between the areas *e*, *i*, *c*, *d* and *e′*, *i′*, *c′*, *d′*, or perhaps by the difference of *d*, *h*, *c* and *d′*, *h*, *c′*, the change which occurs while the lamina of semi-viscous fluid is sinking into flatness. Whilst the exact expression for the profile curve is undecided—and it is to the determination of this that every inquiry on this subject at present tends—I am not aware so far as my own imperfect knowledge extends of any means of stating such difference: that is, of expressing the actual change in the ratio of height to length in precise terms of the diminution in height (viz., *i*, *c* —*i′*, *c′*).

But whatever be the precise function mathematically, the cause suggested will, I think, sufficiently account for all observed circumstances; and it will explain also the peculiar difference noted between the easterly and westerly swell on the southern Australian coast in respect of the ratio of height to length. In those parts the south-east winds are known to extend only, and therefore to act on the swell only, a few hundred miles from the shore; the waves therefore having their genesis within this distance have not space to reach a lengthy decline, or, perhaps, even full maturity. Whereas the south-west winds start from the Pole, and the swell arising therefrom has an unbroken fetch for attainment of the highest possible magnitude, and thousands of miles for the slow process of decline in which it gradually increases its length and diminishes its height. The westerly swell therefore reaches the Australian shore in its declining stage, when the length is great and the height small; the easterly, in its mature or steep stage, when the waves are therefore higher, shorter, and more active, being urged or having been more recently urged by the wind.

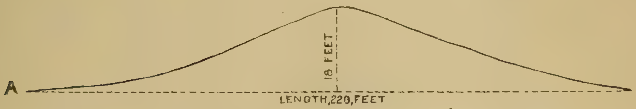
By the fetch of a particular wave at any moment is, of course, meant the distance it has travelled from its genesis as an initial wavelet until then. Let A (Fig. 5) be the point of commencement of the wave (and thence in most cases of the wind also), and A B its path or fetch when it is at the point B. If from points *a*, *b*, *c*, &c., in the fetch ordinates be erected representing the strength or velocity of the wind when the wave was passing those respective

FIG. 1.

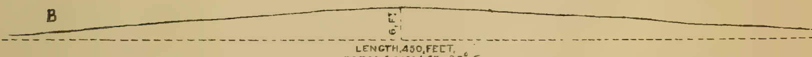


MAGNITUDE AT GENESIS. LAT. 75° S.

FIG. 2.



FORM AND MAGNITUDE IN LATITUDE. 48° S.



FORM &c IN LAT. 36° S.

FIG. 3.

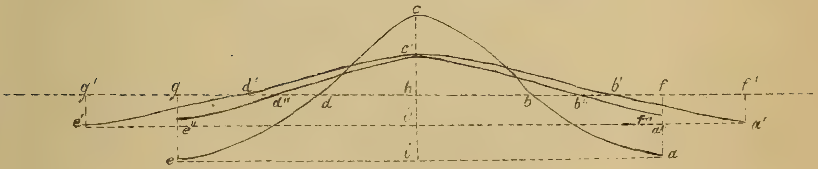


FIG. 4.

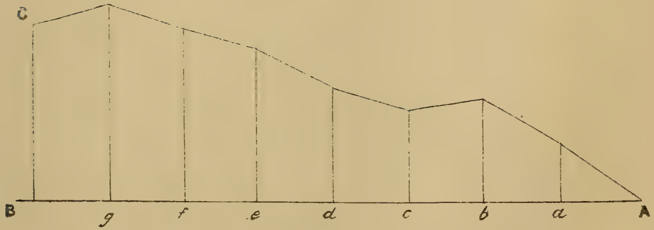


FIG. 5.

points, a certain curve (A C) will be traced out. When this curve is precisely the same as another it is certain that the same form of wave as to height and length will be produced; and, for the same reason, when the curves differ the forms of the waves produced will differ.

Or, instead, let the abscissa A B be a time scale. The curve resulting from the time scale will have a definite relation to that from the distance scale; and it seems pretty certain, as like causes must produce like effects, that the form of the wave produced, as it exists at the point B, will be determined by the nature of these curves, and stand in some definite relation to the area A B C—a relation which, however difficult to determine, shows the infinite variety which the form of the wave (in which the height and length are only particular ordinates) may assume.

ART. XVII.—*Notes on the Newly-found Satellites of Mars.*

BY R. L. J. ELLERY, F.R.S., F.R.A.S.

[Read December 13th, 1877.]

ART. XVIII.—*On the Telephone.*

BY W. C. KERNOT, M.A., C.E.

[Read December 13th, 1877.]

1877.

PROCEEDINGS.

ROYAL SOCIETY OF VICTORIA.

ANNUAL MEETING.

14th March, 1878.

The President in the chair.

Mr. Daniel Howitz, Superintendent of State Forests, was elected a member of the Society.

The election of office-bearers for 1878 took place, with the following results :—

President : R. L. J. Ellery, F.R.S., F.R.A.S.
Vice-Presidents : E. J. White, F.R.A.S.
Geo. Foord, F.C.S.
Hon. Treasurer : Percy de J. Grut, Esq.
Hon. Librarian : James E. Neild, M.D.
Hon. Secretary : E. Howitt, Esq.
Members of Council: H. M. Andrew, J. Bosisto, J. Jamieson, W. C. Kernot, E. J. Nanson, G. H. F. Ulrich, A. C. Allan, R. Barton, J. Duerdin, W. M'Gowan, F. J. Pirani, J. T. Rudall.

The Annual Report and Balance-sheet for 1877 were read and adopted, as follows :—

"*Report of the Council of the Royal Society of Victoria for the year 1877.*

"Your Council has the honour to report that the following papers were read during the Session of 1877.

"On the 20th of April a paper 'On Force' was read by Mr. F. J. Pirani; and another by Mr. S. R. Deverell, entitled 'On some Experiments in Propulsion,' was read by the President.

"On the 10th of May the President read a paper on 'The Present State of Meteorology,' and the discussion on Mr. Pirani's paper 'On Force' was continued.

"On the 14th of June Mr. W. C. Kernot read some notes 'On the Construction of Telescope Tubes,' and Mr. T. E. Rawlinson read one 'On the Coast Line between Warrnambool and Belfast, and the Permanence of Meteorological Phenomena over long Periods.'

"On the 12th of July 'Notes on Barometer Construction' were read by Mr. G. Foord, and the President described a new method of regulating clocks.

"On the 9th of August Mr. Ellery read a description of a new form of galvanic battery, and notes of the disturbance of water in tanks by the late earthquake; and a paper was contributed by the Rev. Julian E. T. Woods, of New South Wales, on 'New Marine Mollusca.'

"On the 13th of September a paper by Mr. F. C. Christy, entitled 'Notes from a Journal in Japan,' was read by Mr. Howitt; and another was read by Mr. Sutherland 'On the Probability that a Connection of Causation will be shown to exist between the Attraction of Gravitation and the Molecular Energy of Matter.'

"On the 11th of October Dr. Jamieson read his paper, entitled 'Experiments on the Comparative Power of some Disinfectants when Vaporised.'

"On the 8th of November Mr. Patching read a paper on 'Heat and Molecular Energy,' and Dr. Jamieson's paper on 'Disinfectants' was discussed. Mr. Etheridge, F.G.S., of the Geological Survey of Scotland, contributed a paper, entitled 'Palæozoic Actinology in Australia;' and Mr. S. R. Deverell's paper, entitled 'On the Ratio of the Length and Height of Sea Waves,' was read by the President.

"On the 13th of December Mr. A. Mica Smith contributed notes of 'Some Experiments in Gold Bullion Assay,' and the President read some notes on the Satellites of Mars. Mr. Kernot then described the Telephone.

"During the year a new Law (No. LIX.) has been added to the Rules, providing for the election of Honorary Corresponding Members not resident in Victoria.

"Volume XIII. of the Society's Transactions is now in the press and nearly ready for issue, and Volume XIV. will be published as soon as possible afterwards.

BALANCE-SHEET.

The Hon. the Treasurer in Account with the Royal Society of Victoria.

Dr.

To Balance from last Balance-sheet				£323	18	3
,, Government Grant, 1877-8				200	0	0
,, Interest on Fixed Deposits				10	8	4
,, Rents				20	15	6
,, Sale of Papers				0	6	3
,, Life Members				21	0	0
,, 15 Entrance Fees				31	10	0
,, SUBSCRIPTIONS—						
1873. 1 country	£1	0	0			
1874. 2 country	2	1	0			
1875. 4 country	4	3	0			
1876. 8 ordinary	16	16	0			
1876. 4 country	4	3	0			
1877. 53 ordinary	111	6	0			
1877. 5 half-ordinary	5	5	0			
1877. 14 country	14	13	0			
				159	7	0
				£767	5	4

Cr.

By Printing and Stationery				£63	8	9
,, Books				12	0	0
,, Bookbinding				4	4	6
,, Freight and Charges on Books				7	3	5
,, Conversazione				12	1	6
,, Interest on Debentures				26	14	10
,, 50 Debentures paid				250	0	0
,, Improvements and Repairs				28	3	11
,, Insurance				2	17	6
,, Rates				6	0	0
,, Gas and Fuel				10	2	9
,, Hall-keeper and Clerical Assistant				25	0	0
,, Collector's Commission				21	0	8
,, Postages and Petty Cash				25	0	0
,, City Council (Pavement)				13	14	7
,, Balance in Bank—						
Fixed Deposit	£200	0	0			
Current Account	59	12	11			
				259	12	11
				£767	5	4

6th March, 1878.

(Signed) P. DE JERSEY GRUT, *Hon. Treasurer.*

Compared with the Vouchers, Bank Pass Book, and Cash Book, and found correct.

(Signed) JOSEPH BOSISTO }
HENRY MOORS } *Auditors.*

6th March, 1878.

STATEMENT OF ASSETS AND LIABILITIES.

ASSETS.	£	s.	d.	LIABILITIES.	£	s.	d.
Balance in Bank	£259	12	11	Due to Publishing Fund	£216	11	2
Estimated Value of outstanding Subscriptions	10	0	0	13 Debentures	65	0	0
Rents due	30	1	9	Interest unclaimed	13	8	9
Interest accrued	3	1	11	Outstanding accounts	10	0	0
Hall, Library, Furniture, &c., insured for	2300	0	0	Balance	2297	16	8
	£2602	16	7		£2602	16	7

"The usual grant-in-aid of the Society for the purpose of assisting it in the publication of its Transactions was voted by Parliament, and has passed to the credit of the Society. Debentures to the amount of £250 have been paid during the last year. The balance in hand amounts to £259 12s. 11d."

The Report and Balance-sheet were adopted.

(Signed) Robt. L. J. Ellery.

ORDINARY MEETINGS.

12th April, 1877.

The President in the Chair—Present, 13 members.

Mr. Edward Bage was elected a member of the Society.

Mr. R. S. Bradley (Grammar School, Stawell) was nominated by Mr. Ellery and Mr. Allan.

Mr. James Macdowall Conroy (Deniliquin), proposed by Mr. Howitt and Mr. Rusden.

Mr. Pirani read his paper "On Force," which was ordered to be printed.

The President read Mr. S. R. Deverell's paper "On some Experiments in Propulsion." Discussion ensued.

The President read a letter from the Secretary of the Scientific Club, Vienna, offering the privileges of honorary membership to such members of the Royal Society of Victoria as might at any time be resident in Vienna.

The Secretaries were instructed to accept this obliging offer, with due acknowledgments of its kindness.

(Signed) Robt. L. J. Ellery.

10th May, 1877.

The President in the Chair—Present, 18 members.

Mr. Bradley and Mr. Conroy were duly elected members of the Society.

Mr. H. S. Patching was nominated by Mr. Harrison and Mr. Ellery.

The discussion on Mr. Pirani's paper, entitled "On Force," was then opened by Mr. Rusden, and various members took part in it.

The President then read his paper "On the Present State of Meteorology," and discussion ensued.

(Signed) Robt. L. J. Ellery.

14*th June*, 1877.

The President in the Chair—Present, 22 members.

Mr. H. S. Patching was duly elected.

Dr. John Fulton was nominated by Mr. Humphreys and Mr. Rusden.

Mr. Louis Le Gould was nominated by Mr. Ellery and Mr. Moerlin.

The President read two messages from the Council; one in reference to the reprinting of such volumes of the Society's Transactions as were out of print; the other in regard to the alteration of the Laws, so as to provide for the Election of Corresponding Members.

The President read a letter from Mr. Louis Le Gould describing a remarkable meteor seen by him.

Mr. Kernot read his paper "On the Construction of Telescope Tubes," and discussion ensued.

Mr. Rawlinson read his paper "On the Coast Line between Warrnambool and Belfast."

Both papers were ordered to be printed.

(Signed) Robt. L. J. Ellery.

12*th July*, 1877.

The President in the Chair—Present, 18 members.

Dr. John Fulton and Mr. Louis Le Gould were duly elected.

Mr. R. E. Joseph was nominated by Mr. Ellery and Mr. White.

The President announced that a special meeting would be held on the evening of the next ordinary meeting, to consider the proposed new rule with reference to Corresponding Members.

Messrs. Nanson, Rawlinson, Rusden, Henderson, Moors, and Sutherland were appointed a committee to consider the republication of the early Transactions of the Society.

The Librarian reported the receipt of foreign publications—English, 93; American, 43; Canadian, 21; French, 3; German, 92; Italian, 27; Russian, 6; Spanish, 13; Dutch, 18; Danish, 11; Batavian, 7; Brazilian, 3; Chinese, 2; Japanese, 1; together with publications from Australia and New Zealand, 42; making a total of 388.

Mr. Ellery then read his "Notes on the Late Earthquake."

Mr. Foord read his notes on "Barometer Construction," upon which discussion followed.

The President then described a method newly invented by Mr. Joseph for regulating clocks by means of electricity.

(Signed) Robt. L. J. Ellery.

SPECIAL GENERAL MEETING.

9th August, 1877.

The President in the Chair—Present, 14 members.

Mr. Rawlinson moved and Mr. Howitt seconded that the following rule be added as No. LIX.:—

"LIX. The Council shall have power to propose gentlemen not resident in Victoria as Corresponding Members of the Society. The Corresponding Members shall contribute to the Society papers which may be received as those of ordinary members, and shall in return be entitled to receive copies of the Society's publications."

The motion was unanimously adopted.

The special meeting then resolved itself into the

ORDINARY MEETING,

9th August, 1877.

Mr. R. E. Joseph, of Swanston-street, was elected an ordinary member of the Society.

The Rev. J. E. T. Woods and Mr. Robert Etheridge were nominated as corresponding members.

The President read the following notes supplementary to his annual address:—

"Reading over my address since its delivery, I am sorry to find that I have made several omissions, which, had I possessed more leisure before our annual meeting, would not have passed uncorrected. The best I can do now is to tell you of them, and to apologise to any concerned for my apparent remissness.

"In the first place, it seems to me that, while referring to our Library and the necessity of making its contents more easily available to our members, I omitted to mention and acknowledge the continued efforts of our Honorary Librarian to bring about such a desirable state of things, and by the omission may have inadvertently attached some blame to Dr. Neild. This, however, was furthest from my intention, for no one knows better than I how much our Librarian has done and is doing in this direction.

"Again, in referring to the progress made in our various science and art departments, I regret to find that I have carelessly omitted reference to several names of persons and instances of progress which the occasion demanded and should have been referred to.

"For instance, in speaking of the prosecution of geological research in this colony, while I mentioned the names of several of our fellow-members who have distinguished themselves, I am exceedingly sorry to find I omitted the name of one who has perhaps most distinguished himself in this direction—namely, Mr. A. W. Howitt, of Gippsland. Our knowledge of the geology of no inconsiderable portion of Gippsland we owe to this gentleman; and his continued researches, prompted solely by his pure love of the science, promise very largely to enrich our geological data of that portion of the colony. This much at least I owe to the gentleman named; and to any others whose labours I have, by necessity or by remissness, omitted to refer to, I tender my sincere apology."

The President then read a note from Mr. G. W. Robinson describing the effects of the late earthquake in disturbing the water contained in certain tanks.

The President then presented the Rev. J. E. T. Woods's paper on "New Marine Mollusca."

The President read his notes on various forms of galvanic battery, and discussion followed.

Dr. James Jamieson was nominated for election by Dr. Neild and Mr. Rawlinson.

Major J. A. Anderson was nominated by the Rev. H. P. Kane and Mr. Howitt.

Mr. K. L. Murray was nominated by Mr. Ellery and Mr. M'Gowan.

(Signed) Robert L. J. Ellery.

September 13*th.*

The President in the Chair.

The following gentlemen were elected ordinary members of the Society:—Dr. James Jamieson, of Latrobe-street West; Major J. A. Anderson, of Brighton Beach; Mr. K. L. Murray, of the Electric Telegraph Department.

The following gentlemen were elected corresponding members of the Society:—The Rev. J. E. T. Woods, of Sydney, and Mr. Robert Etheridge, of Edinburgh. The Right Rev. Charles Perry, D.D., was nominated for election as an honorary member. The President then read a communication received from the Royal Academy of Sciences of Turin respecting the prize established by Dr. Cesare Alessandro Bressa to be given once every four years to any one who shall make the most important discovery or publish the most important work.

The Secretary then read Mr. F. C. Christy's paper entitled "Notes from a Journal in Japan."

A vote of thanks for the paper was moved by Mr. Ellery and Mr. White, and carried.

Mr. Sutherland then read his paper "On the Probability that a Connection exists between the Attraction of Gravity and the Molecular Energy of Matter."

It was resolved that both papers should be printed and circulated for discussion at next meeting.

(Signed) Robert L. J. Ellery.

October 11*th.*

The President in the Chair.

The Right Rev. Charles Perry, D.D., late Lord Bishop of Melbourne, was elected an honorary member of the Society.

The President stated that he had received a telegram from the Astronomer Royal, requesting that search should be made for a satellite of Mars, said to have been discovered at Washington. Search here had been made, but was unsuccessful.

Discussion of Mr. Sutherland's paper, read at the last meeting, then took place.

Dr. Jamieson then read a paper—"Experiments on the Comparative Power of some Disinfectants."

It was resolved that this paper be printed, and discussed at next meeting.

(Signed) Robert L. J. Ellery.

November 8*th,* 1877.

The President in the Chair.

Dr. Jamieson read some notes supplementary to the paper he had read at the last meeting.

An animated discussion followed.

Mr. Patching read his paper on "Heat and Molecular Energy." Discussion followed.

Mr. Etheridge's paper on "Palæozoic Actinology" was then read by Mr. Ulrich. It was ordered to be printed.

Mr. Ellery read Mr. S. R. Deverell's paper on "The Ratio of the Length and Height of Sea Waves." A vote of thanks was awarded to Mr. Deverell.

(Signed) Robert L. J. Ellery.

December 13*th.*

The President in the Chair—Present, 20 members.

Mr. Rawlinson notified his intention of resigning his position as member of the Council.

Mr. Duerdin nominated the existing officers of the Society for election to the same offices at the annual meeting in March; Mr. Duerdin nominated Dr. Jamieson to fill the vacancy in the Council caused by the resignation of Mr. Rawlinson.

These nominations were seconded by Mr. Humphreys.

Mr. Duerdin and Mr. Humphreys nominated Mr. Henry Moors and Mr. J. Bosisto for election as Auditors, and accordingly these gentlemen were duly elected.

Mr. Daniel Howitz, superintendent of forests, was nominated for election as an ordinary member by Mr. Ellery and Mr. J. B. Were.

Mr. Ellery read some notes on the newly-found satellite of Mars.

Mr. Kernot described the ordinary form of the telephone.

(Signed) ROBERT L. J. ELLERY.

LAWS.

Name. I. The Society shall be called "The Royal Society of Victoria."

Objects. II. The Royal Society of Victoria is founded for the advancement of science, literature, and art, with especial reference to the development of the resources of the country.

Members and Honorary Members. III. The Royal Society of Victoria shall consist of Members and Honorary Members, all of whom shall be elected by ballot.

Patron. IV. His Excellency the Governor of Victoria, for the time being, shall be requested to be the Patron of the Society.

Officers. V. There shall be a President, and two Vice-Presidents, who, with twelve other Members, and the following Honorary Officers, viz., Treasurer, Librarian, and two Secretaries of the Society, shall constitute the Council.

Management. VI. The Council shall have the management of the affairs of the Society.

Ordinary Meetings. VII. The Ordinary Meetings of the Society shall be held once in every month during the Session, from March to December inclusive, on days fixed by and subject to alteration by the Council with due notice.

Annual General Meetings. VIII. In the second week in March there shall be a General Meeting, to receive the report of the Council and elect the Officers of the Society for the ensuing year.

Retirement of Officers. IX. All Office-bearers and Members of Council, except the six junior or last elected ordinary Members, shall retire from office annually at the General Meeting in March. The names of such Retiring Officers are to be announced at the Ordinary Meetings in November and December. The Officers and Members of Council so retiring shall be eligible for the same or any other office then vacant.

Election of Officers.

X. The President, Vice-Presidents, Treasurer, Secretaries, and Librarian shall be separately elected by ballot (should such be demanded), in the above-named order, and the six vacancies in the Council shall then be filled up together by ballot at the General Meeting in March. Those members only shall be eligible for any office who have been proposed and seconded at the Ordinary Meeting in December, or by letter addressed to one of the Secretaries, and received by him before the 1st March, to be laid before the Council Meeting next before the Annual Meeting in March. The nomination to any one office shall be held a nomination to any office the election to which is to be subsequently held. No ballot shall take place at any meeting unless ten members be present.

Members in arrear.

XI. No Member whose subscription is in arrear shall take part in the election of Officers or other business of the Meeting.

Inaugural address by the President.

XII. An Address shall be delivered by the President of the Society at either a Dinner, Conversazione, or extra meeting of the Society, as the Council for the time being may determine, not later than the Ordinary Meeting in June in each year.

Vacancies.

XIII. If any vacancy occur among the Officers, notice thereof shall be inserted in the summons for the next Meeting of the Society, and the vacancy shall be then filled up by ballot.

Duties of President.

XIV. The President shall take the chair at all meetings of the Society and of the Council, and shall regulate and keep order in all their proceedings; he shall state questions and propositions to the meeting, and report the result of ballots, and carry into effect the regulations of the Society. In the absence of the President the chair shall be taken by one of the Vice-Presidents, Treasurer, or ordinary Member of Council, in order of seniority.

Duties of Treasurer.

XV. The Treasurer may, immediately after his election, appoint a Collector (to act during pleasure), subject to the approval of the Council at its next meeting. The duty of the Collector shall be to issue the Treasurer's notices and collect subscriptions. The

Treasurer shall receive all moneys paid to the Society, and shall deposit the same before the end of each month in the bank approved by the Council, to the credit of an account opened in the name of the Royal Society of Victoria. The Treasurer shall make all payments ordered by the Council on receiving a written authority from the chairman of the meeting. All cheques shall be signed by himself, and countersigned by one of the Secretaries. No payments shall be made except by cheque, and on the authority of the Council. He shall keep a detailed account of all receipts and expenditure, present a report of the same at each Council Meeting, and prepare a balance-sheet to be laid before the Council, and included in its Annual Report. He shall also produce his books whenever called on by the Council.

Duties of Secretaries.

XVI. The Secretaries shall share their duties as they may find most convenient. One or other of them shall conduct the correspondence of the Society and of the Council, attend all meetings of the Society and of the Council, take minutes of their proceedings, and enter them in the proper books. He shall inscribe the names and addresses of all Members in a book to be kept for that purpose, from which no name shall be erased except by order of the Council. He shall issue notices of all meetings of the Society and of the Council, and shall have the custody of all papers of the Society, and, under the direction of the Council, superintend the printing of the Transactions of the Society.

Meetings of Council.

Quorum.

XVII. The Council shall meet on any day within one week before every Ordinary Meeting of the Society. Notice of such meeting shall be sent to every Member at least two days previously. No business shall be transacted at any meeting of the Council unless five Members be present. Any Member of Council absenting himself from three consecutive meetings of Council, without satisfactory explanation in writing, shall be considered to have vacated his office, and the election of a Member to fill his place shall be proceeded with at the next Ordinary Meeting of Members, in accordance with Law XIII.

Special Meetings of Council.

XVIII. One of the Secretaries shall call a Special Meeting of Council on the authority of the President or of three Members of the Council. The notice of such meeting shall specify the object for which it is called, and no other business shall be entertained.

Special General Meetings.

XIX. The Council shall call a Special Meeting of the Society, on receiving a requisition in writing signed by twenty-four Members of the Society specifying the purpose for which the meeting is required, or upon a resolution of its own. No other business shall be entertained at such meeting. Notice of such meeting, and the purpose for which it is summoned, shall be sent to every Member at least ten days before the meeting.

Annual Report.

XX. The Council shall annually prepare a Report of the Proceedings of the Society during the past year, embodying the balance-sheet, duly audited by two Auditors, to be appointed for the year, at the Ordinary Meeting in December, exhibiting a statement of the present position of the Society. This Report shall be laid before the Society at the Annual Meeting in March. No paper shall be read at that meeting.

Expulsion of Members.

XXI. If it shall come to the knowledge of the Council that the conduct of an Officer or a Member is injurious to the interest of the Society, and if two-thirds of the Council present shall be satisfied, after opportunity of defence has been afforded to him, that such is the case, it may call upon him to resign, and shall have the power to expel him from the Society, or remove him from any office therein at its discretion. In every case all proceedings shall be entered upon the minutes.

Election of Members.

XXII. Every candidate for membership shall be proposed and seconded by Members of the Society. The name, the address, and the occupation of every candidate, with the names of his proposer and of his seconder, shall be communicated in writing to one of the Secretaries, and shall be read at a meeting of Council, and also at the following meeting of the Society, and the ballot shall take place at the next following ordinary meeting of the Society. The assent of at least five-sixths of the number voting shall be requisite for the admission of a candidate.

Members shall sign laws.

XXIII. Every new Member shall receive due notice of his election, and be supplied with a copy of the obligation,* together with a copy of the Laws of the Society. He shall not be entitled to enjoy any privilege of the Society, nor shall his name be printed in the List of Members, until he shall have paid his admission fee and first annual subscription, and have returned to the Secretaries the obligation signed by himself. He shall at the first meeting of the Society at which he is present sign a duplicate of the obligation in the Statute Book of the Society, after which he shall be introduced to the Society by the Chairman.

Conditions of Resignation.

No Member shall be at liberty to withdraw from the Society without previously giving notice in writing to one of the Secretaries of his intention to withdraw, and returning all books or other property of the Society in his possession. Members will be considered liable for the payment of all subscriptions due from them up to the date at which they give written notice of their intention to withdraw from the Society.

Honorary Members.

XXIV. Gentlemen not resident in Victoria, who are distinguished for their attainments in science, literature, or art, may be proposed for election as Honorary Members, on the recommendation of an absolute majority of the Council. The election shall be conducted in the same manner as that of ordinary Members, but nine-tenths of the votes must be in favour of the candidate.

Subscriptions.

XXV. Members of the Society, resident in Melbourne, or within ten miles thereof, shall pay two guineas annually, and Members residing beyond that distance shall pay one guinea annually. The sub-

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* The obligation referred to is as follows :—

Royal Society of Victoria.

I, the undersigned, do hereby engage that I will endeavour to promote the interests and welfare of the Royal Society of Victoria, and to observe its laws, as long as I shall remain a member thereof.

(Signed)

Address

Date

scriptions shall be due on the 1st of January in every year. At the commencement of each year there shall be hung up in the Hall of the Society a list of Members, upon which the payments of their subscriptions as made by Members shall be entered. During July notice shall be sent to Members still in arrears. At the end of each year a list of Members who have not paid their subscriptions shall be prepared, to be considered and dealt with by the Council.

Entrance fees, &c.

XXVI. Newly-elected Members shall pay an entrance fee of two guineas, in addition to the subscription for the current year. Those elected after the 1st of July shall pay only half of the subscription for the current year. If the entrance fee and subscription be not paid within one month of the notification of election, a second notice shall be sent, and if payment be not made within one month from the second notice, the election shall be void. Members, resident in Melbourne, or within ten miles thereof, may compound for all Annual Subscriptions of the current and future years by paying £21; and Members residing beyond that distance may compound in like manner by paying £10 10s.

Life Membership.

Durations of Meetings.

XXVII. At the ordinary meetings of the Society the chair shall be taken punctually at eight o'clock, and no new business shall be taken after ten o'clock.

Order and mode of conducting the business.

XXVIII. At the Ordinary Meetings business shall be transacted in the following order, unless it be specially decided otherwise by the Chairman :—

Minutes of the preceding meeting to be read, amended if incorrect, and confirmed.
New Members to enroll their names, and be introduced.
Ballot for the election of new Members.
Vacancies among officers, if any, to be filled up.
Business arising out of the minutes.
Communications from the Council.
Presents to be laid on the table, and acknowledged.
Motions, of which notice has been given, to be considered.
Notices of motion for the next meeting to be given in and read by one of the Secretaries
Papers to be read.

XXIX. No stranger shall speak at a meeting of the Society unless specially invited to do so by the Chairman. Strangers.

XXX. At no meeting shall a paper be read, or business entertained, which has not been previously notified to the Council. What business may be transacted.

XXXI. The Council may call additional meetings whenever it may be deemed necessary. Additional Meetings.

XXXII. Every Member may introduce two visitors to the meetings of the Society by orders signed by himself. Visitors.

XXXIII. Members shall have the privilege of reading before the Society accounts of experiments, observations, and researches conducted by themselves, or original papers, on subjects within the scope of the Society, or descriptions of recent discoveries, or inventions of general scientific interest. No vote of thanks to any Member for his paper shall be proposed. Members may read papers.

XXXIV. If a Member be unable to attend for the purpose of reading his paper, he may delegate to any Member of the Society the reading thereof, and his right of reply. Or depute other Members.

XXXV. Any Member desirous of reading a paper shall give in writing to one of the Secretaries, ten days before the meeting at which he desires it to be read, its title and the time its reading will occupy. Members must give notice of their papers.

XXXVI. The Council may permit a paper such as described in Law XXXIII., not written by a Member of the Society, to be read, if for any special reason it shall be deemed desirable. Papers by strangers.

XXXVII. Every paper read before the Society shall be the property thereof, and immediately after it has been read shall be delivered to one of the Secretaries, and shall remain in his custody. Papers belong to the Society.

XXXVIII. No paper shall be read before the Society or published in the Transactions unless approved by the Council, and unless it consist mainly of original matter as regards the facts or the theories enunciated. Papers must be original.

XXXIX. Should the Council feel a difficulty in deciding on the publication of a paper, the Council Council may refer papers to Members.

may refer it to any Member or Members of the Society, who shall report upon it.

Rejected papers to be returned.

XL. Should the Council decide not to publish a paper, it shall be at once returned to the author.

Members may have copies of their papers.

XLI. The author of any paper which the Council has decided to publish in the Transactions may have any number of copies of his paper on giving notice of his wish in writing to one of the Secretaries, and on paying the extra cost of such copies.

Members to have copies of Transactions.

XLII. Every Member whose subscription is not in arrear, and every Honorary Member, is entitled to receive one copy of the Transactions of the Society as published. Newly-elected Members shall, on payment of their entrance-fee and subscription, receive a copy of the volume of the Transactions last published.

Property.

XLIII. Every book, pamphlet, model, plan, drawing, specimen, preparation, or collection presented to or purchased by the Society, shall be kept in the house of the Society.

Library.

XLIV. The Library shall be open to Members of the Society and the public at such times and under such regulations as the Council may deem fit.

Legal ownership of property.

XLV. The legal ownership of the property of the Society is vested in the President, the Vice-Presidents, and the Treasurer for the time being, in trust for the use of the Society; but the Council shall have full control over the expenditure of the funds and management of the property of the Society.

Committees elect Chairman.

XLVI. Every Committee appointed by the Society shall at its first meeting elect a Chairman, who shall subsequently convene the Committee and bring up its report. He shall also obtain from the Treasurer such grants as may have been voted for the purposes of the Committee.

Report before November 1st.

XLVII. All Committees and individuals to whom any work has been assigned by the Society shall present to the Council, not later than the 1st November in each year, a report of the progress which has been made; and, in cases where grants of money for scientific purposes have been entrusted to them, a statement of the sums which have been expended, and the balance of each grant which remains unexpended. Every Committee shall cease to exist on the 1st November, unless re-appointed.

Grants expire.

XLVIII. Grants of pecuniary aid for scientific purposes from the funds of the Society shall expire on the 1st November next following, unless it shall appear by a report that the recommendations on which they were granted have been acted on, or a continuation of them be ordered by the Council.

Personal expenses not to be paid.

XLIX. In grants of money to Committees and individuals, the Society shall not pay any personal expenses which may be incurred by the Members.

Alteration of laws.

L. No new law, or alteration or repeal of an existing law, shall be made except at the General Meeting in March, or at a Special General Meeting summoned for the purpose, as provided in Law XIX., and in pursuance of notice given at the preceding Ordinary Meeting of the Society.

Cases not provided for.

LI. Should any circumstance arise not provided for in these laws, the Council is empowered to act as may seem to be best for the interests of the Society.

Sections.

LII. In order that the Members of the Society prosecuting particular departments of science may have opportunities of meeting and working together with fewer formal restraints than are necessary at the Ordinary Meetings of the Society, Sections may be established.

Names and number of sections.

LIII. Sections may be established for the following departments, viz.:—

Section A. Physical, Astronomical, and Mechanical Science, including Engineering.
Section B. Chemistry, Mineralogy, and Metallurgy.
Section C. Natural History and Geology.
Section D. The Microscope and its applications.
Section E. Geography and Ethnology.
Section F. Social Science and Statistics.
Section G. Literature and the Fine Arts, including Architecture.
Section H. Medical Science, including Physiology and Pathology.

Meetings of Sections.

LIV. The meetings of the Sections shall be for scientific objects only.

Members of Sections.

LV. There shall be no membership of the Sections as distinguished from the membership of the Society.

Officers of Sections.

LVI. There shall be for each Section a Chairman to preside at the meetings, and Secretary to keep minutes of the proceedings, who shall jointly prepare and forward to one of the Secretaries of the Society, prior to the 1st of November in each year, a report of the Proceedings of the Section during that year, and such report shall be submitted to the Council.

Mode of appointment of officers of Section.

LVII. The Chairman and the Secretary of each Section shall be appointed at the first meeting of the Council after its election in March, in the first instance from Members of the Society who shall have signified to one of the Secretaries of the Society their willingness to undertake these offices, and subsequently from such as are recommended by the Section as fit and willing.

Times of meetings of Sections.

LVIII. The first meeting of each Section in the year shall be fixed by the Council; subsequently the Section shall arrange its own days and hours of meeting, provided these be at fixed intervals.

Corresponding Members, election of.

LIX. The Council shall have power to propose gentlemen not resident in Victoria, for election in the same manner as ordinary members, as corresponding members of the Society. The corresponding members shall contribute to the Society papers, which may be received as those of ordinary members, and shall in return be entitled to receive copies of the Society's publications.

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LIST OF THE INSTITUTIONS AND LEARNED SOCIETIES THAT RECEIVE COPIES OF THE "TRANSACTIONS OF THE ROYAL SOCIETY OF VICTORIA."

BRITISH.

Royal Society	London
Royal Society of Arts	London
Royal Geographical Society	London
Royal Asiatic Society	London
Royal Astronomical Society	London
Royal College of Physicians	London
Statistical Society	London
Institute of Civil Engineers	London
Institute of Naval Architects	London
The British Museum	London
The Geological Society	London
Museum of Economic Geology	London
Meteorological Society	London
Anthropological Society	London
Linnæan Society	London
Athenæum	London
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"Quarterly Journal of Science"	London
"Journal of Applied Science"	London
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Foreign Office Library	London
Agent-General of Victoria	London
"Nature"	London
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Philosophical Society	Cambridge
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Public Library	Liverpool
Owen's College Library	Manchester
Free Public Library	Manchester

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University Library	Edinburgh
Royal Botanical Society	Edinburgh
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Royal Geological Society of Ireland	Dublin
Royal Dublin Society	Dublin

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Acclimatisation Society	Paris
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Academy of Science	Stockholm
Academy of Science	Upsal
Royal Society	Upsal
The University	Christiania
Imperial Academy	St. Petersburg
Imperial Society of Naturalists	Moscow
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New Zealand Institute	Wellington, N.Z.
Otago Institute	Dunedin, N.Z.

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AND

PROCEEDINGS

OF THE

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VOL. XV.

Edited under the Authority of the Council of the Society.

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WILLIAMS & NORGATE, 14 HENRIETTA STREET, COVENT GARDEN, LONDON;
To whom all communications for transmission to the Royal Society of Victoria
from all parts of Europe should be sent,

10. Novr 1869. Accq recd

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Royal Society of Victoria.

1878.

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Royal Society of Victoria.

ANNIVERSARY ADDRESS

OF

The President,

Mr. R. L. J. Ellery, F.R.S., F.R.A.S., Government Astronomer.

(Delivered to the Members of the Royal Society of Victoria, at their Annual *Conversazione*, held on Thursday, 8th August, 1878.)

Your Excellency and Gentlemen of the
Royal Society,

It appears to be quite probable that in framing the rules of our Society that portion of the duties of president which refers to the delivery of an annual address was imposed principally as a check against undue pride and elation, likely to be engendered by the loftiness of the position. Whether such was actually the case can now only be surmised, but, as far as I am concerned, its effect in this direction is unmistakable; for, as the time again comes round for preparing and inflicting the prescribed punishment on a patient and long-suffering audience, ostensibly brought together for a little social and intellectual enjoyment, I make a deep and silent vow that the mantle and its responsibilities must find other shoulders for the future. My position here to-night affords another instance of how often such rash vows are only made to be broken, for, in spite of my resolve last year, you have again done me the honour of pushing me back into the presidential chair. I take this opportunity of thanking the members for their confidence, and of assuring them of my high appreciation of the trust, and of the duties and penalties attached thereto.

Since we met together on a similar occasion in August last, our Society has entered upon its twenty-first session and year of existence; and a brief account of its doings since that date, as well as of its present position and prospects, first claims our attention.

The painful duty here devolves on me to record the loss by death of two of our members—Mr. W. M. Cooke and Mr. Fred. C. Klemm. Since the *conversazione* in August last, the Society has held ten ordinary meetings, at which papers were read, exhibits made, and scientific subjects discussed. Your Council has also met regularly, and has had long and earnest deliberations on numerous matters concerning the welfare of the Society, some of which I shall presently especially refer to. The original discussions that have occupied the members at the ordinary meetings are as follow :—" On new Marine Mollusca," by Rev. J. E. Tenison Woods, S.J., F.G.S.; " Notes on Japan," by F. C. Christy, C.E.; " On the Probability that a Connection Exists between the Attraction of Gravitation and the Molecular Energy of Matter," by A. Sutherland, M.A.; " On the Comparative Power of some Disinfectants," by Dr. Jamieson; " On Palæozoic Actinology," by Robert Etheridge, F.G.S.; " On the Ratio of the Length and Height of Sea Waves," by S. R. Deverell; " Photographs on the Retina," by Dr. Jamieson; " Experiments in Gold Bullion Assay," by A. M. Smith; " On a New Self-registering Rain Gauge," by R. L. J. Ellery, F.R.S., F.R.A.S.; "On the Strength of Iron Columns," by W. C. Kernot, C.E.; " On a Point of Resemblance in the Respiration of Plants and Animals," by Dr. Jamieson. These have all been printed, and copies in a pamphlet form have been distributed among the members. In addition to these papers, there have been numerous brief notes, oral communications, and exhibits of the highest interest, which have made every meeting throughout the session a busy one. In no period of the history of the Society have our publications been in so forward a state as they are now; and I congratulate members on this fact, for which our thanks are due to the secretaries, who have in the face of difficulties at length been able to carry out the wishes of the Council in this matter. The fourteenth volume, containing our transactions to the end of 1877, was issued a few weeks ago, and is, I believe, already distributed. The papers of the present session are all either printed or in the press; for, as I informed you in my last address, the course had been adopted of printing and issuing a limited number of copies of all original papers immediately after the meeting at which they had been accepted. This plan has been found to work well, as it places the contributions at once in the hands of our members, and greatly facilitates the discussion of important papers, which frequently takes place at the meeting following that at which they have been read. Our library has been largely increased by donations from the numerous European, American, Asiatic, and Australian societies with which we interchange transactions, as well as from individuals and Government departments. The labour of acknowledging and arranging the very numerous contributions which come to us has become so great that your Council are now considering the best method by which this can be punctually done without the work becoming too burdensome to our hon. librarian. The rolls of the Society now number 128 members, 15 of whom are country members, 25 life members, and three corresponding members. This indicates a slight increase over our strength for the past few years, although our ranks are still too thin for so large and prosperous a colony as ours. Nevertheless, the Society may be congratulated on its present financial position. The Council have been able to clear off most of the debt incurred some years ago in altering and adding to the building, and to keep the printing of the transactions up to date. Our finances would, of course, be in the reverse position were it not for the Government grant which Parliament has liberally voted to the Society for the last few years; for, with the limited income derivable from our subscribing members, we could not possibly pay current expenses and for the printing of our transactions as well. As it is, we have a small balance to the good to pay off the remaining debentures coming due next year, amounting to about £70, and to assist in paying for some very necessary repairs and alterations to the building, which cannot much longer be delayed.

A few words concerning the future of the Society, and I will pass on to other subjects. Your Council has received applications from one or two kindred societies in Melbourne for permanent accommodation within this building, and, in futherance of views I expressed in my last address on this subject, have favourably entertained the idea of domiciling other societies devoted to science, literature, and art, under this roof, and have appointed a committee to consider the best means of doing so, whether by adding to the building in accordance with the original plans, or by doing as our architect and fellow-member, Mr. Joseph Reed, suggests—namely, to continue the floor of the library over the theatre and throw the whole upper floor into one chamber, while the space beneath will give two more commodious rooms. Whatever may be done, I trust the exterior of the building will not be overlooked, for it is beginning to have a really dilapidated appearance; and if we are to have, as it appears likely, a magnificent edifice in the Carlton Gardens, we should for shame's sake give a little more decent appearance to the outside of the house of the chief scientific body of the colony.

In considering the comparatively small number of members of which this Society is composed in proportion to our population, prosperity, and intelligence, several members of your Council have from time to time suggested the desirability of broadening its basis, and the Council has given these suggestions earnest consideration. As you are aware, our constitution provides that members shall pay two guineas entrance fee and two guineas annual subscription, except in the case of country members, where the annual subscription is one guinea only. Now, it has been suggested that this subscription is almost prohibitive to many of the young men of our community whose tastes and education lead them towards our ranks, and whose enrolment is much to be desired; and it became a serious question whether the annual subscription should not be reduced. The Council, however, ultimately decided to recommend the Society to add to its constitution the power to admit associates at half fees, whose privileges would, with a few exceptions, be equal to those of members, and a committee has been appointed to devise a scheme which will be laid before a special meeting of the members. If such a course is adopted, I have little doubt we shall soon have a very welcome addition to our active members, and that we shall be able to resuscitate several of the sections for which our constitution provides. You may recollect that in former addresses I advocated a pet idea of mine; and although this has got no further than it was at our last gathering of this kind, I do not intend to abandon it, and hope, with your assistance, yet to see it realised—I mean the occasional delivery in this hall of brief and special lectures for the record or demonstration of new interesting facts in physical and other sciences, by members of the Society to members and their friends.

This will, I think, place you in possession of the principal facts in connection with the Society's affairs; and I will now briefly review the progress made by some of the public departments and societies in Melbourne, whose aims are kindred to our own. At the Observatory the usual work in astronomy, meteorology, &c., has been carried on without interruption. The great telescope has been occupied with its special work—observation of the southern nebulæ—and it continues to perform satisfactorily. I regret to say, however, that the drawings of the nebulæ already observed, and which were being lithographed at the time of my last address, are not yet published. The scheme of intercolonial meteorology, concerning which I spoke at some length last year, is being gradually improved, and, since the completion of the Western Australian line, our weather telegrams embrace the whole of the south coast of Australia, from King George's Sound to Cape Howe. The undertaking, however, labours under a great disadvantage in these colonies as compared with Europe and America, inasmuch as the precedence and prompt despatch which is conceded to weather telegrams in those countries has not yet been secured for ours. In October last telegrams from America and England were received at the Observatory, requesting a look-out for supposed satellites of Mars. Diligent search was made with the great telescope, whenever the weather was favourable, but with no decided results, and it is doubtful if either of the satellites now known to exist was seen at our Observatory. This failure was somewhat unaccountable, as subsequent news informed us that the brightest of the two satellites had been seen by much smaller telescopes than our reflector. It may be stated, however, that, owing to an interruption in telegraphic communication, the telegram referred to was delayed fourteen days. Mars was rapidly increasing his distance from us, and after the message was received a period of cloudy weather still further delayed our search until the planet had receded enormously from the position in which its satellites were discovered, or subsequently seen by any except the most powerful telescopes.

The transit of Mercury across the sun's disc in May last was a noteworthy event, and its later phases were successfully observed at the Observatory, but no new points of interest in connection with this phenomenon were noted. The opposition of Mars on the 6th September last year occurred when that planet was unusually near to the earth, and a remarkably good opportunity presented itself of again determining the solar parallax. In conjunction with European and American observatories, we undertook a series of observations for parallax in declination, and succeeded in securing a fine set of measures, extending from 21st July to 22nd October, the results of which will probably be known by the end of the year.

Encke's comet again returned to perihelion on July 26th. Last mail I received a particular request from Professor Asten, of Pulkowa, that we should endeavour as it came south to obtain as late observations of it as possible. It is now too near the sun to be seen, but we hope to pick it up in a few days. This comet was first observed in 1786, and since that year it has made 28 consecutive revolutions round the sun with remarkable regularity; in only 20 of these, however, has it been observed. In 1822 it was seen only at the Observatory of Paramatta. Great interest is attached to the observation of this comet, owing to the fact that each succeeding revolution is made in less time than the last, thus showing that the comet is diminishing its mean distance from the sun. This would appear to indicate that it experiences resistance in its course, which, if continued, will ultimately cause it to fall into the sun. At the present time its revolution round the sun is accomplished in a period which is more than two days less than at the time of its discovery in 1786.

Some important additions to the literature of botanical science have been made during the past year. Our fellow-member, Baron von Mueller, the Government botanist, has published the tenth volume of the well-known *Fragmenta Phytographiæ Australis,* as well as the first volume of his work on the plants of New Guinea, to which I referred in my last address. The learned baron in this work demonstrates the close affinity existing between the plants of this large island and those of North Australia. A further supplement has lately been added to the work on *Useful Plants Suitable for Cultivation in this Colony,* and another publication which promises to be of great interest and utility has been commenced. This is a description, with illustrations, of the eucalyptus trees, the first eleven plates of which have already been issued. The publication of an illustrated book containing a full description of all the plants hitherto found in Victoria has lately been authorised, and it is now in the press. And last, though not least, I would mention a work on the organic constituents of plants, translated from the German of Professor Wittstein, and published here privately by Baron von Mueller, with many new notes and observations. This book is eminently calculated to assist in the local analysis of our native vegetation, and will, no doubt, prove of great utility in this respect.

Another work, by Mr. Guilfoyle, the curator of the Domain and Botanical Gardens, entitled *Australian Botany,* must not be overlooked, more especially as it is likely to supply a great want felt by young students of this science in the colony.

The National Museum still continues to advance its collections illustrative of the different branches of natural science towards systematic completion, and in several departments it is now no easy matter to obtain the rarities which alone are required to fill up the gaps in the general series of the living and fossil forms of the animal kingdom, as well as in the sections of geology and mineralogy; 42,292 species of the higher classes are catalogued as named in the cases, besides many thousands of the lower classes named, but not as yet entered. The efforts of the director towards perfectly displaying the collections which he has got together, named, and classified, so as to show fairly the principles of classification adopted, are seriously hampered by the non-completion of the building. Parliament voted £4000 for this purpose last year, but difficulties arose and the money has lapsed. It is to be hoped that such a national work as the completion of the museum may not be further retarded from this cause. The collection continues in great beauty and freshness of preservation, and the number of visitors is constantly increasing, 102,572 being recorded for the year ending on 30th June last. To the publication of six of the decades of the *Palæontology of Victoria,* which have been very favourably received by the scientific press of Europe, there has just been added the first decade of the *Zoology of Victoria,* with beautiful illustrations in colours of the snakes, fishes, insects, &c., of the colony, the originals, as in the former work, being all in the national collection. The other decades will quickly follow, and may be expected to give an impetus to the study of the natural history of the colony.

The Public Library and Museums, with the thriving Schools of Technological Science and Fine Arts, which have grown up under its shelter, form an institution of which our community may be most justly proud. Our members will be pleased to hear that in the laboratories there are now 47 students at work. These are chiefly miners, metallurgists, electro-platers, dyers, manufacturing chemists, soap and candle makers, &c.; their studies, of course, have a direct utilitarian bearing, and it is gratifying to learn that several have worked out new processes to apply to their trade. A course of elementary lectures on chemistry has been delivered by Mr. F. Dunn, to which the pupils of the higher classes of the public schools were invited. They were well attended by an average of over 200 adults and scholars, and it is intended to continue the course. The classes for painting in the National Gallery now number 49, and the School of Design 110 students—a fact which is significant of the increasing hold the fine arts are taking upon the community, and a sure indication of its intellectual advancement.

As regards the advancement of medical science in the colony, we need only glance over the past year's proceedings of the Medical Society of Victoria to be assured that this all-important branch of knowledge is not languishing in our midst; and the fact that the Society have lately built a new and commodious hall, in which to hold their meetings and keep their library, is additional evidence of progress. Among the proceedings of the past year, while we see the usual predominance of practical reports of cases, statistics, and more purely utilitarian matter, it is gratifying to find that the larger subjects of chemico-physiology, etiology, and research into the propagation and prevention of disease, have received a share of attention. As an example, I may cite Dr. Day's paper on "The Chemico-physiological Effect of Nascent Oxygen," and Dr. Patrick Smith's able contribution "On the Etiology of Typhoid Fever." No subject in the whole realm of medical science has greater claims for investigation than that involved in the latter paper, especially in our community, where, evidently favoured by climatic vicissitudes, this fell disease seems to be stalking upon us with annually-increasing strides. Any really scientific research, reasoning, or even trustworthy statistics concerning the cause, propagation, and prevention of typhoid fever, should be hailed as a public boon. I therefore refer with pleasure to the fact that the literature of the subject has been reinforced by a very important publication in Melbourne from the pen of Mr. Wm. Thomson, entitled *The Cause and Extent of Typhoid Fever.* The very decided and opposite opinions held among our medical brethren as to the cause and propagation of this dreadful malady, indicate the necessity of increased research into its etiology, which, it is to be hoped, will be prosecuted with the steady view of discovering the truth, rather than of advocating favourite opinions and speculations. Human life is largely concerned in this question, and it takes no great foresight to estimate of what surpassing value any means of preventing and staying the spread of this disease will yet become. The true etiology once found, the hope that it will then be possible to banish typhoid fever from any community is surely not an unreasonable one.

Looking back upon the additions to knowledge that have been made during the past year in the various branches of science, our attention is arrested by several subjects of more than ordinary interest, to one or two of which I would now refer.

The results obtained from the transit of Venus observations have not yet been finally dealt with, although partial deductions from British and French observatories have been published. Last summer the Astronomer Royal reported to Parliament on "The Value of the Mean Solar Parallax Deducible from Observations of the Transit by British Observers," and the resulting solar parallax was stated to be 8″·764. Mr. Stone, of the Cape of Good Hope, who is one of our highest authorities upon this subject, questions the correctness of the conclusions arrived at in this report, and, in an article which appears in the *Monthly Notices* of the Astronomical Society, he gives the result of the observations treated in his own way, wherein the parallax differs sensibly from the Greenwich deductions. In the same periodical, Captain Tupman, who had charge of the Greenwich computations, referring to Mr. Stone's paper, speaks of the method of treatment of the observations of ingress at Greenwich as unsatisfactory. This throws more weight on Mr. Stone's results, which are here compared with the Greenwich and with earlier deductions:—

	Parallax.	Distance.
	″	Miles.
1. Greenwich results from transit of Venus, 1874	8·764	93,400,000
2. Mr. Stone's results from do.	8·884	92,138,000
3. From re-discussion of transit of Venus observations in 1769	8·910	91,870,000
4. From observations of Mars, 1862	8·940	91,561,000
5. M. Cornu's observations of velocity of light ...	8·860	92,388,000
6. Le Verrier's classical deductions from planetary perturbations	8·880	92,180,000

These figures will give an idea of how modern observations approximate to the solar parallax, but they must not be taken as absolutely conclusive, as the results of the American and German expeditions, as well as those of the photographic methods adopted by both British and American parties, have yet to be taken into account. Moreover, the recent opposition of Mars has furnished another excellent opportunity of testing the question, and there can be little doubt that most trustworthy results will be obtained from the combination of the northern and southern observations which were secured from August to November last year, and towards which our Observatory, as already mentioned, has contributed a very complete series of measures. The discovery at Washington by Mr. Asaph Hall of two satellites of a planet hitherto regarded as being companionless, like Venus and Mercury, marks a new era in astronomical science, and adds another laurel to the many already won in the same field by our American cousins. I have already spoken of the fruitless search we made here, and the probable cause of our failure, and I may now add that this fact, in connection with the comparative ease with which the satellites were seen with the 26-in. refractor at Washington, has led to comparisons between large refractors and reflectors unfavourable to the latter; but in this I do not acquiesce, for, during our search, stars, far more minute than the satellites, were traced close up to the edge of Mars, and had we known of or suspected the existence of satellites in August or September, and had favourable weather, I feel confident we should have found them and kept them in tow; as it was, our watch commenced only late in October, in broken weather. "Moonlit" (not "moonless") Mars is undoubtedly accompanied by two satellites at least, and the observers at Washington suspect the existence of a third. The most remarkable feature in connection with these bodies is their exceeding smallness, and their nearness to the primary. The inner satellite cannot be 4000 miles from the surface of Mars, or less than one-sixtieth of our moon's distance from us; and should there be any Martial astronomers with good telescopes, they could not be long in doubt as to whether their moons are inhabited or not. The estimated diameter of the smallest of these bodies is only about seven miles, giving a surface of 154 square miles, equal to a few Australian sheep-runs. The larger and inner satellite is probably about thirty miles in diameter, and with a superficial area of 2826 square miles. The minuteness of these bodies renders it highly improbable that they will again be seen until the next near approach of Mars to the earth, about fifteen years hence. Our knowledge of the constitution of the sun has again been further supplemented by help of the spectroscope. The spectrum of hydrogen gas, in the bright line form in the chromosphere and reversed in the photosphere, has long since been recognised, but the presence of no other of our known gases had as yet been ascertained. Professor Draper, however, about July last year obtained photographs showing bright lines of oxygen at the extreme blue end of the spectrum occupying the region of Fraunhöfer's G line, and between G and H, and, therefore, nearly at the limit of the visible spectrum. Professor Draper also considers that the photographs afford evidence of the existence of nitrogen, which also appears in the form of bright lines. This discovery will necessarily lead to some modification of the hitherto adopted views of the constitution of the sun's surface, and adds another to the already long list of telluric elements found to exist upon our luminary.

In my last address I referred at some length to the then recent invention of the telephone. Since then this wonderful little instrument has been greatly improved, and is now in actual use in Melbourne, not only as a scientific toy, but as a means of communication. We had no sooner become familiar with the telephone than we were astounded by accounts of a still more wonderful apparatus—the "phonograph"—by which, it was stated, sounds and human speech could be automatically imprinted on a sheet of tinfoil and reproduced with all the original intonations at will and at any subsequent time. Still later we hear of the "microphone," by which the faintest sounds can be heard by means of the telephone, highly intensified, and at long distances from their source. All of these instruments are more or less familiar to our members, for they have been exhibited, explained, and commented upon at several of the ordinary meetings, and I believe there are specimens of them all in the building to-night. The principles recognised in the action of the telephone and microphone point to the existence of an entirely new field for experiment in some of the less understood properties of magnetism and electricity; and although their practical applications are as yet limited, there can be but little doubt that they will eventually become of great value to the electrician, physicist, and even to the surgeon; indeed, the value of the microphone in surgical diagnosis has already been demonstrated. While a wonderful future is predicted for the phonograph, at present, if we except its power of giving a peculiar graphic representation of multiple and complex sounds, it cannot be said to be out of the category of scientific toys.

That branch of biological science which has become known as the germ theory still justly occupies the attention of many of the foremost investigators in physics, physiology, and pathology, while diligent inquiries are also being made by many less known but earnest seekers after the truth. The burning part of the question a few years ago was, whether or not the lower class of organic life was ever produced by spontaneous generation; this, I think, may be considered to be finally answered in the negative by the conclusive results of the experiments of Tyndall, Cohn, and others. Some of these results were described by our vice-president, Mr. Foord, at a former *conversazione*, in which it was demonstrated that a temperature of 212 deg. Fahrenheit, long continued, completely sterilised inoculated solutions. The old maxim, *Ex nihilo nihil fit*, therefore, still holds true in the arcana of nature. The most important and interesting phase this question has more recently assumed has reference to the influence exercised by low forms of organic life upon the human body in health and disease. Professor Tyndall's recent experiments show how difficult it is to free the air we breathe and live in from the myriads of microscopic and ultra-microscopic germs, plants, and animals that pollute it, but that, with proper precautions, it is not only possible to do so, but to keep it so. In air thus thoroughly divested of all germs and organic life, animal and vegetable substances which we have generally regarded as possessing inherent properties of decay and corruption are found, when once sterilised by boiling, to remain pure and unchanged for years. There now remains little doubt, therefore, that the decay of animal and vegetable matter is entirely due to parasitic organisms which assert their dominion the instant the vital forces in either cease, or even fall below a certain standard; there is no decay without these, and Professor Tyndall shows how they can be kept from their prey. Under the ordinary circumstances of life these organisms doubtless play a beneficial part in the great scheme of nature, but the subtle and invisible power which has thus been revealed to us is also capable, under certain conditions, of acting most deleteriously upon human health and life, and there is a steadily-growing conviction that they play a most important, if not the only part, in many contagious as well as simply septic diseases. Should this be demonstrated beyond a doubt, which I think far from improbable, the results arrived at by Professor Tyndall unmistakably indicate the direction which any effort at prevention of such diseases must take; and it becomes manifest that no researches in etiology can claim to be scientific or aiming at the truth which ignore the grand work that has been, and is being, done in this branch of biological science. One of the most remarkable achievements in physical science effected during the present year is the liquefaction of oxygen, nitrogen, and hydrogen gases, and the solidifaction of the last named—results approached by the experiments of M. Calletet, in Paris, and about the same time realised in a far more pronounced form by M. Raoul Pictet, at Geneva. Our experience of the three states of matter—the solid, liquid, and gaseous forms, and of the facility with which water, for example, passes from solid ice to the fluid state, and from the latter to the state of vapour—has long since led to the hypothesis which assumes that all material substances which are not decomposed by alteration of temperature are capable, under suitable influencing circumstances, of passing through these three phases; and very much effort has been devoted to bringing refractory gaseous bodies within the boundaries of the assumed law. In 1823, Michael Faraday, at the suggestion of Sir Humphrey Davy, heated hydrate of chlorine in an hermetically sealed glass tube, and made the discovery of liquefied chlorine gas. Faraday made the discovery, and, unaided, puzzled out the proper interpretation of the result of the experiment; but that Davy had a penetrative insight into the nature of the chemico-physical problem involved in it, seems obvious from his own words. "One of three things," he says, "might be expected to happen as the result of the experiment—either that the solid and crystalline hydrate of chlorine would become a fluid, or that a decomposition of water with formation of euchlorine would take place, or that the chlorine would separate in a condensed state." He goes on to point out how much more is to be effected in future liquefaction experiments from pressure obtained in sealed vessels than from refrigeration, and further how the agency of pressure may be assisted by artificial cold in cases where gases approach the state of vapour. Faraday, in the course of his labours, reduced many gases, and Thilorier in 1834 contrived an apparatus for liquefying carbonic acid in quantity, and reducing it to the state of snow, which, as a means of attaining very low temperatures, greatly assisted the course of subsequent experiment, and indeed is now largely used in physical investigation and in the arts. In 1845, by the combination of pressure and refrigeration, Faraday succeeded in adding to the list of gases susceptible of assuming the liquid and solid states; but still oxygen, nitrogen, and hydrogen held out against all experimental coercion, and in that sense remained still in the category of permanent gases. This is how the case has stood until the experiments of M. Calletet, and more especially those of M. Pictet, have been crowned with the success of breaking down the dividing wall between gases and vapours. The collation of Davy's remarks appended to Faraday's paper on the liquefaction of chlorine (as already given) with Pictet's method and his theoretical views, is certainly a matter of interest, but as Mr. Barton during the evening will explain the details of M. Pictet's experiments, and as time presses, I need say no more on this highly interesting subject. One word, however, may be added concerning the converse problem of the liquefaction and vaporisation of refractory solids. Carbon uncombined is known only in the solid state; to melt and vaporise it is a work yet to be accomplished, but with the results recently achieved we are encouraged to hope for further triumphs, and the ultimate confirmation by actual experiment of all that has been premised on theoretical or mathematical grounds concerning the several states of matter; or should we fail in this, we may yet hope for experimental proof of what is defective in the hypothesis, by means comparable to those by which the almost tenable phlogistic hypothesis of Stähl was overturned on the application of the deep-searching experimental method of Lavoisier.

TRANSACTIONS.

ART. I.—*A New Form of Circuit Closer for the Firing of Torpedoes.*

BY R. E. JOSEPH, ESQ.

[Read 11th April, 1878.]

ART. II.—*Photographs on the Retina.*

BY JAMES JAMIESON, M.D.

[Read 11th April, 1878.]

AT the meeting of the Berlin Academy of Sciences, on 23rd November, 1876, there was read a communication from Professor Franz Boll, of Rome, on the subject of some remarkable properties of the retina, which had not till then been described. He experimented first with frogs, in the following manner :—A frog, which had been kept for some time in the dark, was beheaded, and its eye removed as quickly as possible. The front of the eye was cut off with scissors, and the retina lifted from the dark layer behind, when it was seen to be of an intense red colour, which rapidly faded, so that in ten to twenty seconds it had disappeared. For the next thirty to sixty seconds the retina had a satiny lustre, which also gradually disappeared, leaving the structure quite colourless and transparent. Boll found that the colour has its seat in the rods, and not in the cones; and that it is found in all animals in which there is a well-developed layer of rods. Even in the rods it is confined to the outer portion, which is made up of thin plates. Along with these red rods Boll found a smaller number of green ones, which also undergo some changes of shade under the influence of light, but which have not had their properties well investigated; and he had not, indeed, been able to discover whether they occur in any other animals than the amphibia. He tried the effect of exposing the eye to light of different colours, and obtained interesting results helping to explain some of the curious phenomena of colour-blindness, to which reference will be made further on. His communications are to be found in the *Monatsbericht* for November, 1876, and January and February, 1877.

The subject obtained considerable development in the hands of Professor W. Kühne, of Heidelberg, who has published his results in a collected form in the *Heidelberg Untersuchungen*, Vol. I., 1877, with which I am acquainted only at second hand in *Schmidt's Jahrbücher*, No. 10, December, 1877. He found that the colours seen by Boll are not merely owing to refraction, but that there is an actual pigment which he has succeeded in isolating in the form of a solution. His first efforts to obtain *optograms* failed altogether; but he has had more success subsequently by the help of improved methods. One of his experiments was conducted in the following way:—The head of a rabbit, with the one eye fixed open, was held in front of an opening in a window shutter, and after being covered for five minutes with a black cloth, was exposed to the light. The animal was then quickly decapitated, the eye removed under the sodium light, opened, and laid in 5 per cent. solution of alum. The other eye was exposed to the light after decapitation. Both retinas were examined next morning, and found of the usual milky appearance, but close inspection showed on both a sharply defined quadrangular figure of the same form as the opening in the shutter. In the eye which had been acted on during life there was still a reddish colour, but in the other the figured spot was quite white. In another experiment Kühne succeeded in getting a complete picture of a window with one round-topped and six square panes, white on a red ground, the cross markings being also red. The method now followed is, to place the head of an animal, or the extirpated eye in a box, whose lid is formed of a plate of dim glass, on which can be laid figures cut out of black paper. The retina, on which the figure has become printed, is laid on a porcelain plate, and dried over sulphuric acid, when the picture is found to be more permanent. The eyes of other animals than the rabbit have been used, and Kühne has found that of the ox to be sensitive for about an hour after death. The presence of the pigment is not dependent on the retina being in a state of freshness as regards its functional capacity. It is bleached only by light; very quickly (in about thirty seconds) by direct sunlight, and in twenty to thirty minutes by gaslight; whilst in the dark or in the sodium light it does not disappear in less than twenty-four to forty-eight hours. During life, and even for some time after death, the colour is continually renewed, and does not owe its existence therefore to the continuance of the circulation of blood in the eye, but to the layer of epithelium which connects the outer portion of the rods with the choroid.

It was mentioned that Kühne had obtained the red pigment in solution. It is got by adding a clear watery solution of crystallised ox gall to the fresh retina, on which it has a remarkable effect, causing the plates composing the outer section of the rods, to fly asunder like coins from a roll, and then wholly disappear. The solution thus obtained is of a rich carmine hue, and gradually bleaches in the light, passing first into yellow. Monochromatic light also acts on it in the same way, though more slowly, the most active being green and yellowish green (in about fifteen minutes), then blue in about an hour, violet still longer, and pure (spectral) red having very little influence on it.

The part played by retina red in the physiology of vision can in the present state of our knowledge be little more than matter of speculation. That it is indispensable to mere visual perception can scarcely be held, since it is absent, or at least has not yet been found, in the retina of many animals which certainly see—such as the pigeon, the hen, the bat; and further is not to be found in the yellow spot, the seat of direct vision in man, which has no rods. Its importance, however, can scarcely be doubted when we consider that it has been discovered in almost all animals, and also in view of the remarkable influence exerted on it by ordinary white light. Two cases reported by Dr. Adler, of Vienna, also testify to its importance. In one of these an eye which had been blind for several years had no trace of the red colour. In the other case one eye was partially blind, and the affected half of the retina was colourless, the other half showing a distinct rose tint, like that in the sound eye.

It may serve in some way for the perception of colours, the varying effect on it of different kinds of coloured light pointing in that direction. Boll noticed that the microscopic appearance of the coloured rods was very much the same in animals which had been kept for a time under red and green glass, while it differed considerably when the cover had been blue; and he connected this with the well-known fact that colour-blind persons readily confuse red and green, but rarely red and blue. An important question raised is about the probability that in every act of visual perception there is a picture of the object seen printed on the retina by the action of light on this pigment. If this is so, we may suppose that the nerve fibres are stimulated in varying degrees by the colouring matter, according to the extent to which it has undergone the bleaching process. Of course it is easy to point out difficulties attaching to such opinions. It must be regarded as certain, however, that in the retina we have not merely a sensitive surface, like the photographer's plate, but a self-acting photographic workshop, the retina not only receiving an impression, but wiping off the old picture and charging itself in preparation for another. Speculations on the subject for the present have perhaps little value, and exact knowledge is likely to increase slowly, since in animals we can scarcely know with certainty how much is actually seen, and man cannot be made the subject of experiments. Of course new modes of investigation may unexpectedly be discovered, and lead to unexpected extensions of knowledge.

ART. III.—*Sir William Thomson's Electric Replenisher.*

BY F. J. PIRANI, ESQ., M.A.

[Read 11th April, 1878.]

ART. IV.—*Some Experiments in the Gold Bullion Assay.*

BY ALFRED MICA SMITH, B.SC.

[Read 16th May, 1878.]

THE following series of assays were undertaken at the suggestion of Mr. George Foord, of the Melbourne branch of the Royal Mint, and performed there some time ago. The demonstrations which constitute Part I. are here offered as a communication in the hope that they may be of use for reference by some who may not themselves have the opportunity of performing the exercises, as well as by others, who, on going over the same ground for practice, may use these results for comparison with their own. The method adopted was the rigorous system in use in the Melbourne Mint.

PART FIRST.

EXPERIMENTAL DEMONSTRATIONS.

I.

To demonstrate the facts on which "quartation" is based, or to show the limits of the proportion of gold to silver within which it is necessary to keep in order to part an alloy of these metals, at the same time to note the colours of the alloys throughout the operation.

From the data obtained to construct the curve of "surcharge."

Synthetical alloys of gold and silver were prepared, ranging from an alloy containing 5 per cent. of gold up to fine gold, and of the uniform weight of 35 grains each. Twenty places, as detailed in Table A, were cupelled each with copper disc (1⅓ grains) and lead case (84 grains), the cupellations occupying 21 minutes. The colour and appearance of the buttons having been noted, they were flatted, annealed, rolled to the 13-1000th of an inch in thickness, annealed, and coiled according to the usual routine. The parting was then conducted as follows :—

Nos. 1 to 5, inclusive, were parted separately in flasks. Each was boiled in 1½ oz. of 1st acid (sp. gr. 1.17) for ten minutes beyond the time at which the red fumes cease to be evolved.

Washed with distilled water.

Boiled for ten minutes in 1 oz. of 2nd acid (sp. gr. 1.26).

Boiled for ten minutes in 1 oz. of 3rd acid (sp. gr. 1.3).

Washed in two waters, transferred to crucible and annealed.

Nos. 6 to 20 were parted together in the 20 platinum tray.

Boiled in 22½ oz. 1st acid, and for ten minutes after red fumes cease.

Washed in hot distilled water.

Boiled for ten minutes in 15 oz. of 2nd acid.

Boiled for ten minutes in 15 oz. of 3rd acid.

Washed in two successive hot waters, and annealed.

TABLE A.—To Show the Proportions that will Part.

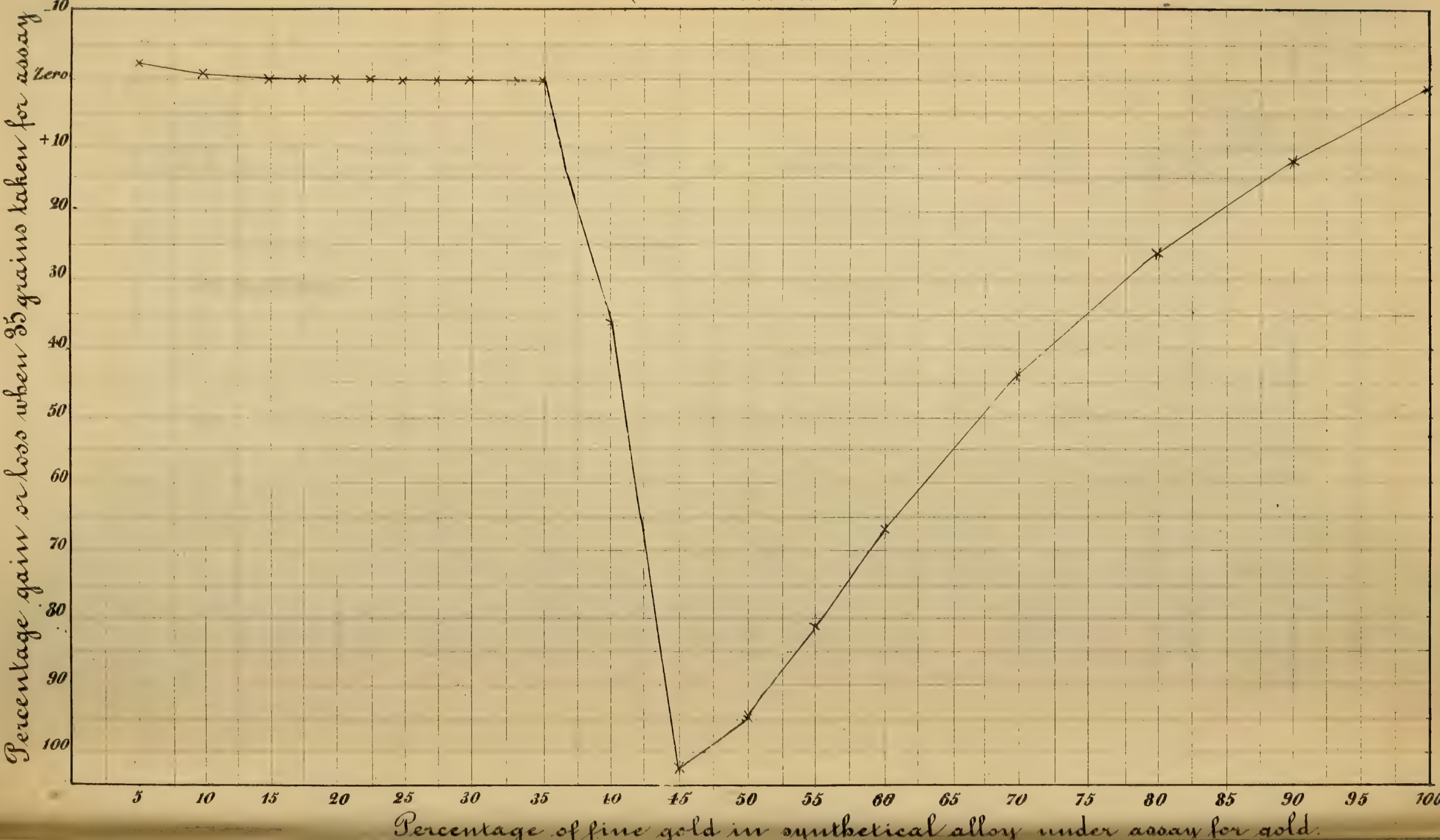
	A.	B.	C.	D.						
No.	Percentage of fine gold desired in synthetical alloy of gold and silver.	Proportion of Gold to Silver.	Grains of Gold in 35 grains of alloy.	Equivalent in Gold of fineness 991.	Fine Silver added to Gold in D. to make 35 grain button.	Error in weighing D.	Observations on the appearance of the button after cupellation.	Weight of Cornets after parting.	Difference from weight of Gold taken.	Percentage, loss or gain (see Diagram I.).
		Gold to Silver		Grains.	Grains.	Thousandths of a grain.		Grains.	Grains.	
1	5	1 to 19	1·75	1·7659	33·2341	0	Dimpled } vegetated, requiring	1·714	− 0·036	− 2·057
2	10	1 ,, 9	3·5	3·5318	31·4682	0	Dimpled } refusion	3·486	− 0·014	− 0·4
3	15	1 ,, $5\frac{2}{3}$	5·25	5·2977	29·7023	0	Smooth and dimpled	5·245	− 0·005	− 0·095
4	$17\frac{1}{2}$	1 ,, $4\frac{7}{10}$	6·125	6·1806	28·8194	0	Smooth and dimpled	$6·127\frac{1}{4}$	+ $0·002\frac{1}{4}$	+ 0·042
5	20	1 ,, 4	7·0	7·0636	27·9364	0	Slightly crystallised	7·004	+ 0·004	+ 0·051
6	$22\frac{1}{2}$	1 ,, $3\frac{1}{2}$	7·875	7·9465	27·0535	$-\frac{1}{2}$	Smooth, but undimpled	7·878	+ $0·003\frac{1}{2}$	+ 0·044
7	25	1 ,, 3	8·75	8·8295	26·1705	$-\frac{1}{2}$	Smooth, but undimpled	8·756	+ $0·006\frac{1}{2}$	+ 0·074
8	$27\frac{1}{2}$	1 ,, $2\frac{3}{5}$	9·625	9·7124	25·2876	0	Smooth and dimpled	9·630	+ 0·005	+ 0·052
9	30	1 ,, $2\frac{1}{3}$	10·5	10·5954	24·4046	0	Smooth, but undimpled	10·506	+ 0·006	+ 0·071
10	$33\frac{1}{3}$	1 ,, 2	11·6	11·7726	23·2274	0	Half crystallised, half smooth	11·6865	+ 0·0199	+ 0·176
11	35	1 ,, $1\frac{9}{10}$	12·25	12·3613	22·6387	− 1	Smooth, but undimpled	12·288	+ 0·039	+ 0·310
12	40	1 ,, $1\frac{1}{2}$	14·0	14·1271	20·8729	$-\frac{1}{2}$	Uniformly crystallised } frosty ap-	19·08	+ $5·080\frac{1}{2}$	+36·291
13	45	1 ,, $1\frac{1}{5}$	15·75	15·8930	19·1070	0	Uniformly crystallised } pearance	31·86	+16·11	+102·286
14	50	1 ,, 1	17·5	17·6589	17·3411	0	†Crystallised, with bright interspaces	$34·064\frac{1}{2}$	+16·5645	+94·654
15	55	1 ,, $\frac{8}{10}$	19·25	19·4248	15·5752	$+\frac{1}{2}$	Do. do. do. (green distinct)	34·842	+15·5915	+80·909
16	60	1 ,, $\frac{7}{10}$	21·0	21·1907	13·8093	0	Crystallised	35·1155	+14·1155	+67·217
17	70	1 ,, $\frac{2}{5}$	24·5	24·7225	10·2775	0	‡Crystallised	$35·232\frac{1}{2}$	+10·7325	+43·806
18	80	1 ,, $\frac{1}{4}$	28·0	28·2543	6·7457	0	Crystallised (brittle)	$35·375\frac{1}{2}$	+ $7·375\frac{1}{2}$	+26·340
19	90	1 ,, $\frac{1}{9}$	31·5	31·7861	3·2139	− 1	Crystallised (very brittle)	$35·464\frac{1}{2}$	+ $3·965\frac{1}{2}$	+12·589
20	100	1 ,, 0	35·0	35·0 *	0·0	0	Gold coloured by the retained copper	35·568	+ 0·568	+ 1·623

* Of fineness ·999962. † Faint green just perceptible. ‡ Gold colour just perceptible.

NOTE.—Cornets Nos. 1, 2, 3, went to pieces—1, finely divided; 2, less finely divided; 3, in pieces, some large, some small.

DIAGRAM I.

Showing the proportions that will part
or the Surcharge on alloys of gold and silver
(from **Table A** lest column)



RESULTS.

PARTING PROPORTIONS.—The alloy containing 15 per cent. of gold (or 1 of gold to $5\frac{2}{3}$ silver) went to pieces, the minuteness of division increasing as the percentage of gold decreased.

The alloy containing $17\frac{1}{2}$ per cent. of gold (or 1 of gold to 4 7-10th silver) did not go to pieces, nor did the alloys with higher percentages of gold.

When the ratio of the gold to the silver was 1 gold to 4 7-10ths silver, or 1 gold to $2\frac{1}{3}$ silver, or between these, the cornet parted well.

SURCHARGE.—With the alloy containing 15 per cent. gold (1 gold to $5\frac{2}{3}$ silver) and those with more silver, there was negative surcharge.

With the alloy containing $17\frac{1}{2}$ per cent. gold (1 gold to 4 7-10ths silver), and those with less silver, there was positive surcharge.

Between the alloys containing 35 per cent. gold (1 gold to 1 9-10ths silver) and 40 per cent. gold (1 gold to $1\frac{1}{2}$ silver) there was a sudden great rise in surcharge exhibited, the maximum being near the alloy containing 45 per cent. gold (1 of gold to 1·22 silver).

In Diagram I. these relations are made visible.

COLOUR.—*Buttons.*—Beginning the examination with No. 1, and passing downwards, the gold could be detected first in the button containing 50 per cent. of gold (·500) by the faint green tinge it exhibited; this colour increased in depth with the percentage of gold until the button containing 70 per cent. of gold (·700) was reached, at which point the warm colour of gold appeared. This again kept deepening until the last, in which the gold was tinged by the residual copper.

Cornets.—After coming from the acids—
Nos. 6 to 13, inclusive, were dark;
Nos. 14 to 19 bright; 20 golden.

After annealing—4 to 12 bright yellow.
13 greenish yellow.
Increasing to 15.
16—19 silvery green increasing.
20 coppery.

II.

To show the progress in parting: the surcharge at the end of stated intervals between the time at which the red fumes cease and the finish of the parting process.

Thirty places prepared (Table B), each 10 grains of fine gold (·99984), with 25 grains of fine silver. Copper and lead case, as before. Cupelled for 21 minutes, flatted, annealed, passed twice between rollers set at 8-1000ths of an inch, annealed, coiled, and placed in thimble tray.

Boiled together in large beakers:—

For 22 minutes in 45 ozs. 1st acid, by which time red fumes off.

For 15 minutes longer in 1st acid (one being removed per minute).

For 10 minutes in 35 ozs. 2nd acid (one being removed per 2 minutes).

For 10 minutes in 35 ozs 3rd acid (one being removed per minute).

Each thimble, as it was removed, was washed in two successive waters, afterwards all washed together before annealing.

The progress is rendered visible in the curve represented in Diagram II.

DIAGRAM II.

Showing the progress in parting
From time of disappearance of red fumes to the end of the process
with 10 grains gold to 25 grains silver
From **Table B** – Surcharge.

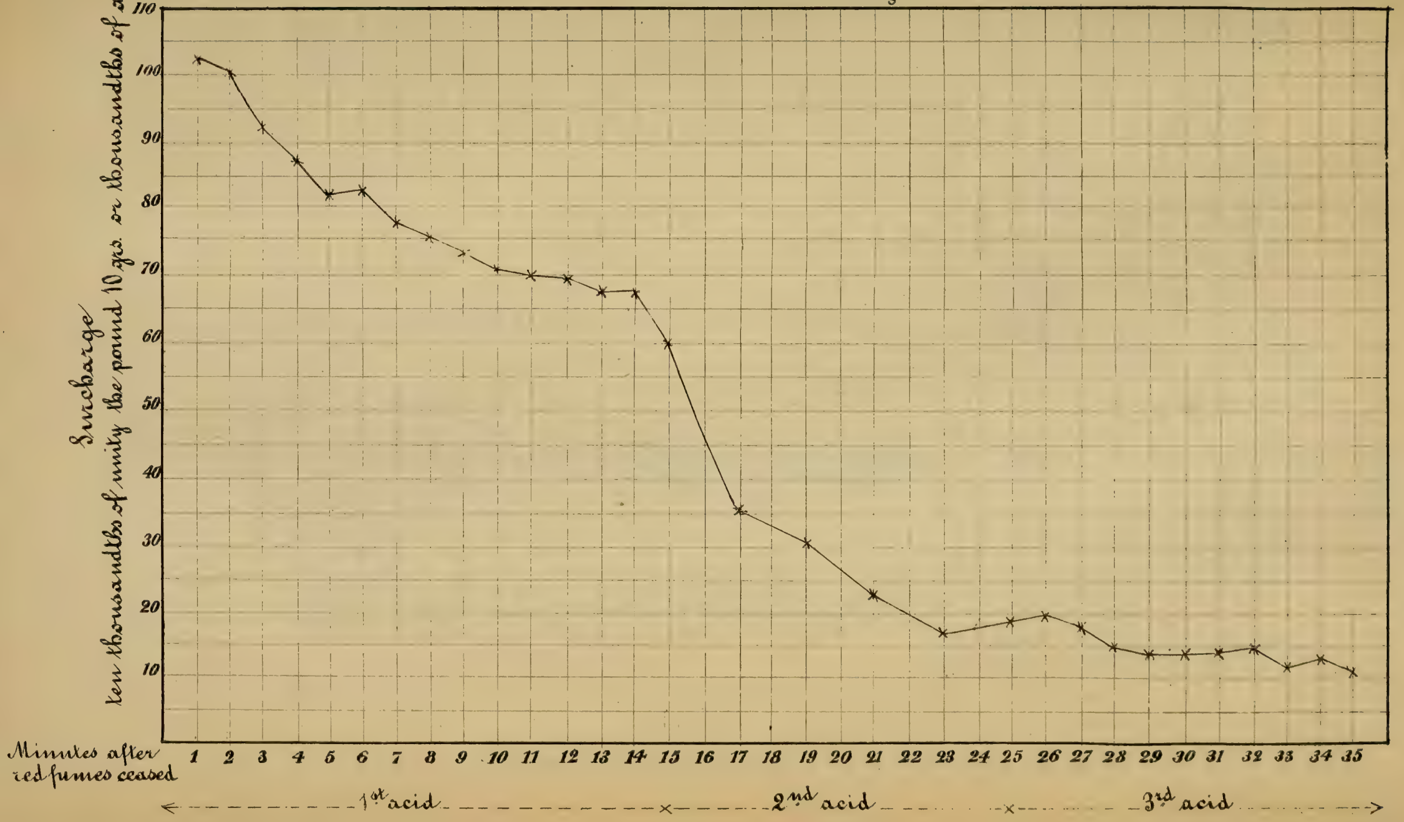


TABLE B.

To show the Progress in Parting.

	No.	Weight of ·99984 Gold taken = 10 grains + or −	When Cornets extracted. Minutes from time of cessation of red fumes.	Weight of Cornets.	Surcharge. (See Diagram II.)
		Thousandths of a grain.		(Unity=10 grains.)	(Unity=10 grains.)
1st Acid.	1	+0	1	1·0101	0·01026
	2	+0	2	1·0099	·01006
	3	+0	3	1·0091	·00926
	4	−¼	4	1·0085½(¾)	·00873
	5	+0	5	1·0080	·00816
	6	+0	6	1·0081	·00826
	7	+0	7	1·0076	·00776
	8	+0	8	1·0073½	·00751
	9	+0	9	1·0071½	·00731
	10	+0	10	1·0069	·00706
	11	+0	11	1·0068	·00696
	12	+0	12	1·0068	·00696
	13	+0	13	1·0066	·00676
	14	+0	14	1·0066¼	·00678
	15	+0	15	1·0058¾	·00603
2nd Acid.	16	+0	17	1·0033½	·00351
	17	+0	19	1·0029	·00306
	18	+0	21	1·0021	·00226
	19	+0	23	1·0015	·00166
	20	+0	25	1·0016½	·00181
3rd Acid.	21	+0	26	1·0017½	·00191
	22	+0	27	1·0016	·00176
	23	+0	28	1·0018	·00146
	24	−¼	29	1·0012(¼)	·00138
	25	+0	31	1·0012	·00136
	26	+0	31	1·0012	·00136
	27	+0	32	1·0013	·00146
	28	+0	33	1·0010	·00116
	29	+0	34	1·0011	·00126
	30	+0	35	1·0009	·00106

III.

To show the progress in parting: The rate at which the silver is dissolved throughout the process of parting.

Twenty-eight places prepared (Table C) each 10 grains (·99984 gold) with 25 grains silver (accurately weighed), copper and lead as before, and cupellation similarly conducted.

Cornets placed in platinum thimble tray and boiled in large beakers.

For 20 minutes in 42 ozs. 1st acid, by which time red fumes off (one removed every two minutes).

For 10 minutes more in 1st acid (one removed every 2 minutes), washed in 1st water.

For 10 minutes in 2nd acid (one removed per 2 minutes).

For 10 minutes in 3rd acid (one removed per 2 minutes), washed in two waters.

Each thimble, as it was removed, was washed in two waters, finally all washed together and annealed.

No. 1 could be readily unrolled, the white of silver visible on the surface.

No. 2, brittle, on being broken, a core of silver revealed.

Nos. 3 and 4, brittle, could be readily crushed up with the fingers, but no silver core.

The progress is rendered visible in diagrams III. and IV., from which it will be seen that solution proceeds very rapidly at first, but more slowly as the process is continued, so much so that the most of the time is consumed in expelling what may be called the last traces of silver.

Whilst the process of parting extended over 50 minutes,

At the end of the 2nd minute 16⅓ out of the 25 grains of silver were dissolved.

At the end of the 4th minute 22 out of the 25 grains of silver were dissolved.

At the end of the 6th minute 24⅓ out of the 25 grains of silver were dissolved.

Forty-four minutes further boiling being required to remove the remaining ⅔ of a grain,

At the end of 20 minutes about 1-10 grain was left.

At the end of 30 minutes about 1-20 grain was left.

At the end of 40 minutes about 1-100 grain was left.

At the end of 50 minutes about 1-200 grain was left, which may be permitted to remain and allowed for as surcharge.

DIAGRAM III.

Showing the progress in parting
from the commencement to the end of the process
with the pound of gold and 2½ of silver
(from **Table C** - Surcharge.)

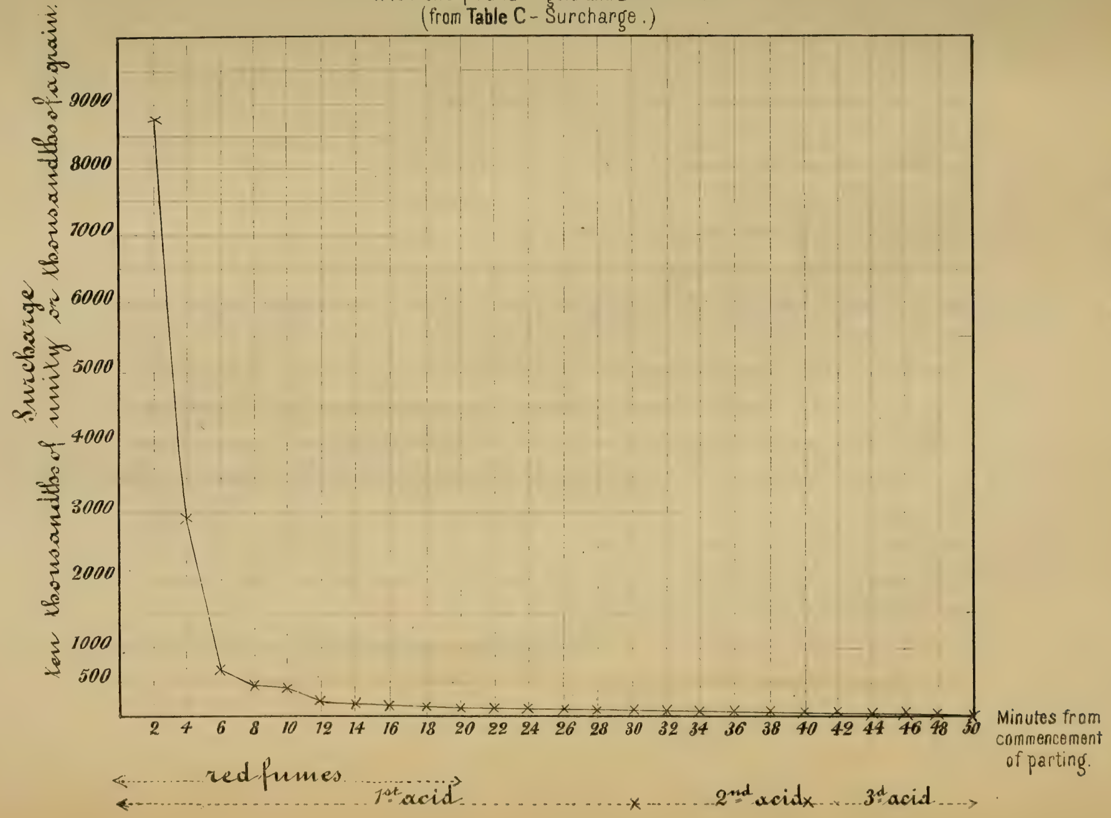
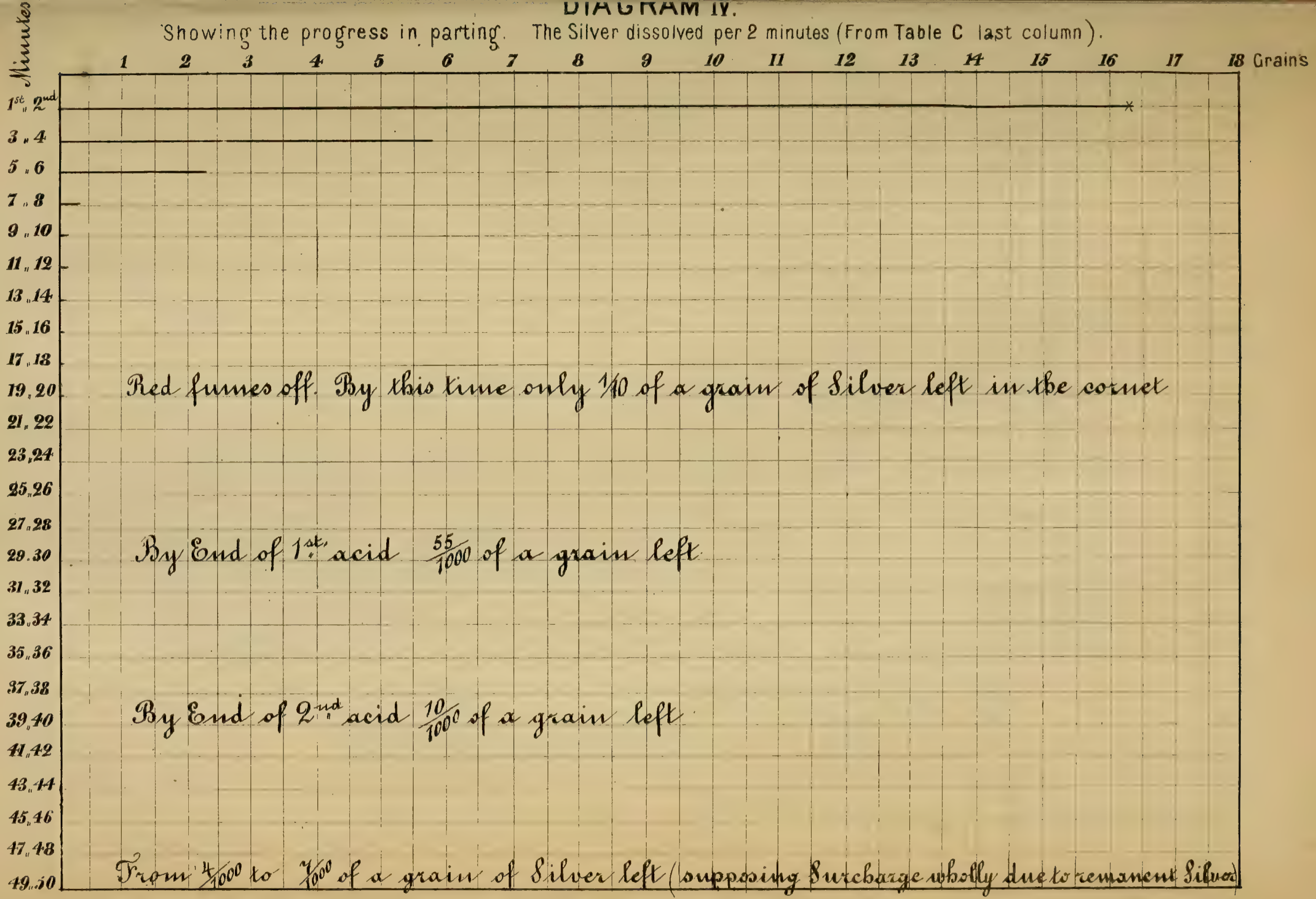


DIAGRAM IV.

Showing the progress in parting. The Silver dissolved per 2 minutes (From Table C last column).



Red fumes off. By this time only 1/10 of a grain of Silver left in the cornet

By End of 1st acid 55/1000 of a grain left

By End of 2nd acid 10/1000 of a grain left

From 4/1000 to 7/1000 of a grain of Silver left (supposing Surcharge wholly due to remanent Silver)

TABLE C.

To Show the Progress in Parting—the Rate at which the Silver is Dissolved.

No.		Weight of ·99984 Gold taken = 10 grains ±	Weight of Fine Silver taken = 25 grains ±	When Cornets extracted—minutes from commencement.	Weight of Cornets.	Surcharge.	Silver dissolved.	Silver dissolved per 2 minutes.
		Thousandths of a grain.	Thousandths of a grain.		Unity=10 grains.	Unity=10 grains.	Grains.	Grains.
1st Acid—red fumes escaping.	1	+ 0	+ 0	2	1·8683	0·8684½	16·317	16·317
	2	− ½	+ 0	4	1·2910¼	·2912¼	22·088¾	5·771¾
	3	+ 0	− ¼	6	1·0685½	·0687	24·314¾	2·226
	4	− ½	− ¼	8	1·0441	·0443	24·558	0·243¼
	5	− ½	+ 0	10	1·0306	·0308	24·693	0·135
	6	− ¼	− ¼	12	1·0200	·0201¾	24·799¾	0·106¾
	7	+ 0	+ 0	14	1·0173	·0174½	24·827	0·027¼
	8	+ 0	+ 0	16	1·0118	·0119½	24·882	0·055
	9	+ 0	+ 0	18	1·0109½	·0111	24·890½	0·008½
	10	+ 0	+ 0	20	1·0094	·0095½	24·906	0·015½
1st Acid—red fumes off.	11	+ 0	− ¾	22	1·0084	·0085½	24·916¾	0·010¾
	12	+ 0	+ 0	24	1·0076	·0077½	24·924	0·007¼
	13	− 1	+ 0	26	1·0065	·0067½	24·933	0·009
	14	− ½	− ½	28	1·0064	·0066	24·935½	0·002½
	15	+ 0	+ 0	30	1·0055	·0056½	24·945	0·009½
2nd Acid.	16	+ 0	+ 0	32	1·0023	·0024½	24·977	0·032
	17	+ 0	− ½	34	1·0017	·0018½	24·982½	0·005½
	18	+ 0	+ 0	36	1·0014	·0015½	24·986	0·003½
	19	+ 0	+ 0	38	1·0012	·0013½	24·988	0·002
	20	+ 0	+ 0	40	1·0009	·0010½	24·991	0·003
3rd Acid.	21	+ 0	+ 0	42	1·0012	·0013½	24·988	
	22	− ¾	+ 0	44	1·0008	·0010¼	24·992	
	23	+ 0	− ½	46	1·0007	·0008¼	24·993½	
	24	− ¾	+ 0	48	1·0007	·0009¼	24·993	
	25	+ 0	− ½	50	1·0004	·0005½	24·995¾	
	26	− ½	− ½	50	1·0006	·0008	24·993½	
	27	− ½	+ 0	50	1·0006	·0008	24·994	
	28	+ 0	+ 0	50	1·0007	·0008½	24·993	

Art. V.—*On a New Form of Self-Registering Rain-gauge.*

By R. L. J. Ellery, F.R.S., &c.

[Read 16th May, 1878.]

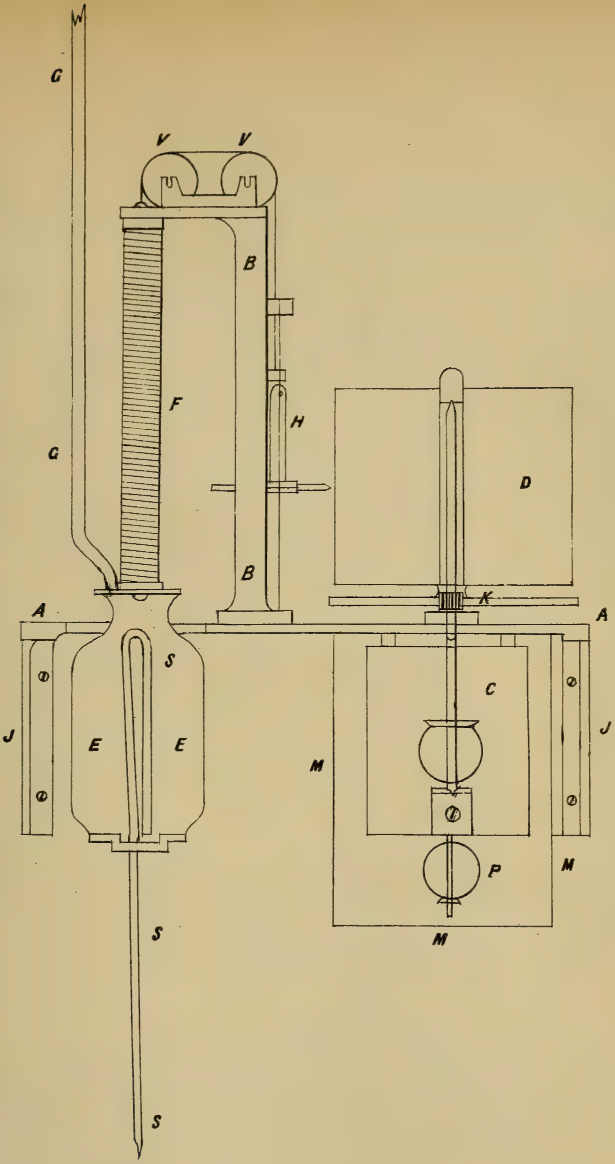
For the ordinary purpose of rainfall observation and record, the common rain-gauge, where the rain collected is measured in a graduated glass measure once or twice a day, is all that is required.

Questions often arise, however, in which the *rate* at which heavy rains fall, or the *time over which the fall may be spread,* becomes an important point, and this is especially the case in cities, large towns and other localities, in connection with drainage, disposal of storm waters, &c. To meet such requirements a self-registering rain-gauge, that will furnish the required information, becomes a valuable and indeed an essential instrument.

Various forms of self-registering rain-gauges are constructed, the best of which are very expensive, while the cheaper ones are generally very defective and untrustworthy.

The form I now submit to the Society can, I think, claim simplicity and economy in construction, a high sensitiveness as well as trustworthiness.

The principle is this. The rain which is collected in a circular area of 10 in. diameter flows at once through the pipe into (G), a small copper vase-shaped vessel (E E) holding about 19·5 cubic inches of water. This vessel is suspended from an iron bracket by two steel spiral springs (F) made of the best pianoforte wire, and most carefully tempered. Inside this vessel is a small glass tube, bent into the form of a siphon (S), and projecting through the bottom for about 10 or 12 inches, forming an intermittent siphon, which, whenever a certain quantity of water has accumulated, rapidly empties the vessel. This acts so delicately that it always requires the same quantity, almost to a single drop, to cause it to overflow, and it will always overflow with this exact quantity. In this gauge it empties itself for every quarter of an inch of rain collected in the receiver—that is, when about 19·5 cubic inches (= $\frac{1}{4}$ of an inch fall) have accumulated. As the rain drops into the vessel from the receiver the suspending spiral springs


G
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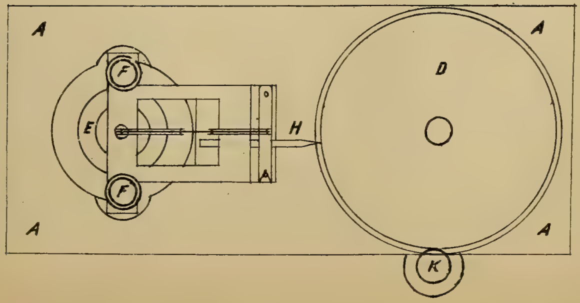
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K

stretch from the increasing weight, until the vase is full, when it is about two inches lower than in its empty position. Immediately it is emptied by action of the siphon, the vessel recovers its original position.

The other parts are—a common clock (C), which rotates a cylinder (D) about 4 inches in diameter once in 24 hours; on this drum is stretched the paper on which the register is made. Attached to the vase is a fine wire running over pulleys (VV) on the top of the bracket, and also attached to a light brass frame (H) that has a free vertical motion guided by two stretched German silver wires. As the vase, therefore, descends with the accumulation of rain, this light brass frame is raised by means of the fine wire. In the frame is a freely-suspended glass pen, charged with an ink made of aniline dye with a little glycerine. The point of this pen, which is horizontal (the surface of the registering cylinder being vertical) rests lightly against the register paper, and marks it with a clear fine line as the barrel rotates by clockwork; this line is straight as long as there is no rain, but becomes more or less curved according to the rapidity of any rainfall; as the vase empties itself the pen at once returns to the zero position, showing an indentation or "tooth," as it were, on the register-paper for every quarter of an inch of rain. A sheet showing two inches of rainfall has therefore eight indentations or "teeth" on its register, and the paper being graduated, any fraction of an inch of rain less than a quarter can be read off, while graduations parallel with the axis of the barrel give the times of any phases of the phenomenon.

Reference to Diagram.

A A. Base-plate of cast iron. B. Pillar and bracket of cast iron. C. Clock. D. Register-drum, or barrel. E. Vase-shaped receiver. F. Spiral springs suspending vase. G. Pipe leading from collector to receiver. H. Pen frame. J. Brackets to support base-plate. K. Pinion taking into large wheel on which the drum is fitted. MM. Dust-tight cover for clock. P. Pendulum-bob. SS. Syphon. VV. Pulleys for fine wire connecting receiver and pen frame.

ART. VI.—*Sir William Thomson's Form of Daniell's Constant Battery.*

BY F. J. PIRANI, ESQ., M.A.

[Read 13th June, 1878.]

ART. VII.—*The Strength of Columns.*

BY W. C. KERNOT, M.A.

[Read 13th June, 1878.]

A COLUMN may be defined as a construction piece exposed to a compression in one direction and otherwise unstrained. Columns as thus defined are of constant occurrence in engineering and architectural structures. About 50 per cent. of the material in an ordinary roof or bridge truss consists of columns; the piston rod, connecting rod, and various other important parts of a steam-engine perform the functions of columns; and immense quantities of cast-iron are employed in the construction of warehouses, theatres, churches, and other buildings in the form of columns. The question of designing a column so as to secure sufficient strength at a minimum cost is therefore one of vast practical importance. Columns vary much in size, shape, and position, but, as a general rule, have one dimension considerably greater than either of the other two; in other words, they are comparatively long and slender pieces of material. Further, they are usually, though not always, straight. Bent columns, however, being of unfrequent occurrence, will not be discussed in this paper. A column is usually compressed in the direction of its length or greater dimension, and it is immaterial, so far as strength is concerned, whether this direction be vertical, horizontal, or inclined.

Columns are divided, according to their mode of fracture, into two great classes. The first of these contains those which fail by direct or simple crushing, unaccompanied by any lateral bending. These are technically termed "short columns," as this kind of fracture usually occurs when the ratio of the length to the least transverse dimension is not particularly large. The "carrying strength" of a short column—that is to say, the greatest load it will bear without fracture—will, provided the centre of stress of each cross section coincide with its centre of gravity, be found by multiplying the area of the least cross section in square inches by the compressive resistance of the material in pounds to the square inch. If, however, the column be loaded with a weight less in any given ratio than its carrying strength, then *the stress in every part of the column will be diminished in the same ratio.* The carrying strength of a short column and the compressive stress upon any part of it under a load less in any given proportion than the carrying strength, can therefore be determined with ease and precision. With regard to such columns I have at present nothing further to say.

The second class includes those columns in which a lateral bending precedes fracture, and of which the fracture is a complex phenomenon, intermediate in its character between that of beams and that of short columns. To these the appellation of "long columns" is given by writers upon the subject, fracture of this kind occurring usually when the ratio of length to least transverse dimension is comparatively large. It will at once be evident that the question of the breaking and safe working load of a long column is one of comparative intricacy.

The question of the breaking load of a long column was first investigated by Euler, whose paper on the subject is to be found in the *Berlin Memoirs* for 1747, and a *résumé* of whose conclusions is given in Unwin's *Machine Design*, p. 48, &c. Unwin states that "Euler's rules assume the elasticity of the bar to be unimpaired. In that case no increase of the load would directly cause bending, but a point is reached at which the equilibrium of the bar becomes unstable. With less loads, the bar, if bent, will restore itself to straightness by its elastic resistance to bending; with greater loads it is unable to do so, and if any flexure is produced, however slight, that flexure will be increased by the action of the load until the bar breaks."

According to this view the strength of a long column of square or circular section is proved to vary directly as the fourth power of its diameter, directly as the modulus of elasticity of the material and inversely as the square of its length between points of inflexion, and the column if originally straight, will remain so until the load reaches this critical amount, when equilibrium becoming unstable, some trivial cause will produce an infinitesimal lateral deflection, which, rapidly increasing, results in fracture.

Euler's rules possess the recommendations of mathematical completeness and consistency, and therein contrast favourably with those of some of his successors.

In 1840 Professor Eaton Hodgkinson communicated to the Royal Society (of England) an account of a very extensive series of experiments on columns of various materials, accompanied by a set of rules empirically deduced therefrom, and in 1857 contributed the results of a further set of trials on a comparatively large and practical scale. In his paper of 1857 he says:—"In commencing experiments in my former research on this subject, and keeping in view the theory of Euler, I sought with great care for the weight which would produce incipient flexure in columns, and more particularly in those of cast-iron. In this metal flexure commenced with very small weights, much smaller than would be useful to load pillars with in practice; and I became convinced that no such point existed in cast-iron, or, at any rate, none that would be useful to the engineer; and my subsequent experiments upon wrought-iron pillars have been attended with very little more success in seeking for the weight producing incipient flexure."

Failing thus to reconcile his observations with Euler's investigation, he abandoned that investigation altogether, and proceeded to obtain a purely empirical formula, based upon no theory whatever, and simply intended to represent in a concise but merely approximate form the average result of a very extended series of experiments. According to these experimental researches the ultimate strength of a solid circular cast-iron column varies directly as the 3.6th power of the diameter and inversely as the 1.7th power of the length between points of inflection.

Professor Gordon, Professor Rankine's predecessor in the chair of engineering at Glasgow, next proposed a formula of more convenient form, and apparently based upon a scientific hypothesis, as to the nature of the stress at the instant of fracture. This formula is stated by Rankine to have been deduced from Hodgkinson's experiments; but I find by actual trial that it gives results by no means perfectly, or even approximately, in accordance with Hodgkinson's rule in the case of large hollow cast-iron columns. The breaking load of the great central column supporting the water tank from which the town of Echuca is supplied, for example, is 1320 tons by Hodgkinson's rule, and only 1030 tons by Gordon's.

More recently still, Professor Cawthorne Unwin, of Cooper's Hill Engineering College, has in his work on *Machine Design* advocated a return to Euler's original formula, to the exclusion of those subsequently arrived at. The ultimate strength of the Echuca column will be 900 tons, according to his version of Euler's results.

Thus it will be seen that most serious differences of opinion exist with reference to the behaviour of long columns under strain, and to the proper algebraical expression for their breaking loads. With the exception of Hodgkinson the writers above referred to appear to base their formula rather upon their opinion of what ought to be than upon their observations of what is. Hodgkinson, on the other hand, abandons in despair the attempt scientifically to explain the facts, and is content carefully to observe and record actual cases of fracture, and empirically to construct a formula having no *a priori* signification, but simply approximating to the average result of his experiments.

The question now suggests itself—Is it possible to reconcile these differences of opinion, and show any approach to harmony, or at any rate explanation of the discrepancies between Euler's *a priori* anticipations and Hodgkinson's observed results. I think it is, and will endeavour to throw some slight further light upon this vexed subject. The first serious discrepancy is as to the behaviour under an increasing load. Euler says that a column originally straight will remain so until its load reaches a certain critical amount, when it will suddenly double up. Hodgkinson says his columns behaved quite differently—commencing to bend under loads very small compared with those required for fracture. These diverse statements may be accounted for as follows:—Euler necessarily assumed that his column consisted of perfectly uniform material, and that the load was applied fairly, its line of action passing through the centre of gravity of each cross section. And I believe that could these conditions be faithfully complied with in practice, Euler's predictions would be verified. Baker, in his work on *Beams, Columns, and Arches,* describes an experiment upon a very long blade of finely-tempered steel, which behaved in a manner very closely approximating to that predicted by Euler; and I have myself obtained corresponding results from a straight piece of clock-spring very carefully loaded. The reason why Hodgkinson's cast-iron rods began to bend so soon was, I believe, this, that the material was not homogeneous, or the load possibly applied slightly eccentrically. Let us suppose that a solid circular column is softer and more elastic on one side than the other. The smallest load will now bend it; for even if at first the centres of pressure and of figure are perfectly coincident, the more elastic side will yield more than the other; this will cause the bar to bend, the more elastic material being on the concave side; this bending will cause the centre of pressure of each cross section to deviate from the centre of figure *toward the softer or more elastic side of the bar*, thus throwing a greatly increased portion of the total pressure on that part of the column most affected by it. In this way a perceptible flexure may be produced by a load minute compared with that necessary for fracture. That this is the true explanation of the anomaly is, I think, rendered certain by two facts observed by Hodgkinson. The first of these is that the amount of flexure produced by the same load on columns of the same size and material varied very greatly, thus indicating that it depended upon some slight accidental peculiarity in apparently similar bars. Some of the bars tested bent visibly under less than one-fifth of their breaking load, while others remained straight until two-thirds of that load was applied, thus approximating to Euler's theoretical case. The second fact is that certain hollow columns through defective casting were much thicker on one side than the other, and that these when tested bent so that the thick side was concave and the thin convex; the greater hardness and higher co-efficient of the elasticity of the thin and more rapidly cooled side of the casting more than compensating for its deficiency in substance. If, then, the softer side became concave in these hollow columns, much more would it tend to do so in solid ones, where the countervailing influence of extra thickness was absent.

The next discrepancy is this:—Euler predicted that the strength would vary as the 4th power of the diameter; Hodgkinson found it to be the 3.6th. Now this is nothing more than I think might have been expected by any one acquainted with the softness of large castings as compared with small ones. The strength of large castings is never quite so great as that of smaller of the same material, and the difference between the 3.6th and 4th powers of the diameter appears to me to be a reasonable allowance for the effect of this variation of hardness and strength.

To reconcile the square of the length given by Euler with the 1.7th power of Hodgkinson is perhaps not quite so easy, and I should prefer not to express an opinion with regard to it at present.

To determine the breaking load of a pillar is, however, after all, only a means to an end, only a step towards obtaining that practically valuable result, *the safe working load;* and the next question that arises is—What is the factor of safety to be? what proportion of the breaking load can we safely apply in actual construction? And this question appears to me—and I would wish to say it with all due deference to such eminent names as Rankine, Unwin, Stoney, and Baker—to have been hitherto answered in an utterly unreasonable and illogical manner. These writers, one and all, apply a factor of safety to the case of a long column in the same manner as they would apply it in the case of a tie-rod or beam, altogether overlooking the fact that under any ordinary working load the column is either not bent at all, or at any rate is not bent nearly so much as it is immediately before fracture, and that consequently the stress is not only less, but is distributed over each cross-section with a much nearer approach to uniformity.

In a tie rod a double load implies double tensile stress—the stress is increased but its distribution is unaffected; in a beam we believe the same to be the case, but in a long column a double load not only means double average stress on any given cross section, but also increased flexure, causing a very large increase in the ratio in which the maximum stress exceeds the average. In fact, a double load may involve quadruple, sextuple, or even tenfold stress, according to the proportions of the column and the amount of its flexure.

By a very simple and conclusive mathematical process I find that in a certain column, tested by Hodgkinson, the maximum compression upon any part was 43,820 lbs. per square inch under a load of 124,000 lbs., but only 24,500 lbs. per square inch under a load of 109,000. In other words, an increase of 14 per cent. in the load caused an increase of no less than 78 per cent. in the maximum stress. Now, the true factor of safety is the ratio of the ultimate resistance of the material to fracture to the *maximum stress endured by any part* of the piece under strain, and taking the ultimate resistance to crushing of the material at 90,000 lbs. per square inch, the true factor of safety was 2.04 under a load of 124,000 lbs., and no less than 3.67 under the slightly diminished load of 109,000.

We are therefore led to the following conclusions :—

1. That in a column of perfectly uniform material, loaded in a perfectly symmetrical manner with a load less than that required to produce unstable equilibrium, there will be no flexure, and the stress will be independent of the length, and may be but a very small fraction of the ultimate compressive resistance of the material, even under a load closely approximating to that which would destroy the column.
2. That actual columns will approximate more or less closely in their behaviour to the above theoretical case, according to their proportions, the nature of the material, and the mode of applying the load.
3. That the true factor of safety of a long column cannot be found by dividing its breaking load by its working load, but is a function of its flexure also; and that this flexure depends on slight accidental peculiarities in the material, or in the way of applying the load, and is therefore not calculable.
4. That the true factor of safety, or the ratio in which the ultimate resistance of the material exceeds its working stress in a long column, is always greater, and generally very much greater, than the ratio in which its breaking load exceeds its working load; and that consequently the present method of dimensioning errs on the side of safety, and involves waste of material.
5. That in order to arrive at a rational method of dimensioning, we must determine by numerous experiments, under practical conditions, the greatest probable flexure, under working loads, of columns of different materials and of various cross sections.

The term breaking load as applied to long columns appears to me objectionable and likely to lead to confusion, not being properly analogous to that of tension rods and beams. I would suggest the term "critical load" as preferable, on the analogy of the "critical angle" in optics.

ART. VIII.—*A New Point of Resemblance in the Respiration of Plants and Animals.*

BY JAMES JAMIESON, M.D.

[Read 13th June, 1878.]

RESPIRATION in plants consists just as it does in animals, in the inhalation of oxygen and the exhalation of an approximately equivalent quantity of carbonic acid. This process, though masked under ordinary circumstances by the more active deoxidizing action of the green parts of the plant, seems, according to recent investigations, to be constantly going on, and to be as necessary to the life and health of the plant as of the animal. The deoxidizing action of the green organs, carried on by means of the chlorophyll contained in them, is tolerably well known, and consists in the splitting up of carbonic acid into oxygen and carbonic oxide. The oxygen is wholly, or in great part, set free in the air, while the carbonic oxide seems to enter into some kind of combination with the chlorophyll, as a preliminary to the formation of more complex compounds, and especially of the various hydro-carbons. A series of investigations on this point are contained in a paper by Adolf Baeyer, in the *Chemisches Centralblatt*, 1871, pp. 27—38, and also translated in a slightly condensed form in the *Journal of the Chemical Society*, 1871, pp. 331—341. My object is not, however, to enter into any details on this process, which is one of assimilation, but rather to consider the mechanism of respiration in the proper sense of the word, which is essentially associated with processes of regressive metamorphosis. Some observations which I have made seem to throw light on the chemistry of the respiratory function in plants, and I desire therefore to report the result of them, incomplete and fragmentary as they are.

For the proper understanding of the particular point on which I wish to lay stress, and which, after consulting the best accessible authorities, I am led to believe is new, or at least very little known, it will be necessary to mention certain facts connected with the better-known chemistry of the function of respiration in the higher animals. The red colour of blood is due to the presence in it of large numbers of discs or corpuscles, infiltrated with a red colouring matter of very complex constitution called hæmoglobin. These red corpuscles take up oxygen while the blood is passing through the capillaries of the lung, the oxygen entering into loose combination with the hæmoglobin. As the blood flows in the systemic circulation through all parts of the body, the oxygen is gradually given off, and enters into definite combinations with the tissues undergoing disintegration; one of the main ultimate products of the oxidation process being carbonic acid, which is taken up by the blood and carried to the lungs, there to be exchanged for a fresh supply of oxygen. The following passage from *Hermann's Physiology* (English translation, p. 47) gives shortly what is generally admitted as to the properties of the oxygen contained in the blood, though there is not perfect unanimity on all points, as I will afterwards show:—"As blood when saturated with oxygen takes up exactly as much of that gas as corresponds to the amount which its hæmoglobin can combine with, it follows that all the loosely combined oxygen of the blood is linked to hæmoglobin. The oxygen of the blood is given up so readily to oxidizable substances that it has been thought to be present in the form of active oxygen, or ozone O_3. The following properties of blood appear to favour this view:—(1.) Both the blood corpuscles and hæmoglobin are so-called 'ozone-transferrers'—that is, they possess the power of immediately transferring ozone from substances in which it is present (as turpentine which has been kept for a long time) to readily oxidizable substances (ozone reagents, such as tincture of guaiacum, which becomes blue by oxidation—Schœnbein, His.); for this reaction the presence or absence of oxygen in the blood is of no importance (for instance, it may be saturated with CO). (2.) Blood and hæmoglobin can themselves ozonize oxygen, so that in presence of air they can cause guaiacum tincture to become blue (A. Schmidt); if the blood itself contains oxygen the presence of air is not necessary; it is necessary if the blood has been saturated with CO (Kühne and Scholz). On the activity of its oxygen depends the decomposition of sulphuretted hydrogen by blood. It is therefore very probable that the oxygen naturally contained in blood is present in the form of ozone, or in some similar condition."

With regard to the first of the properties, viz., the power possessed by hæmoglobin of acting as an "ozone-transferrer," there is no room for difference of opinion, that quality indeed being made the basis of a valuable test for blood, with which the name of Dr. Day, of Geelong, is associated. Tincture of guaiacum and peroxide of hydrogen may be brought together without any change of colour appearing; but as soon as a minute trace of blood or hæmoglobin is added a deep blue is struck. The presence of ozone in the blood, as first asserted by Professor Alexander Schmidt in 1862, and confirmed by W. Kühne (*Lehrbuch der Physiologischen Chemie,* 1868, p. 214) and others, has been doubted by some physiologists, and indeed quite lately by Dr. Michael Foster in his *Textbook of Physiology,* first edition, 1877, p. 240. As there is not yet by any means unanimity of opinion as to the nature of ozone and its characteristic reactions, the dispute may be mainly about names, there being really agreement that the oxygen in the blood is more active, *i.e.,* combines more readily with reducing substances, than the ordinary form existing in the atmosphere. The transformations undergone by oxygen in the vegetable economy do not seem to have been traced in the same way. For the purpose of discovering the present state of knowledge on the subject I have gone through the most likely sections in Sachs' *Textbook of Botany,* in Watts' *Dictionary of Chemistry,* including the supplements, and in the *Dictionnaire de Chemie,* of Wurtz, as well as through the articles most likely to touch on the subject in the *Journal of the Chemical Society,* and the *Chemisches Centralblatt* for the last few years, and have been able to find nothing but the vaguest statements. My own observations were first made some years ago in the course of a series of experiments mainly designed to test the reliability of the guaiacum test for blood, the results being embodied in a paper in the *Australian Medical Journal* for October, 1869. At that time I did not see the full bearing of these observations on the subject now under discussion; but having occasion again to take the matter up recently I have been able to reach more definite conclusions. The recent experiments have been made chiefly with fruits of different sorts, especially apples and pears, though what is true of them holds good of most other fresh vegetable structures and expressed juices. If a drop of tincture of guaiacum be allowed to fall on a freshly cut surface of an apple or pear, which has not been too long pulled and is not decayed, it will generally be found that a blue colour is quickly struck. Again, if a few crumbs of biscuit or other cooked starch are sprinkled on a similar surface, and a little of a strong solution of iodide of potassium added, the starchy particles will become gradually brown and then black from the formation of iodide of starch. Here, then, we have the recognised reactions characteristic of the presence of ozone. The rapidity and intensity of these reactions will be found to vary with different articles or different specimens of the same article; and they may fail altogether, as in very watery fruits, such as some grapes, though even with these the guaiac reaction may be perceptible in a green berry from the same bunch. I have not observed this reaction with the soft pulpy fruits which quickly decay, such as the strawberry or peach, perhaps because the specimens were not fresh enough, while with the apple and pear both reactions may be obtained though the fruits have been pulled for a considerable time.

With reference to the agent providing these reactions it may certainly be said:—(1.) That it is not merely ordinary oxygen absorbed and dissolved in the vegetable juice; and this, both on account of these reactions and from the fact that Cahours (*Comptes Rendus*, 1864, LVIII., pp. 495 and 653) could obtain carbonic acid gas and nitrogen, but never oxygen, from expressed fruit juices. (2.) It is not newly-formed oxygen, separated by the chlorophyll, which may possibly in part be diffused into the structures below the surface as well as liberated into the atmosphere, since Pellucci has shown (*Chemisches Centralblatt*, 1872, p. 356) that the oxygen developed under water in sunlight by various plants does not act on starch and iodide of potassium like ozone, agreeing therein with the results obtained by Mulder and others, *v.* Hoppe-Seyler's *Physiologische Chemie*, 1877, p. 47. These reactions are also given by sections of pulled fruits, which, though capable of carrying on a process of respiration for a time, no longer liberate oxygen; and also by underground organs like the potato, turnip, &c., which never perform that function. (3.) It is not probable, in spite of these reactions, that the substance is actually dissolved ozone, since it is scarcely conceivable that it could continue to co-exist for any length of time with the complex mixture of solid and dissolved organic matters contained in fruits. We are therefore in a manner shut up to the conclusion—(4.) That the oxygen is in a form of loose combination, as it is in the blood, and therefore capable of being slowly given off in a very active form to combine definitely with oxidizable substances. Cahours *(op. cit.)* and others often since have found that fruits, during their period of growth, appropriate carbon and give off oxygen, like other green parts of the plant; but that when ripening they cease to do so, and begin to inhale oxygen and give off carbonic acid; the chemical changes taking place during the process of maturation being essentially oxidation phenomena. It is also well established that many fruits, such as the apple, the pear, and the orange, continue the maturation process after separation from the parent stem, acting in a manner like independent organisms. If placed in a close vessel containing air, a portion of the oxygen gradually disappears, and is replaced by carbonic acid. A difficulty was felt by Cahours in explaining the continued exhalation of CO_2 from fruits enclosed in an atmosphere of nitrogen or hydrogen, which he could ascribe only to some fermentation. Fremy, in a note to the communication of Cahours, tries to explain it as being due to the slow process of combustion going on in the interior of the fruit, which is no doubt true; but is at the same time rather an insufficient explanation, without some account such as is here given of the state in which the oxygen exists while that slow combustion is going on, the full explantion being that the oxygen is stored up in loose combination, to be given off as required for the formation of oxidation products and among them CO_2.

With reference to the substance with which the oxygen is temporarily combined I cannot speak very definitely; it is certain, however, that in fresh fruits and other vegetable substances there is an element which is possessed of the same ozone-transferring property as hæmoglobin. If a fresh section does not supply spontaneously the blue colour on the application of tincture of guaiacum, it can be brought out by the addition of a drop of solution of peroxide of hydrogen; and if it had appeared spontaneously, the peroxide has the effect of rendering the blue more intense. I have found that in fruits, when long-kept, the ozone reaction is gradually enfeebled, the power of inhaling oxygen being lost and the amount stored up gradually consumed. On the other hand, the ozone-transferrer may still be detected when the fruit has become over-ripe and has entered on the stage of incipient decay, disappearing entirely, however, in parts which have become actually rotten. When fruits, &c., are cooked either with moist or dry heat, both this substance and the active oxygen are destroyed, no blue colour being produced by guaiacum alone or on the addition of peroxide of hydrogen. It is known that other substances contained in the animal economy, and belonging to the protein group, such as fibrin, myosin, globulin, act like hæmoglobin in the way of carriers of ozone. I conclude, therefore, from analogy, as well as from its properties above described, that in fresh vegetable substances there is contained an ingredient, probably albuminous, which acts as an ozone-transferrer, and may be presumed to be the agent with which oxygen enters into loose combination. It certainly is not chlorophyll, which has been compared with hæmoglobin (by Baeyer in his paper referred to above) on account of the property which they possess in common of combining with CO. The difference in function, however, is well marked, chlorophyll causing the elimination of oxygen, while hæmoglobin enters into combination with it. In addition, the substance whose nature I am considering exists abundantly in the interior portions of fruits and in many other structures, such as the potato, turnip, &c., which never contain chlorophyll. I think it probable that considerable difficulty will be found in isolating this substance, both on account of its destructibility and because it is almost uniformly diffused through fresh vegetable structures. It is probably intimately associated with the vascular tissue, since I have found that the ozonic reaction, as well as the ozone-transferring function, in fruits are most marked and persistent near the core, where the vessels from the stalk are more abundant than in the outer, more purely cellular, parts. A perhaps more doubtful opinion is that this substance is attached to the small cells or granules, called by Sachs "aleurone grains," which, according to him, are mainly proteinaceous. They resemble somewhat in size the red blood corpuscles, and I have sometimes thought that minute sections of fruits, which had been rendered blue by guaiacum, when examined under the microscope showed the most intense colouration at the spots where these aleurone grains occurred in groups.

Whether what I have ventured to advance by way of opinion prove to be correct or not, the following points have, I think, been established:—(1) That the oxygen inhaled by plants as well as by animals enters first into some form of loose combination whereby it is ozonized or rendered active; and (2) that plants contain a substance, other than chlorophyll, having some important points of analogy with the hæmoglobin of animals, acting like it as an ozone-transferrer. It cannot, however, yet be regarded as more than fair presumption that this substance is that with which oxygen becomes loosely combined.

Art. IX.—*Note of the Great Meteor of June 8th*, 1878.

By R. L. J. Ellery, F.R.S.

[Read 11th July, 1878.]

There is one point in connection with the apparition of the great daylight meteor of June 8, 1878, which is remarkable and interesting—that is the apparent exactness with which different observers, hundreds of miles apart, erroneously localise certain phases of the phenomenon, and the imaginary *nearness* to the observers at which these phases occurred, leading one to the conclusion that usual human experience in judging of distance, &c., is altogether at a loss in the case of such phenomena as this. The meteor appeared about 3 p.m. on June 8, and was seen at Sydney, off the N.S.W. coast at sea, at Yass, Braidwood, Cooma, Omeo, over many parts of Gippsland, at Geelong, Ballarat, Seymour, &c., &c., and by sifting all the reports, and allowing for difference of local time, *all about the same time.* There can be no doubt it reached its minimum distance from the earth somewhere in the zenith of Kosciusko, and passed nearly over the zeniths of Cooma and Omeo. From Seymour it was seen in the east, about 30° high; from this its height may be roughly estimated as over 100 miles, while by two different observers at different places a bursting-up of the meteor was witnessed, followed at an estimated interval of from 10 to 15 minutes by loud explosions—most probably one explosion and its aerial echoes. This would give us an estimate of its distance from these observers of nearly 200 miles.

At Cooma, Yass, and about that district, it was firmly believed to have come to the earth in the neighbourhood, and to have fell by the side of Jellimatong; indeed, it was reported that fragments were picked up in that district. The explosion seemed to be quite close to the observers, and was called by some an earthquake.

Now from Mr. Christian Ogilvie, at Omeo, I received a very interesting account of the meteor as seen in the Omeo district by numerous observers, and here also the explosion was localised at the mountain called the "Brothers." Two observers, five miles from the mountain, in different directions, describe it "*as if the mountain had burst,*" and "*like the crash of an enormous falling rock, followed by thunder.*"

It is not probable, I think, that there could have been two explosions of this meteor, but that whoever witnessed the apparition and heard the explosion, estimated it to have taken place in his immediate vicinity, although there can be little doubt that the meteor was at no time during its appearance within 80 or probably 100 miles of the earth. Observers at Seymour describe having seen the meteor burst, though no sound, of course, reached that district.

ART. X.—*The Perception of Colour.*

BY JAMES JAMIESON, M.D.

[Read 17th October, 1878.]

A FEW months ago, in a short communication to this Society ("Photographs on the Retina," 11th April, 1878), I endeavoured to give an account of what was then known of the properties of the colouring matter called retina-purple. More extended observations have tended to establish further the importance of photo-chemical processes in the act of vision. That the retina contains colouring matter, capable of undergoing rapid changes under the action of light, and that pictures of objects can be printed on the retina by help of it (optograms of Kühne), would alone be sufficient to suggest its functional importance. The well-known persistence of visual impressions, *i.e.*, the fact that after looking at an object, especially a bright one, we can still see it if the eye is immediately closed, the outlines gradually becoming less distinct till the picture fades away, is best explained by the alternate destruction and restitution of the retina-purple by light and in the dark. Boll has found the colour of the human retina deeper and more intense after a night's sleep than later in the day; and in this may be found an explanation of the great sensitiveness to light of an eye which has been long in the dark. The transparent retina has become more fully saturated with the pigment, and more tumultuous chemical changes go on, with correspondingly intense stimulation of the optic nerve. This varying sensitiveness of the retina at different times of the day has been made the subject of exact experiment by M. Auguste Charpentier (Academy of Sciences, 20th May, 1878, *v. Gazette Medicale*, 23, 1878). He found that the difference of acuteness in the rested and active eye holds good with all kinds of light. For instance, the eye which has been kept dark for 15 to 20 minutes experiences a luminous sensation, with a minimum of green light equal to 16, while the eye which has been active requires a minimum of 121; the comparative amounts of red light under the same conditions being 12 and 50, and of blue 16 and 400. As Charpentier argues, it is impossible to conceive of this difference of sensibility being due to fatigue, in any proper sense of the word, since the eye which had been in exercise had merely been performing its normal function. The explanation, as he says, is to be found in the comparative amount of retina-purple under the different conditions investigated by him, the sensitiveness to light being in direct proportion to the chemical changes in the pigment produced by that light. In a further note to the Academy (27th May, 1878, *Gazette Medicale,* 24, 1878), M. Charpentier reported that according to his direct observations it seems to result, that where there is less of the red substance in the retina there is less luminous sensibility, and that when the red is in excess that sensibility is exaggerated. These facts taken together seem to put beyond doubt that the retina-purple plays a very important, perhaps essential, part in the physiology of vision.

When we proceed to apply the knowledge recently gained in a more special way, difficulties increase. I propose, however, to consider in how far the discoveries of Boll and Kühne throw light on the very difficult question of the perception of colour, and before doing so it is necessary to indicate shortly the generally accepted view on that subject. Early in the present century Dr. Thomas Young proposed a theory which has been, with slight modifications, adopted by Helmholz, and accepted generally by physiologists. It is to the effect that in every spot of the retina capable of receiving colour impressions there must be a number of distinct nerve terminations, each sensitive to the impression produced by a single colour. An analysis of the components of white light led him to fix on three as the least possible number of these nerve terminations capable of being acted on by red, green, and violet respectively. By the combination of these three colours, or of two of them in varying proportions, either white light or any intermediate colour can be produced. White light is the combined sensation resulting from the equal stimulation of all three nervous elements; and so with varying degrees of stimulation of one or more, the particular colour perception results, yellow, for instance, being the colour perceived when the terminations for red and green are about equally stimulated, and the one for violet little or not at all. This hypothetical explanation of the phenomena has been almost universally accepted as a satisfactory one, since with the help of the minimum of secondary hypotheses it could be applied so as to account for certain peculiarities and abnormalities of the colour-sense. The theory as a whole of course rests on the doctrine of the specific energy of different nerves and nerve terminations; the doctrine, namely, that each nerve responds only to one particular stimulus, the optic nerve to light, the auditory nerve to sound, and so on. On the Young-Helmholz theory it is assumed that, in addition to the specific energy of the optic nerve, as a whole, there are fibres or fibre-terminations endowed with specific energies adapting them for receiving different colour impressions. It might be questioned in how far such an extension of the doctrine is allowable, unless we are prepared to accept a similar differentiation of the elements of the other nerves of special sense. It would perhaps be applying the *reductio ad absurdum* test to such an extension of the doctrine, to what might be called secondary specific energies, to assume that there must be in the olfactory nerve, or its surface endings, a special element susceptible only to the stimulus of one odorous substance, one each for every possible smell between otto of roses and assafœtida. I do not know that it is allowable to make that extension of the doctrine in the case of the optic nerve, merely because we can indicate a possible minimum number of elements in it so endowed, while in the case of the other special senses there is no approach to such a limitation. I make this criticism with all humility, knowing that it is in opposition to the opinion of the most eminent physiologists. It is certain, however, that histology gives no support to the theory of three or more distinct percipient elements existing together in all parts of the retina, all the rods and cones in one part of the retina of the same animal being of similar construction, so far as can be shown by the microscopes at present in use. A difference in the index of refraction of different elements would perhaps be sufficient, without any difference of form; but that is merely another hypothesis framed to obviate a difficulty in accepting an opinion which is itself hypothetical. A simpler, and therefore more feasible, view of the phenomena of colour perception is to regard it as the result of photo-chemical changes in the retina; though, in the present state of our knowledge, it may be somewhat premature to attempt to apply it for the explanation of all the peculiarities of that function, normal and abnormal. In my last communication the suggestion could only be ventured that the retina-purple may serve in some way for the perception of colours. The great difficulty then lay in the circumstance that Boll and Kühne agreed in stating, that the colouring matter was not to be found in the cones; and yet the *macula lutea* is the part of the retina most sensitive to colour, that sensitiveness being most marked in the *fovea centralis,* which contains only cones and no rods. There are sufficient reasons, however, for supposing that there was error in denying the presence of retina-purple in that region, or in the cones generally. The layer of pigment cells on which the rods and cones rest is the source of supply of the purple, which it seems to manufacture and store up. Now these cells are more abundant behind the yellow spot than at any other part of the retina. Dr. Schmidt-Rimpler has reported (Archiv für Ophthalmologie, xxi., 3, 1876) that in perfectly fresh human eyes he found the *macula lutea* of a reddish-brown colour, which gradually faded, giving place to the usual yellowish hue; the last speck of red, however, being seen in the centre of the *fovea.* That Kühne did not detect the red colour in the cones is probably to be explained by the delicate points of these structures allowing of its more rapid disappearance than from the broader based rods; this explanation being made more probable by the fact that the transformations of the retina-purple under the influence of light go on slowly, and are therefore most easily observed in the amphibia and cartilaginous fishes, whose retinal rods are unusually large. It was necessary to dispose of this preliminary difficulty, since the result of growing knowledge of the structure and functions of the organ of vision has been to connect colour impressions specially with the cones.

If it be granted that retina-purple plays an important part in the act of vision, as has been shown, we are in a position for considering facts and arguments in favour of its importance in the perception of colour. The first point in favour of that view is the fact that light of different colours acts differently on it. An experiment of Kühne's shows this in a very unmistakable manner. He arranged frogs' retinas on a screen, and exposed them simultaneously to the whole length of the solar spectrum. He found the bleaching process begin with, and pass successively through greenish-yellow, yellowish-green, bluish-green, greenish-blue, blue, indigo, and violet; later, through pure yellow and orange; much later, through ultra violet; and finally, through red. He found that the human retina is bleached by blue to violet in twelve minutes, by green in twenty-five minutes, and by red only in about eight hours. He further found that the various stages in the transformation of the pigment, from red through orange to yellow, as well as the ultimate disappearance of all colour, are passed through with varying rapidity. Green light rapidly brings about the change to yellow, but complete decomposition is then slower; while with violet light the change to yellow is made very slowly, but from that point the advance to complete transparency is rapid. Whether the transformations of the retina-purple differ in kind as well as in the rapidity of their production, under the influence of light of different colours, has not been determined, very little being yet known with regard to its chemical constitution; and even less is known of the nature and function of the green colour found in certain rods in the retina of the frog, though it also varies under the action of different kinds of monochromatic light. It is established that the photo-chemical changes in the retina are not the same under the stimulus of different colours, and it is therefore fair matter of hypothesis that the sensation of colour is produced by the action of different modifications of the retina-purple or other pigments on the fibres of the optic nerve. Absolute demonstration of this mode of production of sensations of colour is, for obvious reasons, difficult, perhaps impossible of attainment; but its claim to acceptance will be all the greater if it throws a clearer light on, or gives a simpler explanation of the phenomena, than the current theory. MM. Landolt and Charpentier have shown (*Gazette Medicale,* 10, 1878), that before any colour is recognised for what it is, a variety of phases are passed through, the first being a simple luminous sensation; and that gradually the chromatic character of the light is perceived. It has also been long known that a different length of time is required for the perception of different colours, red requiring the longest time. On the theory of Young, it is not easy to see why this should be the case; why a nerve termination, specially adapted for the perception of one colour, should respond more slowly to the stimulus of that colour than a second nerve termination does to another colour, by which alone it is acted on. On the photo-chemical theory it meets with a simple explanation in the varying action of different rays on the pigmentary matter of the retina, red light transforming it most slowly. In the same way when we take the remarkable abnormality of vision, known as Daltonism, the superiority of the photo-chemical hypothesis is apparent. In the vast majority of cases red is the colour which is not seen, there being cases in which very intense red can be detected, but not duller shades. On Young's theory this is to be explained only on the supposition that one of the three new elements, whose existence is postulated, is awanting, or has wholly or partially lost its excitability; but no explanation is afforded of the fact, that it is almost always the element susceptible to red which is thus defective. On the hypothesis of photo-chemical action the explanation is much simpler and more easily acceptable. The least refrangible (red) rays have least action on the pigment of the retina, even when isolated; they are also normally absorbed in great proportion by the transparent media of the eye; and it is only necessary to suppose a slight increase of that resistance to their passage to account for their total absorption, the same increase of resistance having a slighter effect on the more refrangible rays. In this way the partial or total blindness to red would be accounted for, the perception of other colours being inappreciably impaired.

There is another point which at first seemed to throw serious difficulty in the way of this view of the mechanism of the production of impressions of colour. The retinas of most birds and reptiles have none of this retinal colour, and yet there is reason to suppose that birds at least have a well-developed colour sense. There had long ago been observed in the rods and cones of the retinas of these animals spherical fatty drops of red and yellow colour, which have been supposed by physiologists to be of importance in colour perception, but they differ from the retinal purple in that light has not much effect in bleaching them. An investigation of their nature and properties by Dr. Capranica (*Annales d'Oculistique,* lxxviii., p. 144, 1877) has revealed, however, that as regards solubility and reactions the colouring matter contained in these globules agrees completely with that in the pigment layer of the frog's retina, and that the difference between the red and yellow is only one of concentration. When dissolved in alcohol, chloroform, or sulphuret of carbon, this pigment is decolorised by the action of light, the different forms of monochromatic light acting on it as on retina-purple, with which it has therefore the closest affinities. The photo-chemical sensibility, according to Capranica, depends on the amount of fatty matter associated with it. These isolated coloured globules may therefore be presumed to play the same part as the more diffused colour in the retina of the mammalia.

Enough has been said, I think, to make it at least highly probable that the perception of colours is essentially connected with photo-chemical processes, and the admission of this interpretation has the further advantage that it brings this function into closer analogy with other special senses, the optic fibres being stimulated by particles of chemical substances just as the olfactory and gustatory nerves are by particles of odorous and sapid substances, and the auditory nerve terminations by mechanical pressure or the impact of the minute bodies known as otoliths.

In addition to the references given in this and the previous communications, I may state that the data on which the argument in this paper is based have been obtained mainly from the following authorities :—

(1.) A review of the literature on retina-purple in the *American Journal of the Medical Sciences,* July, 1878.

(2.) Wilhelm Schoen. *Die Lehre vom Gesichtsfelde und seinen Anomalien,* 1874.

(3.) Hermann. *Human Physiology* (English translation), 1875.

(4.) Wilhelm Wundt. *Lehrbuch der Physiologie,* 1868.

ART. XI.—*On the supposed Intra-Mercurial Planet.*

BY R. L. J. ELLERY, F.R.S.

[Read 14th November, 1878.]

THE announcement that during the total eclipse of the 29th July last, visible in the United States of America, Professor Watson had discovered an unknown body near the sun, supposed to be an intra-mercurial planet, has revived the almost dormant question of the existence of such a body, and awakened fresh interest in the earlier observations of the supposed planet Vulcan. It will be known to some of you, no doubt, that long since, the celebrated Leverrier demonstrated that Mercury's perihelion moved 40 seconds per century faster than it should do, taking into account the gravitating action of only the known planets of the system. This he most easily accounted for by supposing that there were between Mercury and the sun a group of small planets. Adopting this theory, various recorded observations of the passage across the sun's disc of dark round bodies, at a more rapid rate than ordinary sun spots, were adduced as evidence of the existence of such planets; but the untrustworthiness of some of these observations, and the failure of experienced observers to detect the phenomena while scrutinising the sun's surface at the very times the reputed passages occurred, has hitherto so weakened the only proofs adduced—except the theoretical one of Leverrier's—that he alone, I believe, out of all experienced astronomers, still had strong faith that intra-mercurial planets or a planet would yet be discovered. On March 21st, 1877, a transit of the supposed body across the sun's disc was announced as probable by Leverrier, and a systematic search was kept up by all the principal observatories of the world during the days indicated, but nothing was discovered. The American astronomers, probably made more sanguine by the recent discovery by one of them of the satellites of Mars, seized the opportunity of the late eclipse for examining systematically the immediate vicinity of the sun during the moments of totality, at which times it is possible to detect comparatively small stars very close to him, except in the rays of the corona.

Professor Watson, a well-known and experienced astronomer, who observed the eclipse at Rawlins, Wy., devoted himself to this work, and by help of specially contrived and extemporised accessories to his equatorial, made a methodical search, which according to accounts already to hand appears to have been, in some degree at least, successful. The first announcement that Professor Watson had discovered Vulcan was received with incredulity, and our veteran English Astronomer Royal thought it highly probable that θ *Cancri* had been mistaken for the sought-for planet; you will remember also I stated at a former meeting that although the discovery of an intra-mercurial planet had been notified, it was not by any means received by astronomers as established. More recent advices, however, add considerably to the probabilities that Professor Watson has actually discovered a planet moving inside the orbit of Mercury. The chart shown will give you an idea of the position of the body, as well as that of θ *Cancri* when observed, which at once disposes of Sir George Airy's suggestion that that star had been mistaken for a planet. Professor Watson says "that while searching with his specially-fitted telescope he came across *a ruddy star of the four and a-half magnitude which had a perceptible disc,* the magnifying power being only 45." He says also "it was much brighter than θ *Cancri,*" which is the fifth magnitude. It has been suggested that the object seen might have been a comet, but Professor Watson specially remarks that "there was no appearance such as would be expected if it had been a comet;" and further, that he feels warranted in believing it to be an *intra-mercurial planet.* Although I do not think this observation alone will establish the existence of a new planet beyond all doubt, it at all events makes it highly probable, and will stimulate astronomers to avail themselves of every possible chance of ratifying Professor Watson's observation. A Mr. Swift, a well-known American observer of comets, also saw a "strange star," and although the positions he gives do not quite agree with those of Professor Watson, his observation is admitted to be in a great measure corroborative. It is pointed out in *Nature,* No. 463, that a search along the Ecliptic within 10° or 12° each side of the sun with large refractors provided with long *dew caps, blackened inside,* will afford the best and probably only chance of recovering Professor Watson's planet, until the total eclipse of 1882.

ART. XII.—*The Sounds of the Consonants, as Indicated by the Phonograph.*

BY ALEX. SUTHERLAND, M.A.

[Read 14th November, 1878.]

ON its first discovery, the phonograph was hailed with much satisfaction by those who are devoted to the study of music as a physical science, but a few months of actual experience have shown that their hopes were by no means likely to be fulfilled. As a means of registering sounds the phonograph is not to be compared with methods that have long been known; the phonautograph of Leon Scott, the manometric flame of König, the graphic method of Duhamel, all give results that are more easy of interpretation than the phonograms printed by the new instrument on tin-foil. It is almost impossible to see, much less properly to estimate, the minute and delicate curves contained in the dots which make up the phonogram. A microscope gives little assistance, for when one looks down into an indentation presenting intricate surfaces of curves in three dimensions, the unaided eye can distinguish little of any importance in its appearance.

Various contrivances have already been adopted for the purpose of examining these indentations more thoroughly; one observer has made careful sections of the tin-foil, and by magnifying these to the extent of about 400 diameters has been able to verify the results already obtained by other instruments. Jenkins and Ewing in their recent articles in *Nature* described multiplying arrangement which they have used with success in order to obtain magnified tracings of the marks obtained by singing the vowels into the phonograph. In this way they have made careful analyses of the wave forms which constitute the vowel sounds ū and ō when sung in different notes. But they cannot claim to have discovered a single new fact. The truth seems to be that while the tin-foil on which the phonograms are imprinted is impressed with moderate ease, there is yet enough of mechanical resistance to destroy altogether the finer sorts of vibrations.

Now we know from Helmholtz's researches that while the pitch and intensity of a note depend on the rapidity and amplitude of its vibrations, its richness, and indeed all that serves to give character to the note, depend on the number and kind of secondary vibrations with which the main vibration is attended. Thus if the note is attended by its octave, that is, if in addition to the vibrations which give the note itself, there are present a secondary set of vibrations of twice the rapidity, then we have a sound which the ear recognises at once as musically the same note, and yet it perceives a richness and fulness which was not present in the simple tone. If to this double set of vibrations there be added a third set, three times as rapid as the first, there is again a change in the quality of the tone; and while a musician would say that the note was the same, the ear would nevertheless declare that though the pitch and intensity were the same, the character is notwithstanding quite different.

It was from the consideration of this last element, the quality of the note, that Helmholtz was able to originate the theory now generally accepted as to the nature of the vowel sounds. Every set of vibrations given off either by the human voice or by any musical instrument tends to strengthen itself by the addition of a series of harmonics, the first being twice as rapid as itself, the next three times, the next four times, the next five times, and so on. Thus, if the sound be C we may have this note strengthened by the addition of the C above, by the G above that, by the next C, the next E, the next G again, and so on.

Now it is possible by means of resonators to strengthen any one of these secondary vibrations, and so completely alter the character of the note produced; if a person were to sing the same note through funnels of different shapes the sounds would still be recognised by the ear as the same note, but each would have its own distinctive character.

This is all that takes place when a vowel is pronounced by a human voice; a certain note is emitted by the larynx, the mouth is shaped into a resonator so as to strengthen certain of the harmonics of that note. If the mouth is partially opened, and the cavity made somewhat round by the action of the under-jaw, we have the second partial tone strengthened and made equal, or in some cases more intense than the fundamental note; the result is that the primary vibration is followed by a second equal to it, and so the phonogram gives for the long sound of ō a series of dots arranged in pairs; in the word "mole," pronounced in a deliberate way but without dwelling unnecessarily on the syllable, there are about ninety of these pairs of vibrations to make up the vowel sound.

 or  or 

The long sound of *ū* or *oo* as in "roof" consists of the fundamental note strengthened by its third partial, that is if the vowel be spoken on the note C, there will be added to this a series of vibrations corresponding to the G of the octave above. The marks produced consist of a series of pear-shaped dots closely contiguous, the broad end representing the place where the fundamental is reinforced by its second harmonic, the narrow end representing the secondary smaller vibrations.

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In the word "roof," pronounced with moderate rapidity, there are between forty and fifty of these impressions to represent the vowel sound.

The vowel ā, as in "far," consists of the fundamental note strengthened by both the second and third partials; hence its phonograms partake of the characters both of ō and of ū. A slightly pear-shaped dot is followed after a definite interval by a much smaller dot. In the word "far" it takes from 150 to 170 of these pairs to give the vowel sounds.

The sound "awe" has altogether four partials, the fundamental tone together with its three first harmonics; its phonogram seems to consist of two pear-shaped dots of which the second is slightly less than the first.

The remaining vowels I have made no effort to analyse, but their phonograms, so far as I can make out, are—

ŏ 

ē 

ŭ 

It is plain, then, that while music can be produced by simply reproducing the fundamental vibration we can hope to reproduce a vowel sound only by adding to that fundamental its proper harmonic. Now, for the first and second harmonics the phonograph does this with sufficient distinctness, hence we get the vowels ō and ū and ā well enunciated; but when we come to produce the vowels ē, āū, ŭ, &c., the results are vague, for the vibrations are too feeble to register themselves properly on the tin-foil, and so, while the fundamental note is loudly sounded, the vowel is almost beyond recognition.

The ear has the power of analysing all these vibrations, but when the sound is drawn by any of the graphic methods the eye does not recognise each of them as a distinct vibration, but sees a single set of vibrations, whose lines are broken and varied by the super-position of the smaller sets. In the phonograms, as seen on the tin-foil, we see the fundamental vibrations marked as a row of prominent dots; the harmonics appear either as smaller dots between, or as variations in the thickness and depth of the main juncture. This is the origin of the pear-shaped dots which recur so often, and also of the dashes which seem as though drawn out in some places and thickened in others. Among the consonants we have to distinguish two very different classes. The sibilants and liquids have wave-forms of their own which are no less constant and definite than those of the vowels; but the remainder which form the real consonants have no wave vibrations peculiar to themselves; perhaps it might be more correct to say that they have no vibrations whatever, but exist only as modification of the vowel sounds.

First, as to the liquids.—Of all the letters there is none that gives so marked a phonogram as R. This consists of groups of dots varying from four to ten, according to the amount of roughness put in the letter, and these groups are separated by intervals equal to about four of their wave lengths. The dots are similar in shape to those of the vowel ŭ, and so we reach the conclusion that the liquid *r* is nothing more or less than the vowel ū interrupted twenty or thirty times in a second.

The letter *l* has a simple sound; its phonogram consists of a series of bars, with smooth surfaces, that is, there are no harmonics visible, ▬ ▬ ▬; the curve dips into the tin foil, and then rises by an unbroken sweep. This is what we should expect; for in pronouncing this sound the mouth is closed by the tongue being placed close against the palate, while the breath issues through the narrow passage then left. The larynx produces its note, consisting of the fundamental vibrations with its harmonics, but there is now no resonating cavity to strengthen any one of these harmonics, and so the letter *l* passes forth as an almost purely musical note; none of the harmonics being strengthened, they are unable to make any impression on the tin-foil, and so we have nothing more than a series of simple dashes.

M seems likewise to consist of a series of dashes, but at the end of every dash there occurs a small dot indicating, I suppose, the existence of some harmonic. The sound of this letter is made by allowing the breath to pass through the nose, and the nasal cavity must in some manner act as a resonator, giving prominence to certain of the partials, but this effect is weak in comparison with the similar action by which the mouth produces the vowel sounds. At the same time the nasal cavities cannot have all to do in the production of the sound of *m*, for if while sounding this letter we raise the tongue and so contract the cavity of the mouth, even though the latter is still kept shut, we change from the sound of *m* to that of *n*, in which the long dash is divided into a shorter dash, followed by a dot, so that the phonogram of *n* is a short dash with two dots.

The phonograph is of little use in the determination of wave-forms for sibilants. It is difficult to obtain records of these sounds, and their excessive minuteness makes it difficult to decide as to their shape. They seem, however, to consist of an excessively numerous series of small dots.

The remaining consonants are all formed in the same way, that is by either checking or letting go the breath; at the beginning of a syllable, we suddenly permit the sound to escape, at the end we suddenly stop it, and the ear recognises these sudden changes as consonants. The change may take place in three ways, either sharply and instantaneously, in which case we have the hard consonants p, t, k, or rather more gradually, which gives the softer sounds of b, d, g, or it may take place by stopping or commencing the sound without at the same time stopping or commencing the breathing. If we stop a sound at the end of a syllable, but allow the breath still to pass out, we have the sounds of f, v, th, or *ch*. The phonograms placed on the table show the differences between these three classes of consonants. With the explosive consonants p, t, k, the vowel sounds commence sharply; with the soft consonants b, d, g, there is a gradual swell in the intensity of the dots, showing that the vowel sound was at first permitted to escape by degrees. With the aspirates f, v, th, and *ch*, a series of indeterminate marks either precedes or follows the vowel sound, showing that the breath was escaping before or after the vowel had sounded.

Now, the difference between the corresponding consonants in these three classes is much more difficult to make out. Why, we may ask, should the sudden stoppage of a sound by the lips be recognised as the letter *p*, and the sudden stoppage of the same sound by the teeth and tongue be recognised as the letter *t*, or if the tongue and palate be employed to do exactly the same thing why should we recognise the resulting consonant as *k?*

An examination of the phonograms gives some clue to this distinction. It will be found that on pronouncing a syllable beginning with *p* such as "pa" before the vowel sound has properly begun, there will be found a few marks which do not really belong to that vowel but have more affinity to the vowel ŭ; the explanation is that if the lips are closed, and we open them to emit the full sound ā, we do not at once reach the necessary resonating cavity, we have to pass through the intermediate stages. Now these intermediate stages are the resonating cavities which give the various sounds of ŭ, and though these are very few in comparison with the subsequent vowel vibrations they are sufficient to be recognised by the ear, and so we can tell at once that it must have been the lips which permitted the sudden passage of the sound.

When the consonant is produced by the tongue and teeth, as in the letter *t*, before the vowel commences we have the marks corresponding to ĕ short; and when the consonant is k, the vowel is preceded by marks corresponding first to the long ē, and then to ā, as in "may."

Hence the formation of all the consonants. They are either hard, soft, or aspirated; and the ear judges as to whether they are formed by the lips, teeth, or palate, by observing the vowels through which the sound glides before dwelling on the main vowels.

Thus we find that all sounds, to which the human voice gives rise, consist of vibrations of fixed periods, with their harmonics; the presence of these harmonics determines the nature of the vowel, and moreover enables us to decide by the ear as to which of the consonants has been uttered.

ART. XIII.—*Experiments made on a Sample of Pig Iron received from the British and Tasmanian Iron Company, Port Lempriere, Tasmania.*

BY J. COSMO NEWBERY AND FREDERIC DUNN.

[Read 12th December, 1878.]

DURING the month of November, 1876, a sample of pig iron was sent to the laboratory for examination and report. Upon treating a portion of this iron (which had been very finely ground) in a flask and boiling on the sand-bath with nitrohydrochloric acid ($1\frac{1}{2}$ parts of hydrochloric to 1 of nitric acid) the iron was readily attacked. When all action had ceased the supernatant liquor was carefully decanted off from the residue. The latter was found to have a peculiar bronze-like appearance. This powder was at first believed to be "nitride of titanium," but upon further investigation was found to be a compound of chromium iron and carbonaceous matter.

The pig iron which was found to contain the most chromium was coarse-grained and crystalline, having a white lustre somewhat resembling "spiegeleisen" in appearance, but its lustre was of a less brilliant white colour, and the crystal plates very rough.

This sample gave a residue on treatment with nitrohydrochloric acid of 9·38 per cent. of a bronze-coloured chromium compound (calculated to the total pig), whereas a sample of pig iron which was of much finer grain and granular in structure gave 1·52 per cent. of the same peculiar compound.

The pig iron when treated with hydrochloric and sulphuric acids gave different results to that obtained by nitric acid.

TREATMENT WITH HYDROCHLORIC ACID.

The pig iron was broken up into pieces about the size of a bean, placed in a flask, and boiled with hydrochloric acid. After all effervescence had ceased the vessel was taken off the sand-bath and transferred to a quiet place, in order that the small particles might settle at the bottom. The supernatant liquor was then decanted off, the residue was re-treated with hydrochloric acid, decanted again, and residue well washed and dried. A magnet was then passed through it so as to take up any particles of metallic iron which might be left undecomposed. Upon examining the residue with the microscope, peculiar bronze-coloured, star-like crystals were observed. Owing to the large amounts of silica and carbonaceous matter which are left, it is very difficult to separate the little bronze-like stars. These stellate forms contain a large percentage of chromium as a component part in combination with iron. A sufficient quantity has not yet been obtained for a quantitative analysis. This difficulty is due to their solubility in boiling hydrochloric acid.

If they are boiled with nitric acid they lose their bronzy appearance, and become silvery white; are very slowly dissolved by this acid.

They are very slowly acted upon by sulphuric acid.

TREATMENT WITH NITRIC ACID.

Small pieces of pig iron, if boiled with nitric acid, leave silvery white plates. When these appear the acid solution was carefully decanted off and the plates well washed with distilled water, and re-treated with nitric acid, and boiled. They were washed out into a suitable vessel and dried. These plates are not magnetic, so that any undissolved iron could be removed by a magnet.

A large proportion of these metallic silvery-looking plates are dissolved, owing to their long-continued boiling in this acid.

The following are the analyses which have been made of various samples of this compound:—

	(1)	(2)	(3)	(4)	(5)	(6)
Percentage of iron	87·44	83·92	84·78	84·60	84·69	84·44
„ chromium ...	12·71	16·07	15·73	15·40	15·90	15·56
„ carbon	trace	—	—	—	—	—
	100·15	99·99	100·51	100·00	100·59	100·00

No. 1. Is the analysis of the first sample of silvery white plates obtained. The plates were not thoroughly freed from undissolved iron, hence the high percentage.

No. 2. This sample was re-treated for some time in nitric acid, washed well with distilled water, dried, and the magnet passed through the mass, and is therefore the purest sample. The iron and chromium were estimated by a process founded on that given by "Crooke's Select Methods in Chemical Analysis."

Nos. 1, 3, 5. The chromium and iron were estimated in these samples by the fusion method, which is described in *Fresenius' Quantitative Chemical Analysis.*

Nos. 4, 6. The iron in these samples was carefully determined by a standard solution of permanganate of potash, and the chromium estimated by loss.

In appearance these non-magnetic scales resemble osmiridium, being of a greyish silvery white, and are brittle.

Hydrochloric acid readily dissolves these plates, forming an emerald green solution. Long boiling is required, however, to get a complete solution.

Towards the end of the operation, small particles having a bronze-like appearance float in the liquid; these can only be dissolved by continued boiling in the concentrated acid.

Sulphuric acid readily attacks the plates. They are not acted upon by acetic acid.

A portion of these plates were boiled in a flask with nitric acid for a very long time, and were entirely dissolved.

There is not the slightest doubt that a large percentage of these silvery plates are dissolved, owing to the long-continued boiling which the pig iron receives during its solution in nitric acid.

TREATMENT WITH SULPHURIC ACID.

Stellate forms are obtained if the pig iron be treated in the same manner as is described under the "hydrochloric acid treatment."

TREATMENT WITH NITROHYDROCHLORIC ACID.

A portion of the finely pulverised iron was treated in a flask with hot nitrohydrochloric acid until a bronze-like powder made its appearance; water was then added to stop the action of the acid, and the powder separated and collected: the iron residue was again treated with acid.

The bronze powder thus obtained was purified by re-treating with nitrohydrochloric acid and well washing.

If the bright bronze powder be left exposed to moist air it becomes slightly tarnished and shows a beautiful iridescence.

If boiled in nitric acid for a short time, it loses its peculiar bronzy appearance and is converted into those silvery white non-magnetic scales, the same as those obtained in the residue, after boiling the pig iron in nitric acid.

The filtrate from these plates was tested to see if any chromium had gone into solution; only a slight reaction was obtained.

The bronze powder upon treating with sulphuric acid and boiling is readily attacked, carbonaceous particles being liberated; the continued action of the sulphuric acid on the latter causes the evolution of fœtid hydrogen, the solution assuming a brownish black appearance, which upon further boiling assumes a green colour.

Hydrochloric acid dissolves this powder, but the peculiar bronzy appearance remains to the last. The solution is of a fine emerald green colour; carbonaceous particles separate during the solution; a peculiar hydrocarbon smell is evolved.

Acetic acid fails to dissolve this bronze powder, and is therefore useful in separating any free iron which may be mechanically mixed with it.

A portion of the powder was ground in an agate mortar with water; it loses its bronze-like appearance, becoming steel-grey, carbonaceous matter being liberated (this shows that the carbonaceous matter is merely mechanically disseminated), the non-magnetic, metallic particles being left behind.

Upon analysis the bronze powder was found to contain in 100 parts:—

Percentage of iron	81·12
„ chromium	15·09
„ carbonaceous matter ...	4·11
„ silica	0.53
	100·85

The analysis shows that when separated from the carbon it has the same composition as the nitric acid residue. The following is its composition, after deducting the carbon and silica:—

Percentage of iron	84·32
„ chromium	15·68
	100·00

TREATMENT WITH ACETIC ACID.

100 grains of finely-ground pig iron were placed in a flask and gently boiled with acetic acid.

During solution the acid at first readily attacks the iron, the liquid assuming a green tint, afterwards passing into a deep brown.

After treating the finely-divided iron two or three times with fresh portions of acetic acid, the liquid becomes nearly colourless, holding very little iron in solution, and not any chromium; on further boiling with acetic acid chromium was taken into solution.

When all action had ceased there were obtained 37 grains of insoluble pig iron, of which 2·20 grains were non-magnetic, metallic particles, the remaining 34·8 grains being magnetic.

Upon analysis the non-magnetic portion gave in 100 parts:—

Percentage of iron	85·39
„ chromium	14·90
„ carbon	trace
	100·29

On treating these particles with nitric acid they are converted into those silvery white plates. They correspond exactly to the non-magnetic particles mentioned under the heading of "Treatment with the Magnet."

Upon treating a portion of the magnetic particles in boiling hydrochloric acid, a few bronze-like stars were obtained, corresponding to those mentioned under the "Hydrochloric acid treatment;" treating a portion also in boiling nitric acid for a short time the silvery white plates are obtained. These, upon analysis, gave in one hundred parts:—

Percentage of iron	84·60
„ chromium	15·40
	100·00

Treatment with the Magnet.

The pig iron was ground to a very fine powder. One hundred grains were then placed upon a glazed sheet of paper and the magnet held in close proximity to the mass, when the magnet became covered with metallic particles. These were shaken on to a sheet of paper, thus separating the magnetic from the non-magnetic. The magnetic particles were then ground to a finer state of division, and re-treated with the magnet several times.

By this treatment there were obtained 2·48 per cent. of bright, metallic, non-magnetic particles, and 7·43 per cent. of slightly magnetic, metallic particles.

Upon analysis one hundred parts of the non-magnetic particles contained :—

Percentage of iron	85·44
„ chromium	14·95
„ carbon	trace
	100·39

Upon boiling a portion of these particles in nitric acid, they were converted into the silvery plates.

One hundred parts of the slightly-magnetic particles gave, upon analysis :—

Percentage of iron	87·55
„ chromium	11·28
„ silica and undetermined	1·17
	100·00

A number of these slightly magnetic particles were boiled in a flask with nitric acid (1 part of nitric acid with 2 parts of distilled water) until the solution ceased to be coloured by the dissolved iron. Those silvery white particles as mentioned under the "Nitric Acid Treatment," were obtained.

Upon analysis these plates gave in 100 parts :—

Percentage of iron	84·44
„ chromium	15·56
	100·00

This shows that a large percentage of those non-magnetic silvery plates are left in the magnetic mass, even after very careful treatment with the magnet; this no doubt is owing to the plates being impregnated with the surrounding particles of metallic iron.

The quantity of star-like forms in the hydrochloric and residue did not suffice for an exact analysis, but their behaviour with acids shows that they differ in composition from the silvery plates.

One sample of the iron gave minute prismatic needles in place of plates, upon treatment with nitrohydrochloric acid.

These examinations show that the assumption that the chromium is alloyed or combined with the whole mass of the iron is incorrect, but that at any rate, most, if not the whole, of it is as two or more definite compounds of iron and chromium diffused through the mass of iron. Different portions of the same pig iron contain variable percentages of these compounds.

The sample of pig iron from which these results were obtained gave 8·98 per cent. of chromium in one part, and 6·63 per cent. in another.

ART. XIV.—*Formation of Hyalite by the Action of Ammonia.*

BY J. COSMO NEWBERY, B.SC.

[Read 12th December, 1878.]

IN the examination of building stones used in Melbourne I have noticed that the greatest amount of decay takes place during the summer months, December, January, and February, and that the stones which harden on exposure harden most during those months; also, that taking two portions of the same stone, saturating one part with water, and leaving the other dry, the wet stone hardens first, the hardening taking place from the outside inwards.

Analysis of the outer portions of these hardened stones shows an excess of silica, more or less hydrous, and nearly always giving distinct traces of ammonia.

In the Geological Survey Reports, Nos. 4 and 5, I have called attention to some of these peculiar passages of silica from the inner to the outer parts of the stone, and shown that all our freestones, except those already hardened by exposure, are acted on with considerable rapidity by ammonia and carbonate of ammonia. Some are hardened by this action and some are disintegrated. Those which are destroyed fall gradually away, the cementing material being decomposed by the ammonia, and the quartz grains are left free to fall or be washed away by the rain.

In the stones which are not destroyed but harden, some other action takes place; the cementing material between the sand grains is not softened, but it changes from a dull opaque or white clayey cement to a vitreous or quartz-like material, eventually, as may be seen on the surface of many of our sandstone ranges, to a dense quartzite.

On the Grampian range, at the Blue range at Mansfield, and at Freestone Creek in Gippsland, the rocks are usually very hard silicious sandstones at the surface, and give when crushed and washed little or no clayey matter; but a few inches, or at most a few feet, from the surface on the same beds the character changes, and on crushing and washing the cementing material may be obtained as a nearly white clayey material like kaolin.

I have to a limited extent succeeded in changing clayey sandstones to hard silicious sandstones by causing them to absorb ammonial solutions in such a manner that the liquid was absorbed at one end of the stone and evaporated at the other, and obtained an outer surface hard and silicious like that found in nature.

With stones containing silicia in a hydrous form, like the Oamaru, New Zealand, limestone, the passage is most marked. In a few weeks the outer or evaporating surface gave upon analysis twice as much silica as the interior of the stone.

Thus, besides mere transfer of silica, the ammoniacal solutions of silica are capable of producing actual metamorphism, changing the character and structure of the silicate rocks.

Some eighteen months ago I placed some clean infusorial earth from Talbot in a solution of ammonia. The whole of the earth was composed of the transparent forms of diatoms, *i.e.*, nearly pure hydrous silica. Recently examining the contents of the bottle, I find that a portion of the silica has been dissolved in the ammonia, giving a solution containing ·771 per cent. of silica; at 212 it lost 0·1 per cent., and 0·01 on heating to about 350. The amount of hydrous silica in solution is therefore over 500 grains to the gallon, far in excess of that held in solution in the waters of the hot springs of New Zealand.

The solution of silicate of ammonia may be boiled till all excess of ammonia has been expelled, and according to Pribram (*Watts' Sup.*), 1 equivalent of ammonia is left in solution with 80 of silica.

This boiled solution, in contact with bases, forms crystallisable hydrous silicates. When evaporated to dryness it deposits the silica as a film, which shrinks and cracks as the last of the water is driven off.

In this solution of silica, held in solution by ammonia, which we may obtain from almost any, if not all, of our springs or subterranean waters, we have, no doubt, one of the active agents of metamorphic action. Just above the surface of the liquid on the sides of the vessel I find a botryoidal coating of hydrous silica, in all respects identical with the mineral hyalite.

In this artificial hyalite there are some infusorial forms which have been entrapped. Most of them seem to be partly dissolved; some are mere skeletons of the original form.

In the mineral hyalite from our basaltic formations my assistant, Mr. Dunn, finds distinct traces of ammonia, and as we know ammonia is present in all our subterranean waters, we have a means of accounting for these films or crusts of botryoidal silica, and probably for the veins and masses of chalcedony and opal found in the decomposed volcanic rocks.

A curious change has taken place in the residue of the infusorial earth from which the solution was made. All the forms of diatoms have vanished, and instead I find a fine granular powder. The mass has shrunk considerably, and is covered by a friable film.

1878.

PROCEEDINGS.

ROYAL SOCIETY OF VICTORIA.

ANNUAL MEETING.

13th March, 1879.

The President in the chair—Present, 14 members.

The Annual Report and Balance-sheet for 1878 were read, as follows:—

"*Report of the Council of the Royal Society of Victoria for the year* 1878.

"Your Council has the honour to report that the following papers were read during the session of 1878:—

"On the 11th of April Dr. Jamieson read a paper on 'Photographs on the Retina;' Mr. Josephs exhibited a new form of circuit closer for torpedo firing; and Mr. Pirani exhibited Sir William Thompson's electric replenisher.

"On the 16th of May Mr. Foord read a *résumé* of Mr. A. M. Smith's paper on 'Gold Bullion Assay;' Mr. Kernot exhibited a phonograph constructed by Mr. Kirkland.

"On the 14th of June Mr. Pirani exhibited Sir William Thomson's new form of Daniell's constant battery; Mr. Kernot read a paper on the 'Strength of Columns;' Dr. Jamieson read a paper on 'Some Points of Resemblance in the Respiration of Plants and Animals;' and Mr. Sutherland exhibited a phonograph.

"On the 11th of July Dr. Wilkie submitted a paper on the cycloid curve; Mr. Pirani exhibited a microphone.

"On the 12th of September Mr. Ellery read a paper on a proposed new method of employing photography in military surveys.

"On the 17th of October Dr. Jamieson read a paper on 'The Perception of Colour.'

"On the 14th of November Mr. Ellery read a paper on 'The Supposed Intramercurial Planet,' and Mr. Sutherland read a paper on 'The Sounds of the Consonants as Indicated by the Phonograph.'

"On the 12th of December Mr. Cosmo Newbery read a paper on the 'Occurrence of Chromium in the Iron Ore of Tasmania,' and another 'On the Formation of Hyalite by the Action of Ammonia on Infusorial Earth;' Mr. Ellery exhibited the singing and the sensitive flame.

"Volume XIV. of the Society's transactions was issued on the 11th of July, and duly forwarded to members and to the Societies entitled to receive it. Volume XV. is now in the press, and will be ready for issue in April.

"During the past year the Society has made provision for the admission of associates, who shall have all the privileges of membership except that of voting, but shall pay no entrance fee, and shall pay an annual subscription of one guinea per annum. Six gentlemen have been elected associates of the Society."

BALANCE-SHEET.

The Hon. Treasurer in Account with the Royal Society of Victoria.

DR.

To Balance from last Balance-Sheet				£259	12	11
,, Government Grant, 1878-9				200	0	0
,, Interest on Fixed Deposit				10	0	0
,, Rent				1	1	0
,, Sale of Papers				1	2	6
,, Entrance Fees				18	18	0
,, One Life Subscription				21	0	0
,, Subscriptions—						
47 Ordinary	£98	14	0			
5 Half do.	5	5	0			
15 Country...	15	15	0			
1 Half do.	0	10	6			
Arrears	26	5	0			
				146	9	6
				£658	3	11

CR.

By Printing and Stationery				£133	12	6
,, Books				4	16	0
,, Freight and Charges on Books and Transactions				4	15	0
,, Conversazione				10	3	6
,, Ten Debentures paid				50	0	0
,, Interest on Debentures				6	6	0
,, Insurance				2	17	6
,, Rates				4	13	4
,, Gas and Fuel				6	10	6
,, Furniture				3	17	6
,, Repairs				3	15	4
,, Hall-keeper and Clerical Assistant				25	0	0
,, Collector's Charges				17	7	8
,, Postages and Petty Cash				27	0	0
				£300	14	10
,, Balance in Bank—						
Fixed Deposit	£200	0	0			
Current Account	157	9	1			
				357	9	1
				£658	3	11

Compared with the Vouchers, Bank Pass-book, and Cash Book, and found correct.

5th March, 1879.

H. MOORS }
JAMES E. GILBERT } AUDITORS.

P. DE JERSEY GRUT, HON. TREASURER.

STATEMENT OF LIABILITIES AND ASSETS.

Dr. LIABILITIES.	£	s.	d.	ASSETS. Cr.	£	s.	d.
To Publishing Fund	£216	11	2	By Balance in Bank	£357	9	1
,, Three Debentures outstanding	15	0	0	,, Estimated Value of Outstanding Subscriptions ...	20	0	0
,, Interest unclaimed	12	12	0	,, Rents due	81	19	1
				,, Hall, Library, Furniture, &c., insured for ...	2300	0	0
	£244	3	2				
Balance	2515	5	0				
	£2759	8	2		£2759	8	2

PUBLISHING FUND.

Dr.	£	s.	d.	Cr.	£	s.	d.
To Balance	£216	11	2	By Royal Society of Victoria	£216	11	2

The Report and Balance-sheet were both adopted.

Nominations were received for the election of officers, which was postponed till next meeting.

A committee, consisting of Messrs. Ellery, Foord, Pirani, Joseph, Kernot, Sutherland, and Dr. Jamieson, was appointed to report on the desirability of instituting a course of lectures.

ORDINARY MEETINGS.

11*th April*, 1878.

R. L. J. Ellery, Esq., F.R.S., in the chair.—Present, 25 members.

G. F. H. Ulrich, Esq., F.G.S., resigned his position on the Council.

Mr. A. Sutherland was elected Honorary Secretary in place of H. K. Rusden, Esq., resigned.

Mr. R. E. Joseph exhibited a new form of circuit closer for use in the firing of torpedoes.

Dr. Jamieson read a paper on "Photographs on the Retina of the Eye."

Mr. Pirani exhibited Sir William Thomson's replenisher, and explained its action.

(Signed) ROBT. L. J. ELLERY.

16*th May*, 1878.

R. L. J. Ellery, Esq., F.R.S., in the chair—Present, 21 members.

H. Moors, Esq., was elected a member of the Council, in place of G. F. H. Ulrich, Esq., resigned.

Mr. J. B. Cohen and Mr. W. M. Madden were duly elected members.

Mr. Ulrich was elected a corresponding member.

Mr. G. Foord read a paper by Mr. A. M. Smith on "Gold Bullion Assay," and a short discussion ensued.

Mr. Ellery described and exhibited a new form of self-registering rain gauge.

Mr. Kernot exhibited a phonograph constructed by Mr. Kirkland, jun., but stated it had not yet been successful in speaking.

(Signed) ROBT. L. J. ELLERY.

14*th June*, 1878.

R. L. J. Ellery, Esq., in the chair—Present, 21 members.

Mr. F. J. Pirani exhibited three cells of Sir William Thomson's new force of Daniell's constant battery.

Mr. Kernot read a paper on the "Strength of Columns." It was resolved that this paper be printed and discussed at the next meeting.

Dr. Jamieson read his paper on a "New Point of Resemblance in the Respiration of Plants and Animals." It was resolved that this paper also should be printed and discussed at the next meeting.

Mr. Sutherland exhibited a phonograph, which made some rudimentary efforts at speech, and the meeting then closed.

(Signed) Robt. L. J. Ellery.

11*th July*, 1878.

R. L. J. Ellery, Esq., in the chair—Present, 18 members.

Mr. C. F. Clough was elected a member of the Society.

A discussion then took place on Mr. Kernot's paper on the "Strength of Columns."

Dr. Wilkie submitted a paper on the Cycloid Curve, which was accepted as read.

Mr. Ellery described the great meteor which had recently been visible over a large part of Australia.

Mr. Pirani exhibited a microphone, and a series of interesting experiments were made with it.

(Signed) Robt. L. J. Ellery.

12*th September*, 1878.

R. L. J. Ellery, Esq., in the chair—Present, 12 members.

Six gentlemen were nominated for membership.

It was resolved that a special meeting should be held to consider the recomendation of the Council as to the admission of associates to the Society.

The postponed discussion then took place on Dr. Jamieson's paper.

Mr. Ellery read a note descriptive of a new method of employing photography in military surveys.

(Signed) Robt. L. J. Ellery.

17*th October*, 1878.

R. L. J. Ellery, Esq., in the chair.

The following gentlemen were elected ordinary members :—F. R. Godfrey, Esq., Dr. Browning, Dr. Le Fevre, A. R. Walker, Esq.

Dr. Thornton, Bishop of Ballarat, was elected a country member.

Sir Samuel Wilson was elected a life member.

Dr. Jamieson read his paper " On the Perception of Colour."

(Signed) Robt. L. J. Ellery.

14*th November*, 1878.

R. L. J. Ellery, Esq., in the chair.

Resolved—That in future *ad interim* members of Council shall retain office only so long as those members whom they replace would have retained it.

Mr. Ellery read a paper on "The Supposed Intramercurial Planet."

Mr. Sutherland read a paper on the " Sounds of the Consonants as Indicated by the Phonograph."

Discussion on this paper was held over till next meeting.

(Signed) Robt. L. J. Ellery.

12*th December*, 1878.

R. L. J. Ellery, Esq., in the chair—Present, 12 members.

Six gentlemen were elected associates, namely—Mr. Challen, Mr. Allman, Mr. Goldstein, Mr. Morris, Mr. Olliver, and Mr. Kirkland, jun.

A discussion then took place on Mr. Sutherland's paper, in which Mr. Pirani and several other members joined.

Mr. Cosmo Newbery read his paper, entitled, " On the Occurrence of Chromium in the Iron Ore of Tasmania," and also a paper on the " Formation of Hyalite by the Action of Ammonia on Infusorial Earth."

Mr. Ellery exhibited the singing and the sensitive flame.

(Signed) Robt. L. J. Ellery.

LAWS.

Name. I. The Society shall be called "The Royal Society of Victoria."

Objects. II. The Royal Society of Victoria is founded for the advancement of science, literature, and art, with especial reference to the development of the resources of the country.

Members and Honorary Members. III. The Royal Society of Victoria shall consist of Members and Honorary Members, Corresponding Members, and Associates, all of whom shall be elected by ballot.

Patron. IV. His Excellency the Governor of Victoria, for the time being, shall be requested to be the Patron of the Society.

Officers. V. There shall be a President, and two Vice-Presidents, who, with twelve other Members, and the following Honorary Officers, viz., Treasurer, Librarian, and two Secretaries of the Society, shall constitute the Council.

Management. VI. The Council shall have the management of the affairs of the Society.

Ordinary Meetings. VII. The Ordinary Meetings of the Society shall be held once in every month during the Session, from March to December inclusive, on days fixed by and subject to alteration by the Council with due notice.

Annual General Meetings. VIII. In the second week in March there shall be a General Meeting, to receive the report of the Council and elect the Officers of the Society for the ensuing year.

Retirement of Officers. IX. All Office-bearers and Members of Council, except the six junior or last elected ordinary Members, shall retire from office annually at the General Meeting in March. The names of such Retiring Officers are to be announced at the Ordinary Meetings in November and December. The Officers and Members of Council so retiring shall be eligible for the same or any other office then vacant.

Election of Officers.

X. The President, Vice-Presidents, Treasurer, Secretaries, and Librarian shall be separately elected by ballot (should such be demanded), in the above-named order, and the six vacancies in the Council shall then be filled up together by ballot at the General Meeting in March. Those members only shall be eligible for any office who have been proposed and seconded at the Ordinary Meeting in December, or by letter addressed to one of the Secretaries, and received by him before the 1st March, to be laid before the Council Meeting next before the Annual Meeting in March. The nomination to any one office shall be held a nomination to any office the election to which is to be subsequently held. No ballot shall take place at any meeting unless ten members be present.

Members in arrear.

XI. No Member whose subscription is in arrear shall take part in the election of Officers or other business of the meeting.

Inaugural address by the President.

XII. An Address shall be delivered by the President of the Society at either a Dinner, Conversazione, or extra meeting of the Society, as the Council for the time being may determine, not later than the Ordinary Meeting in June in each year.

Vacancies.

XIII. If any vacancy occur among the Officers, notice thereof shall be inserted in the summons for the next meeting of the Society, and the vacancy shall be then filled up by ballot.

Duties of President.

XIV. The President shall take the chair at all meetings of the Society and of the Council, and shall regulate and keep order in all their proceedings; he shall state questions and propositions to the meeting, and report the result of ballots, and carry into effect the regulations of the Society. In the absence of the President the chair shall be taken by one of the Vice-Presidents, Treasurer, or ordinary Member of Council, in order of seniority.

Duties of Treasurer.

XV. The Treasurer may, immediately after his election, appoint a Collector (to act during pleasure), subject to the approval of the Council at its next meeting. The duty of the Collector shall be to issue the Treasurer's notices and collect subscriptions. The

Treasurer shall receive all moneys paid to the Society, and shall deposit the same before the end of each month in the bank approved by the Council, to the credit of an account opened in the name of the Royal Society of Victoria. The Treasurer shall make all payments ordered by the Council on receiving a written authority from the chairman of the meeting. All cheques shall be signed by himself, and countersigned by one of the Secretaries. No payments shall be made except by cheque, and on the authority of the Council. He shall keep a detailed account of all receipts and expenditure, present a report of the same at each Council Meeting, and prepare a balance-sheet to be laid before the Council, and included in its Annual Report. He shall also produce his books whenever called on by the Council.

Duties of Secretaries.

XVI. The Secretaries shall share their duties as they may find most convenient. One or other of them shall conduct the correspondence of the Society and of the Council, attend all meetings of the Society and of the Council, take minutes of their proceedings, and enter them in the proper books. He shall inscribe the names and addresses of all Members in a book to be kept for that purpose, from which no name shall be erased except by order of the Council. He shall issue notices of all meetings of the Society and of the Council, and shall have the custody of all papers of the Society, and, under the direction of the Council, superintend the printing of the Transactions of the Society.

Meetings of Council.

XVII. The Council shall meet on any day within one week before every Ordinary Meeting of the Society. Notice of such meeting shall be sent to every Member at least two days previously. No business shall be transacted at any meeting of the Council unless five Members be present. Any Member of Council absenting himself from three consecutive meetings of Council, without satisfactory explanation in writing, shall be considered to have vacated his office, and the election of a Member to fill his place shall be proceeded with at the next Ordinary Meeting of Members, in accordance with Law XIII.

Quorum.

XVIII. One of the Secretaries shall call a Special Meeting of Council on the authority of the President or of three Members of the Council. The notice of such meeting shall specify the object for which it is called, and no other business shall be entertained.

Special Meetings of Council.

XIX. The Council shall call a Special Meeting of the Society, on receiving a requisition in writing signed by twenty-four Members of the Society specifying the purpose for which the meeting is required, or upon a resolution of its own. No other business shall be entertained at such meeting. Notice of such meeting, and the purpose for which it is summoned, shall be sent to every Member at least ten days before the meeting.

Special General Meetings.

XX. The Council shall annually prepare a Report of the Proceedings of the Society during the past year, embodying the balance-sheet, duly audited by two Auditors, to be appointed for the year, at the Ordinary Meeting in December, exhibiting a statement of the present position of the Society. This Report shall be laid before the Society at the Annual Meeting in March. No paper shall be read at that meeting.

Annual Report.

XXI. If it shall come to the knowledge of the Council that the conduct of an Officer or a Member is injurious to the interest of the Society, and if two-thirds of the Council present shall be satisfied, after opportunity of defence has been afforded to him, that such is the case, it may call upon him to resign, and shall have the power to expel him from the Society, or remove him from any office therein at its discretion. In every case all proceedings shall be entered upon the minutes.

Expulsion of Members.

XXII. Every candidate for election as Member or as Associate shall be proposed and seconded by Members of the Society. The name, the address, and the occupation of every candidate, with the names of his proposer and of his seconder, shall be communicated in writing to one of the Secretaries, and shall be read at a meeting of Council, and also at the following meeting of the Society, and the ballot shall take place at the next following ordinary meeting of the Society.

Election of Members and Associates.

The assent of at least five-sixths of the number voting shall be requisite for the admission of a candidate.

Members shall sign laws.

XXIII. Every new Member or Associate shall receive due notice of his election, and be supplied with a copy of the obligation,* together with a copy of the Laws of the Society. He shall not be entitled to enjoy any privilege of the Society, nor shall his name be printed in the List of Members, until he shall have paid his admission fee and first annual subscription, and have returned to the Secretaries the obligation signed by himself. He shall at the first meeting of the Society at which he is present sign a duplicate of the obligation in the Statute Book of the Society, after which he shall be introduced to the Society by the Chairman. No Member or Associate shall be at liberty to withdraw from the Society without previously giving notice in writing to one of the Secretaries of his intention to withdraw, and returning all books or other property of the Society in his possession. Members and Associates will be considered liable for the payment of all subscriptions due from them up to the date at which they give written notice of their intention to withdraw from the Society.

Conditions of Resignation.

Honorary Members.

XXIV. Gentlemen not resident in Victoria, who are distinguished for their attainments in science, literature, or art, may be proposed for election as Honorary Members, on the recommendation of an absolute majority of the Council. The election shall be conducted in the same manner as that of ordinary Members, but nine-tenths of the votes must be in favour of the candidate.

Subscriptions.

XXV. Members of the Society, resident in Melbourne, or within ten miles thereof, shall pay two guineas annually, Members residing beyond that dis-

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* The obligation referred to is as follows :—

Royal Society of Victoria.

I, the undersigned, do hereby engage that I will endeavour to promote the interests and welfare of the Royal Society of Victoria, and to observe its laws, as long as I shall remain a Member or Associate thereof.

(Signed)

Address

Date

tance and Associates shall pay one guinea annually. The subscriptions shall be due on the 1st of January in every year. At the commencement of each year there shall be hung up in the Hall of the Society a list of Members and Associates, upon which the payments of their subscriptions as made by Members and Associates shall be entered. During July notice shall be sent to Members and Associates still in arrears. At the end of each year a list of those who have not paid their subscriptions shall be prepared, to be considered and dealt with by the Council.

Entrance fees, &c.

XXVI. Newly-elected Members shall pay an entrance fee of two guineas, in addition to the subscription for the current year. Newly-elected Associates shall not be required to pay any entrance fee. Those elected after the 1st of July shall pay only half of the subscription for the current year. If the entrance fee and subscription be not paid within one month of the notification of election, a second notice shall be sent, and if payment be not made within one month from the second notice, the election shall be void. Members, resident in Melbourne, or within ten miles thereof, may compound for all Annual Subscriptions of the current and future years by paying £21; and Members residing beyond that distance may compound in like manner by paying £10 10s. Associates on seeking election as Members shall have to comply with all the forms requisite for the election of Members, and shall pay an entrance fee of two guineas.

Life Membership.

Durations of Meetings.

XXVII. At the ordinary meetings of the Society the chair shall be taken punctually at eight o'clock, and no new business shall be taken after ten o'clock.

Order and mode of conducting the business.

XXVIII. At the Ordinary Meetings business shall be transacted in the following order, unless it be specially decided otherwise by the Chairman:—

Minutes of the preceding meeting to be read, amended if incorrect, and confirmed.
New Members to enroll their names, and be introduced.
Ballot for the election of new Members.
Vacancies among officers, if any, to be filled up.
Business arising out of the minutes.
Communications from the Council.

Presents to be laid on the table, and acknowledged.
Motions, of which notice has been given, to be considered.
Notices of motion for the next meeting to be given in and read by one of the Secretaries.
Papers to be read.

Strangers.

XXIX. No stranger shall speak at a meeting of the Society unless specially invited to do so by the Chairman.

What business may be transacted.

XXX. At no meeting shall a paper be read, or business entertained, which has not been previously notified to the Council.

Additional Meetings.

XXXI. The Council may call additional meetings whenever it may be deemed necessary.

Visitors.

XXXII. Every Member may introduce two visitors to the meetings of the Society by orders signed by himself.

Members may read papers.

XXXIII. Members and Associates shall have the privilege of reading before the Society accounts of experiments, observations, and researches conducted by themselves, or original papers, on subjects within the scope of the Society, or descriptions of recent discoveries, or inventions of general scientific interest. No vote of thanks to any Member or Associate for his paper shall be proposed.

Or depute other Members.

XXXIV. If a Member or Associate be unable to attend for the purpose of reading his paper, he may delegate to any Member of the Society the reading thereof, and his right of reply.

Members must give notice of their papers.

XXXV. Any Member or Associate desirous of reading a paper shall give in writing to one of the Secretaries, ten days before the meeting at which he desires it to be read, its title and the time its reading will occupy.

Papers by strangers.

XXXVI. The Council may permit a paper such as described in Law XXXIII., not written by a Member of the Society, to be read, if for any special reason it shall be deemed desirable.

Papers belong to the Society.

XXXVII. Every paper read before the Society shall be the property thereof, and immediately after it has been read shall be delivered to one of the Secretaries, and shall remain in his custody.

XXXVIII. No paper shall be read before the Society or published in the Transactions unless approved by the Council, and unless it consist mainly of original matter as regards the facts or the theories enunciated. **Papers must be original.**

XXXIX. Should the Council feel a difficulty in deciding on the publication of a paper, the Council may refer it to any Member or Members of the Society, who shall report upon it. **Council may refer papers to Members.**

XL. Should the Council decide not to publish a paper, it shall be at once returned to the author. **Rejected papers to be returned.**

XLI. The author of any paper which the Council has decided to publish in the Transactions may have any number of copies of his paper on giving notice of his wish in writing to one of the Secretaries, and on paying the extra cost of such copies. **Members may have copies of their papers.**

XLII. Every Member and Associate whose subscription is not in arrear, and every Honorary Member, is entitled to receive one copy of the Transactions of the Society as published. Newly-elected Members shall, on payment of their entrance-fee and subscription, receive a copy of the volume of the Transactions last published. **Members to have copies of Transactions.**

XLIII. Every book, pamphlet, model, plan, drawing, specimen, preparation, or collection presented to or purchased by the Society, shall be kept in the house of the Society. **Property.**

XLIV. The Library shall be open to Members and Associates of the Society and the public at such times and under such regulations as the Council may deem fit. **Library.**

XLV. The legal ownership of the property of the Society is vested in the President, the Vice-Presidents, and the Treasurer for the time being, in trust for the use of the Society; but the Council shall have full control over the expenditure of the funds and management of the property of the Society. **Legal ownership of property.**

XLVI. Every Committee appointed by the Society shall at its first meeting elect a Chairman, who shall subsequently convene the Committee and bring up its **Committees elect Chairman.**

report. He shall also obtain from the Treasurer such grants as may have been voted for the purposes of the Committee.

Report before November 1st.

XLVII. All Committees and individuals to whom any work has been assigned by the Society shall present to the Council, not later than the 1st November in each year, a report of the progress which has been made; and, in cases where grants of money for scientific purposes have been entrusted to them, a statement of the sums which have been expended, and the balance of each grant which remains unexpended. Every Committee shall cease to exist on the 1st November, unless re-appointed.

Grants expire.

XLVIII. Grants of pecuniary aid for scientific purposes from the funds of the Society shall expire on the 1st November next following, unless it shall appear by a report that the recommendations on which they were granted have been acted on, or a continuation of them be ordered by the Council.

Personal expenses not to be paid.

XLIX. In grants of money to Committees and individuals, the Society shall not pay any personal expenses which may be incurred by the Members.

Alteration of laws.

L. No new law, or alteration or repeal of an existing law, shall be made except at the General Meeting in March, or at a Special General Meeting summoned for the purpose, as provided in Law XIX., and in pursuance of notice given at the preceding Ordinary Meeting of the Society.

Cases not provided for.

LI. Should any circumstance arise not provided for in these laws, the Council is empowered to act as may seem to be best for the interests of the Society.

Sections.

LII. In order that the Members and Associates of the Society prosecuting particular departments of science may have opportunities of meeting and working together with fewer formal restraints than are necessary at the Ordinary Meetings of the Society, Sections may be established.

Names and number of Sections.

LIII. Sections may be established for the following departments, viz.:—

Section A. Physical, Astronomical, and Mechanical Science, including Engineering.

Section B. Chemistry, Mineralogy, and Metallurgy.
Section C. Natural History and Geology.
Section D. The Microscope and its applications.
Section E. Geography and Ethnology.
Section F. Social Science and Statistics.
Section G. Literature and the Fine Arts, including Architecture.
Section H. Medical Science, including Physiology and Pathology.

Meetings of Sections.

LIV. The meetings of the Sections shall be for scientific objects only.

Members of Sections.

LV. There shall be no membership of the Sections as distinguished from the membership of the Society.

Officers of Sections.

LVI. There shall be for each Section a Chairman to preside at the meetings, and Secretary to keep minutes of the proceedings, who shall jointly prepare and forward to one of the Secretaries of the Society, prior to the 1st of November in each year, a report of the Proceedings of the Section during that year, and such report shall be submitted to the Council.

Mode of appointment of Officers of Section.

LVII. The Chairman and the Secretary of each Section shall be appointed at the first meeting of the Council after its election in March, in the first instance from Members of the Society who shall have signified to one of the Secretaries of the Society their willingness to undertake these offices, and subsequently from such as are recommended by the Section as fit and willing.

Times of meetings of Sections.

LVIII. The first meeting of each Section in the year shall be fixed by the Council; subsequently the Section shall arrange its own days and hours of meeting, provided these be at fixed intervals.

Corresponding Members.

LIX. The Council shall have power to propose gentlemen not resident in Victoria, for election in the same manner as ordinary Members, as Corresponding Members of the Society. The Corresponding Members shall contribute to the Society papers which may be received as those of ordinary Members, and shall in return be entitled to receive copies of the Society's publications.

Privileges of Associates.

LX. Associates shall have the privileges of Members in respect to the Society's publications, in joining the Sections, and at the Ordinary Meetings, with the exception that they shall not have the power of voting for the election of Officers; they shall also not be eligible as Officers of the Society.

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OF

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WILLIAMS & NORGATE, 14 HENRIETTA STREET, COVENT GARDEN, LONDON.
To whom all communications for transmission to the Royal Society of Victoria
from all parts of Europe should be sent.

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TRANSACTIONS

AND

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Royal Society of Victoria.

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THE AUTHORS OF THE SEVERAL PAPERS ARE SOLELY RESPONSIBLE FOR THE SOUNDNESS OF THE OPINIONS GIVEN AND FOR THE ACCURACY OF THE STATEMENTS MADE THEREIN.

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OF THE

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